Generalising the Astrometric Uncertainty Function in the Era of the Rubin Observatory's LSST

Tom J Wilson (he/him) and Tim Naylor
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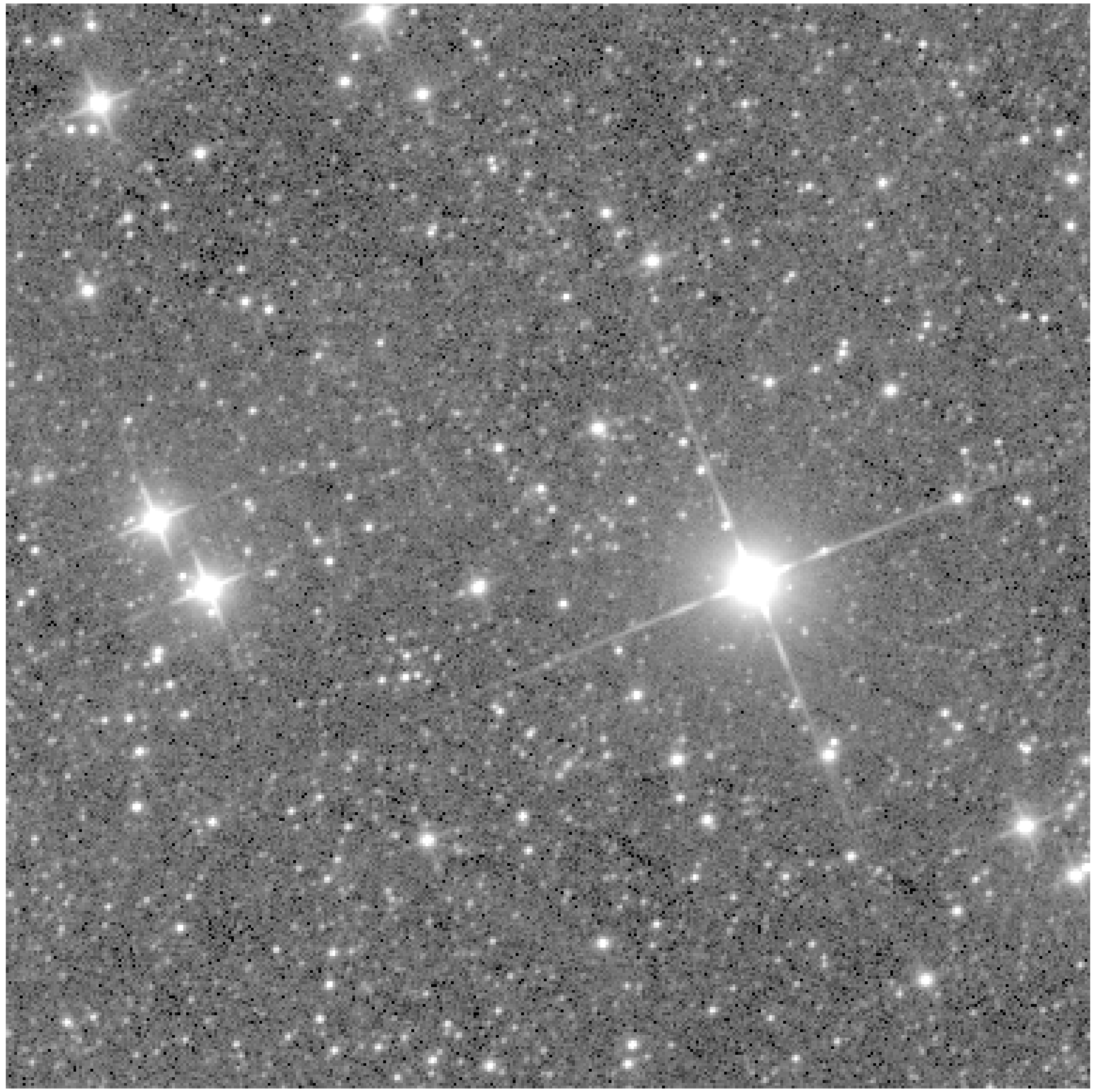
University of Exeter

Photometric Observations



WISE - Wright et al. (2010)

WISE W1

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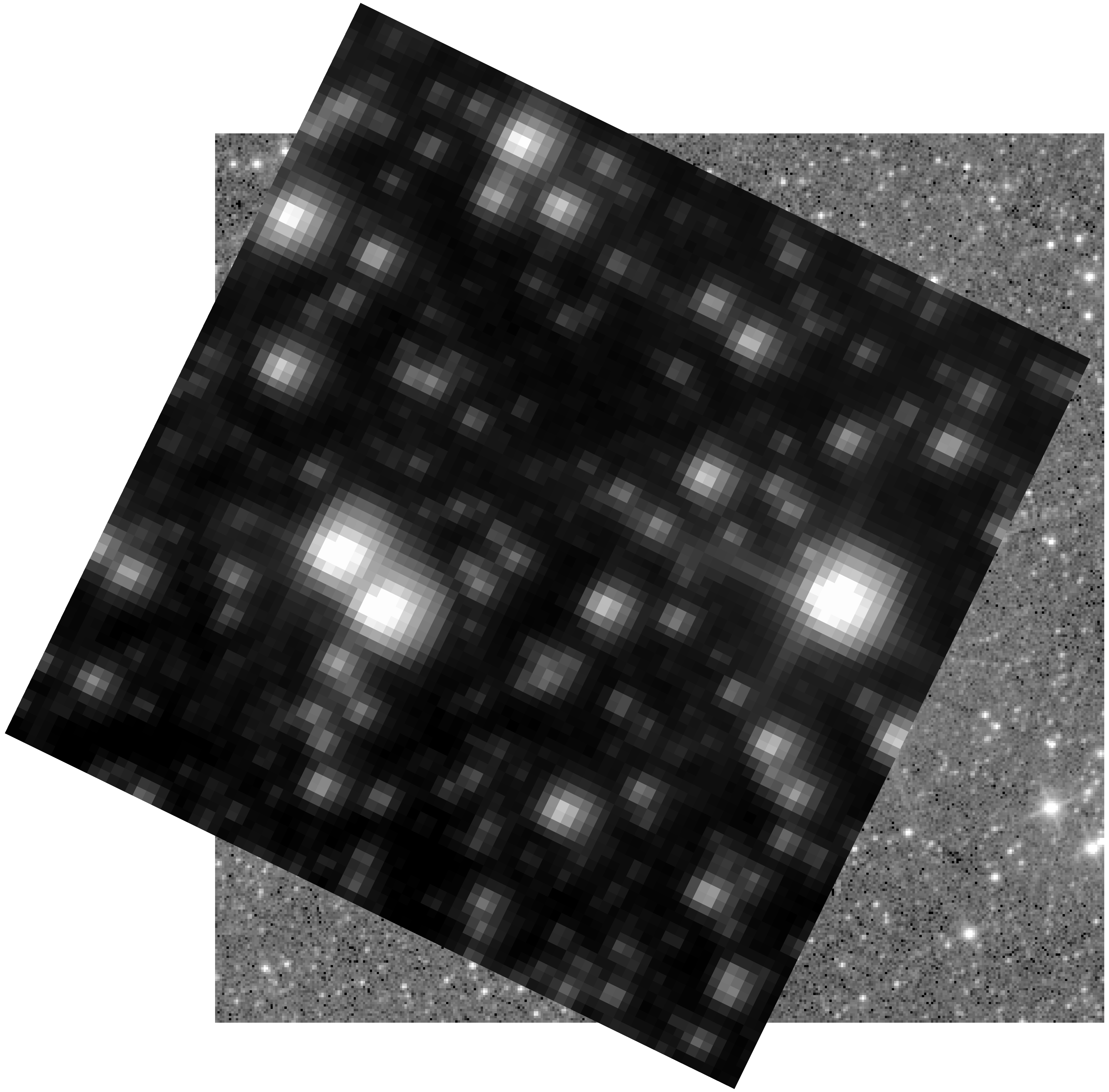

WISE - Wright et al. (2010)
TESS - Ricker et al. (2015)

TESS T

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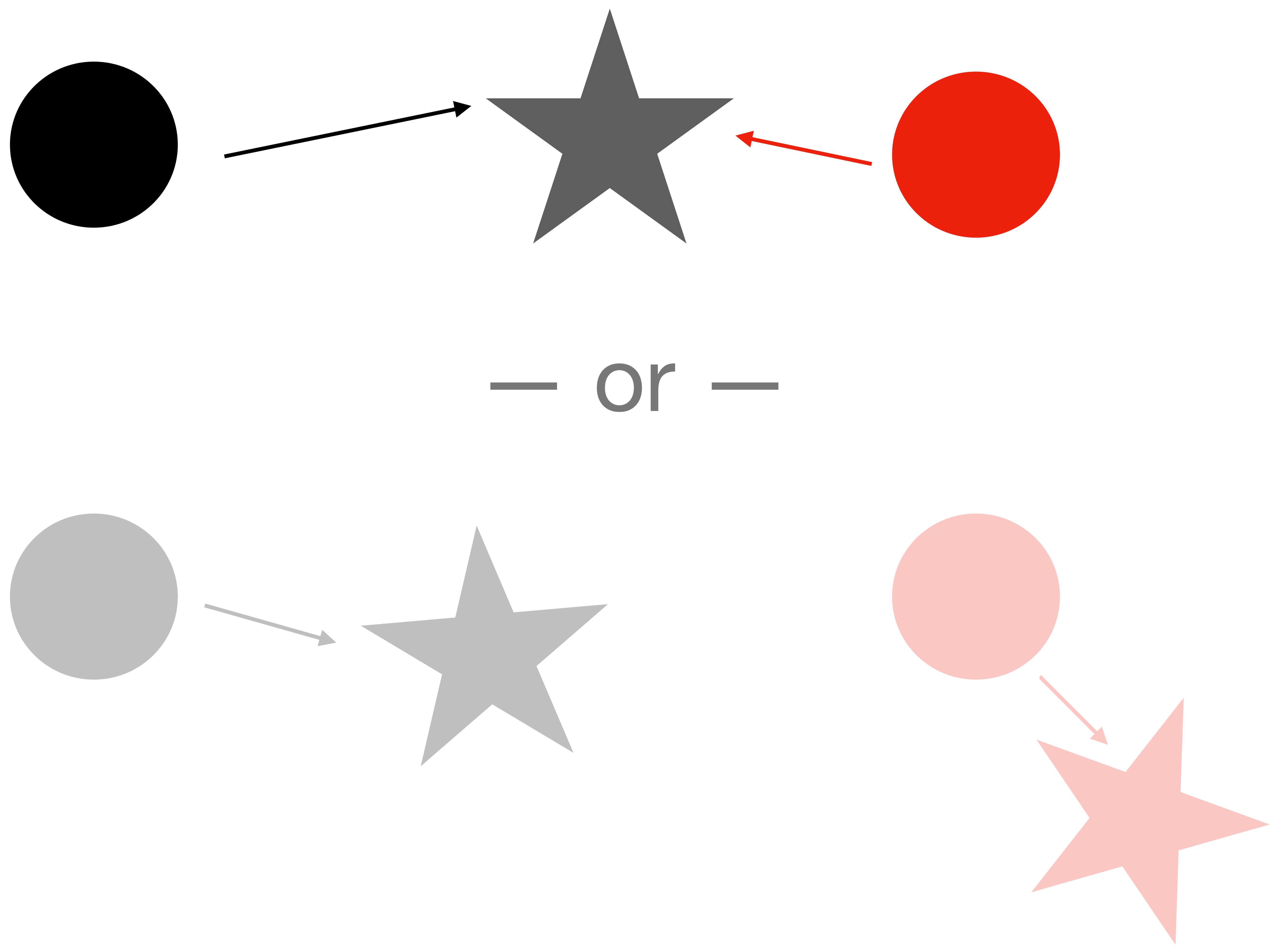
Counterpart Assignment



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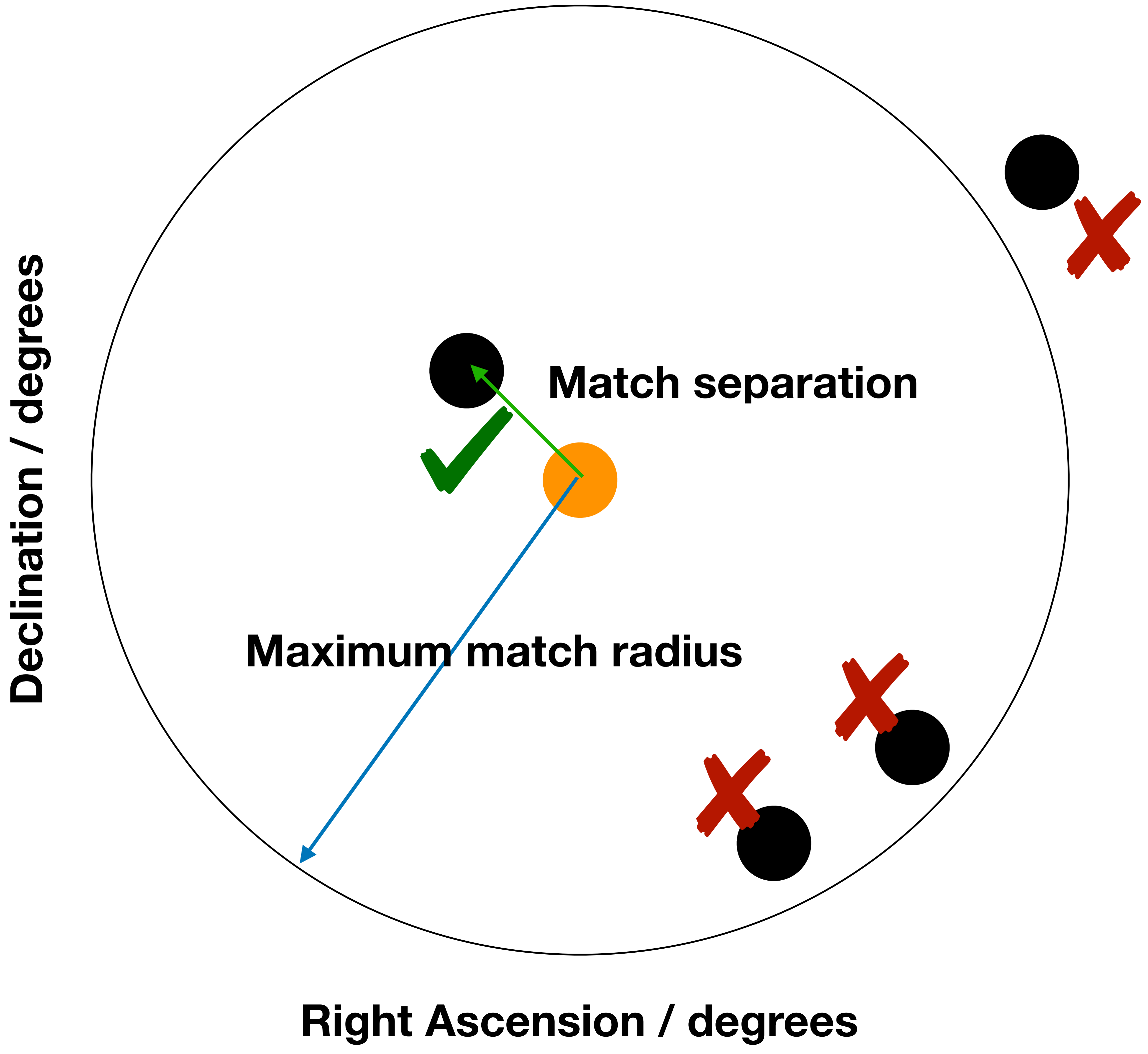
Catalogue Cross-Matching

How to assign the most likely counterparts — or decide if two sources are counterparts or not?

Nearest neighbour/ proximity matching



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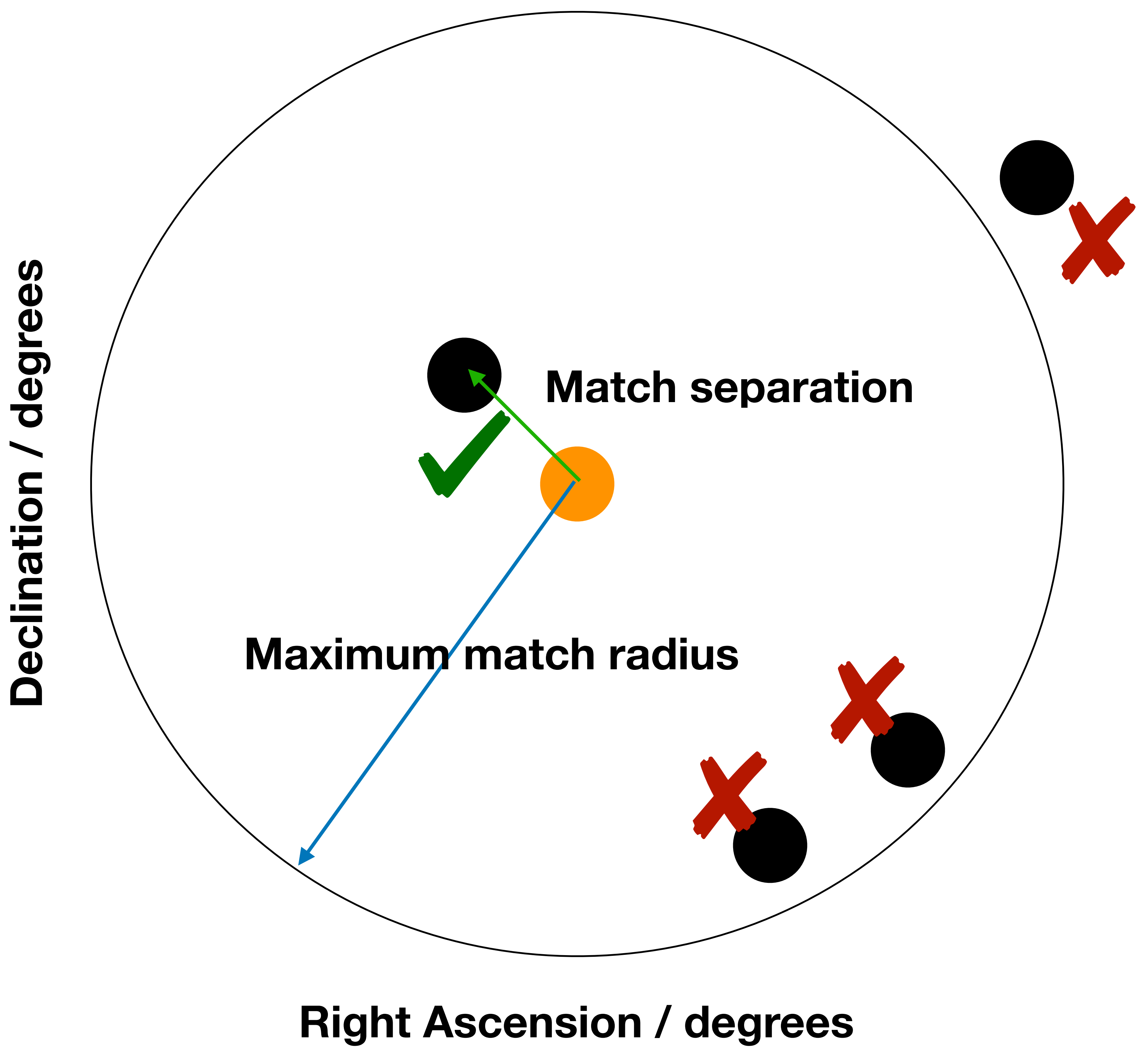

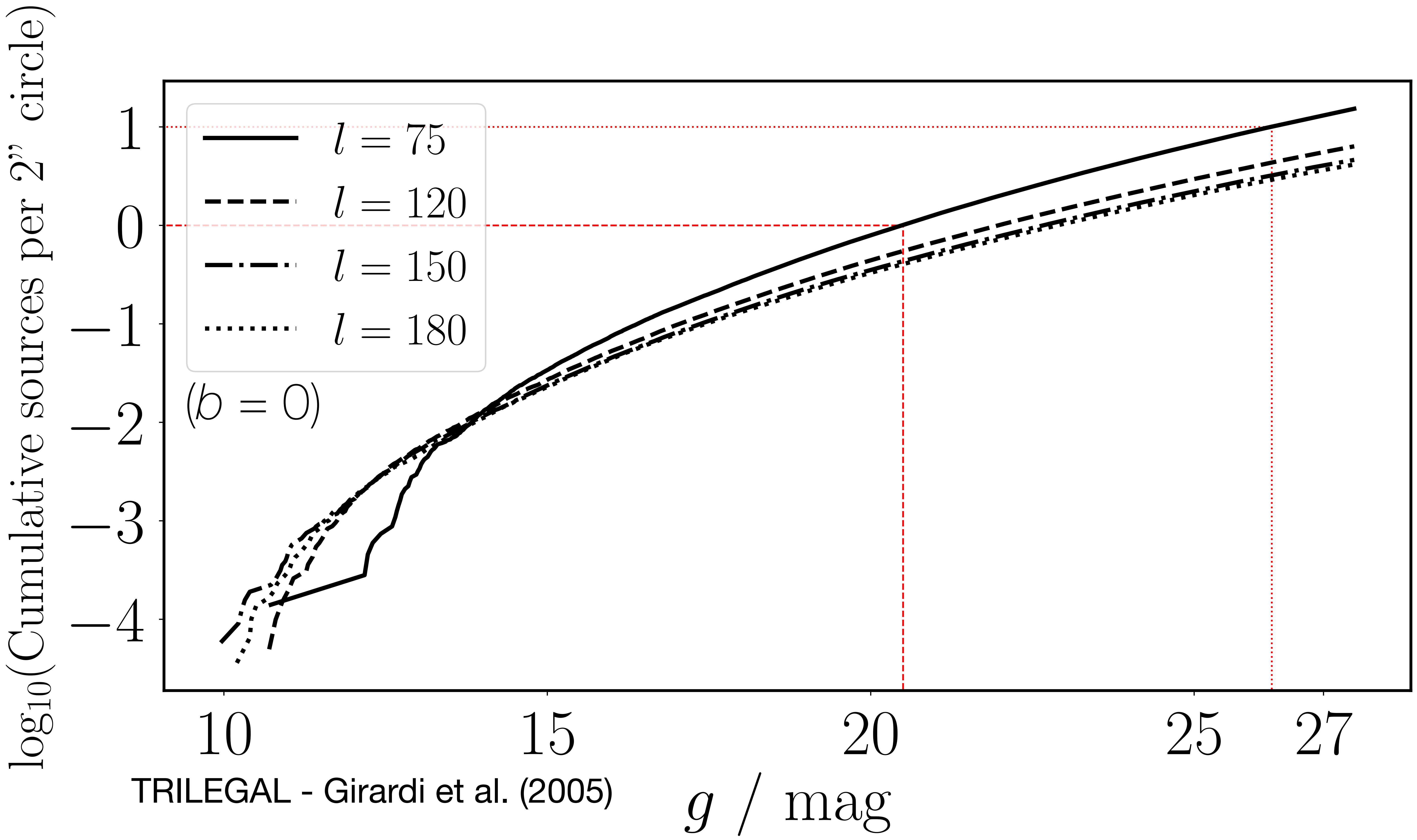

TRILEGAL - Girardi et al. (2005)

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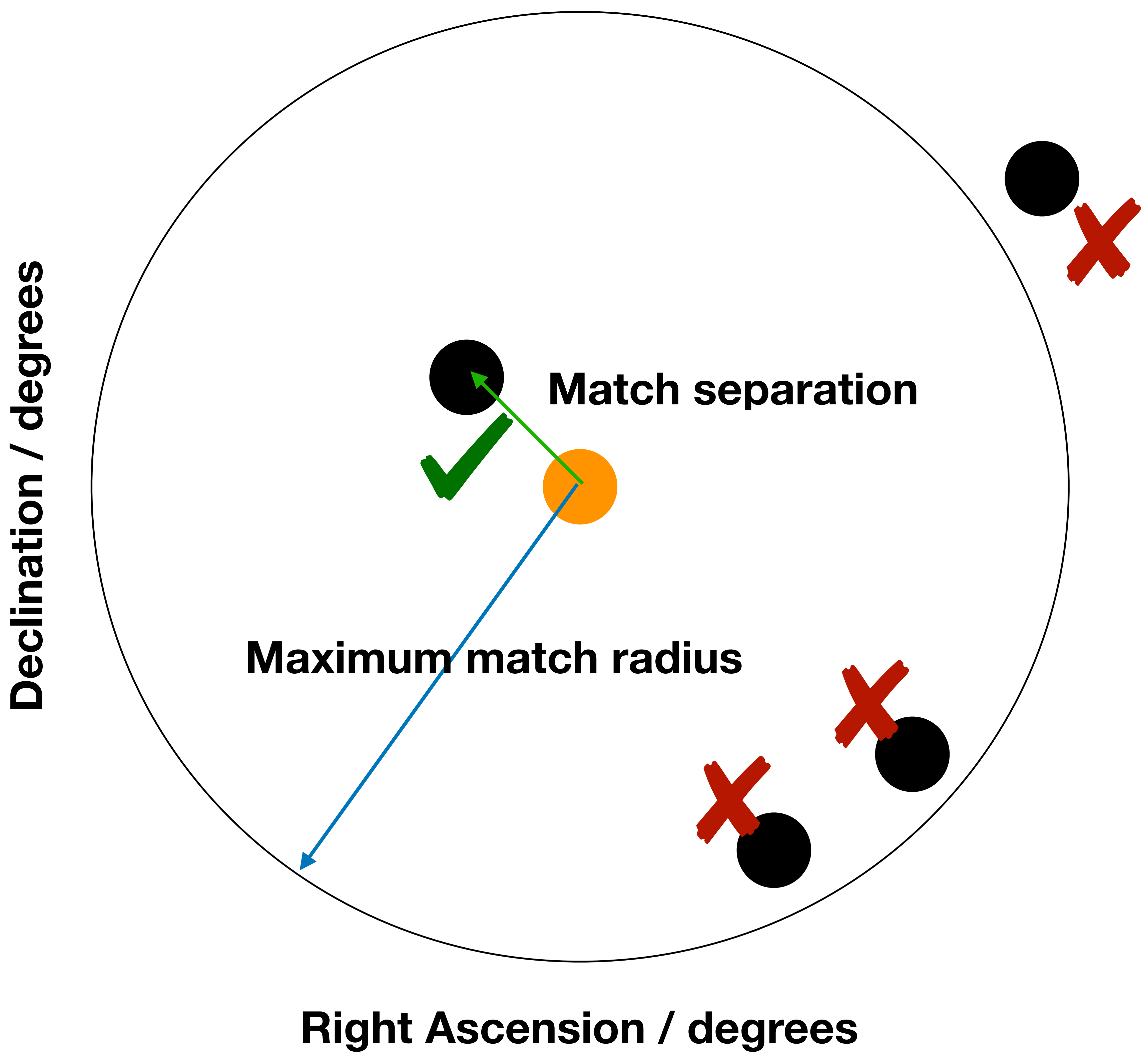

Probabilistic matching

$$dp_{\mathrm{id}} = Qr \exp\left(\frac{-r^2}{2}\right) dr.$$

Wolstencroft et al. (1986)

$$p(D|H) = \int p(\boldsymbol{m}|H) \prod_{i=1}^{n} p_i(\boldsymbol{x}_i|\boldsymbol{m}, H)\, d^3 m$$

Budavári & Szalay (2008)

$$dp(r|c) = 2\lambda r \times e^{-\lambda r^2} dr$$

de Ruiter, Willis, & Arp (1977)

$$R_j = \frac{L_j}{\sum_i L_i + (1 - Q)} \qquad L = \frac{q(m, c)\, f(x, y)}{n(m, c)}$$

Sutherland & Saunders (1992)

$$P(i) = \frac{\frac{X c(m_i)\, g(\Delta x_i, \Delta y_i)}{N f(m_i)}}{1 - X + \sum_j \frac{X c(m_j)\, g(\Delta x_j, \Delta y_j)}{N f(m_j)}}$$

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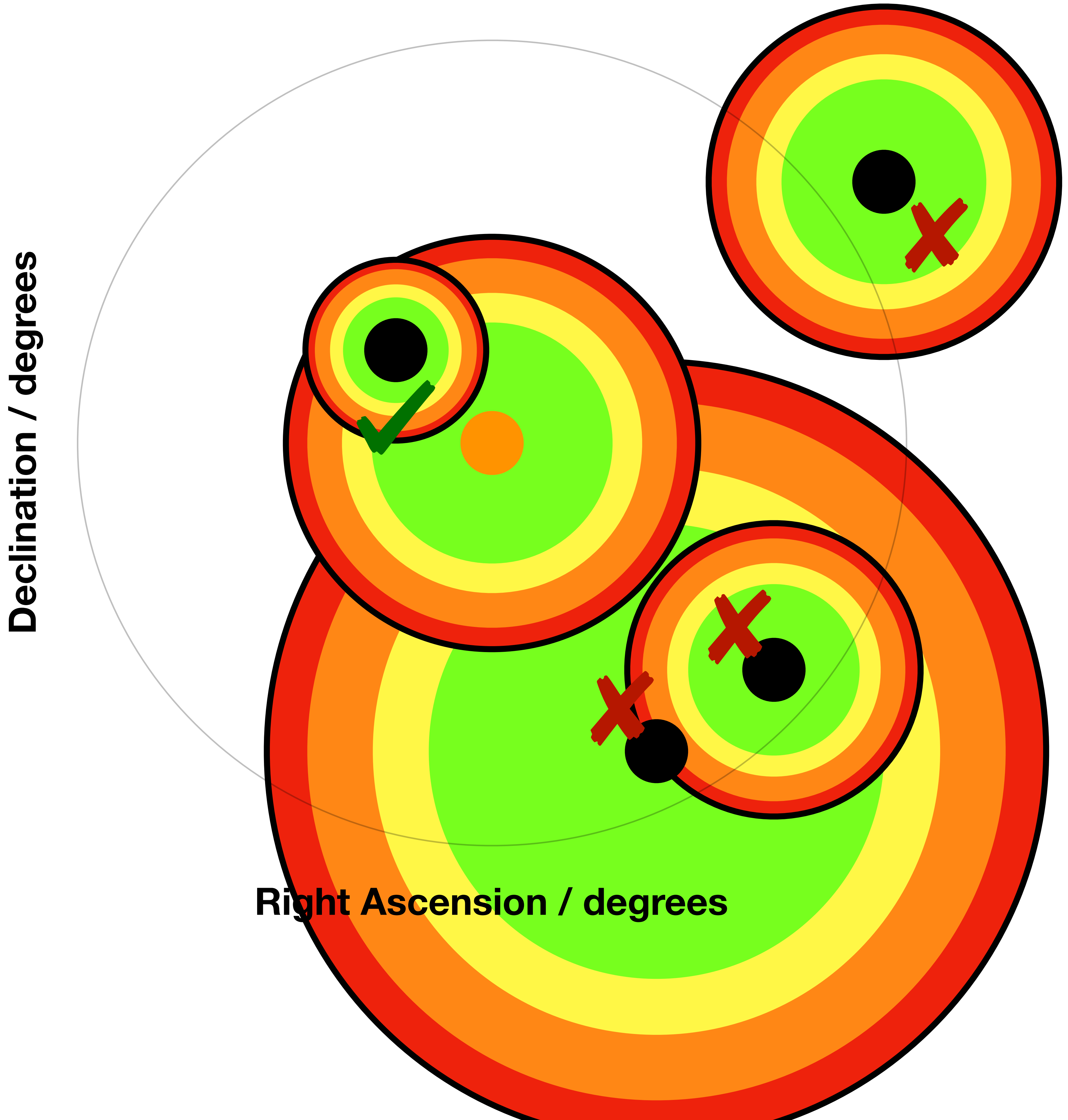

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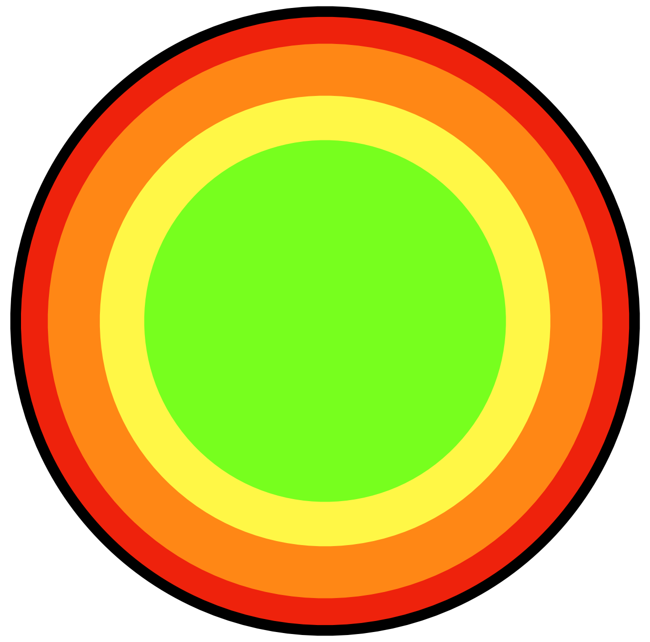
One assumption made in all of these works: positional errors of sources are Gaussian!



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Separation Likelihood Function

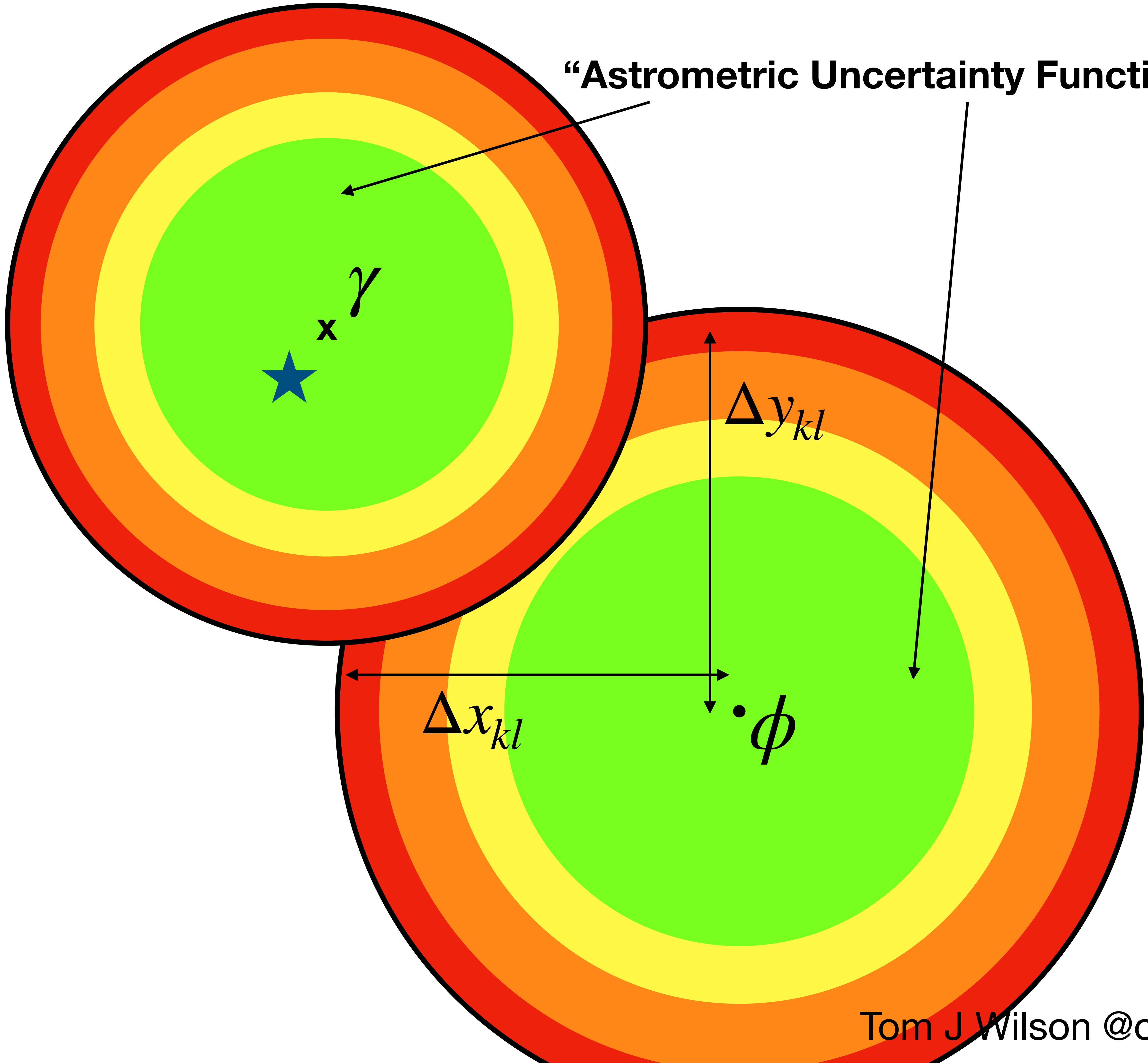
$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0)\,\mathrm{d}x_0\,\mathrm{d}y_0$$

$$= N_\mathrm{c} \times (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl})$$

Wilson & Naylor (2018a)

$$g(\Delta x, \Delta y, \sigma) \propto (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2}\frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right)$$ where $\sigma^2 = \sigma_1^2 + \sigma_2^2$

Convolution



Wilson & Naylor (2018b)

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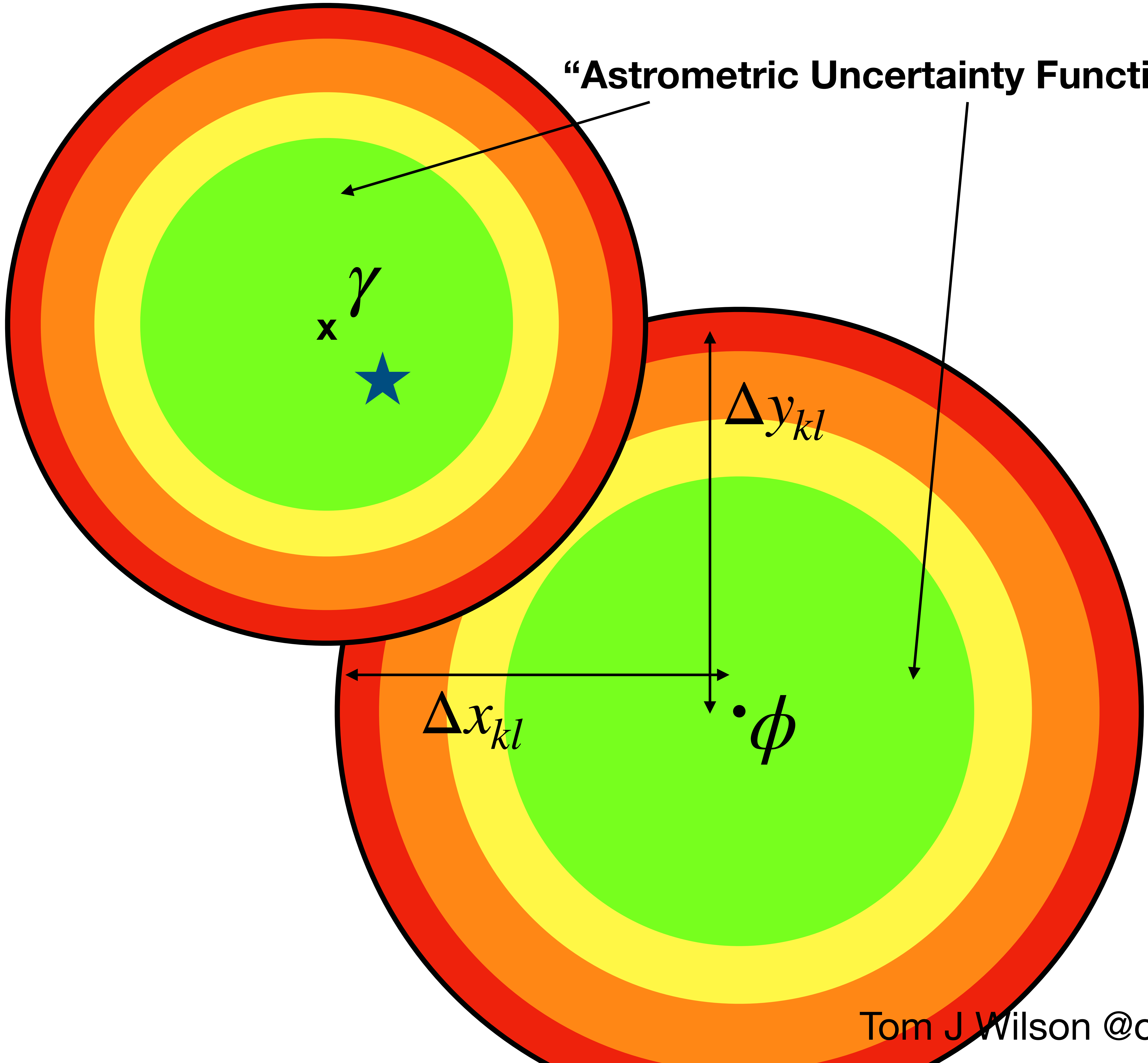
Convolution

"Astrometric Uncertainty Functions"



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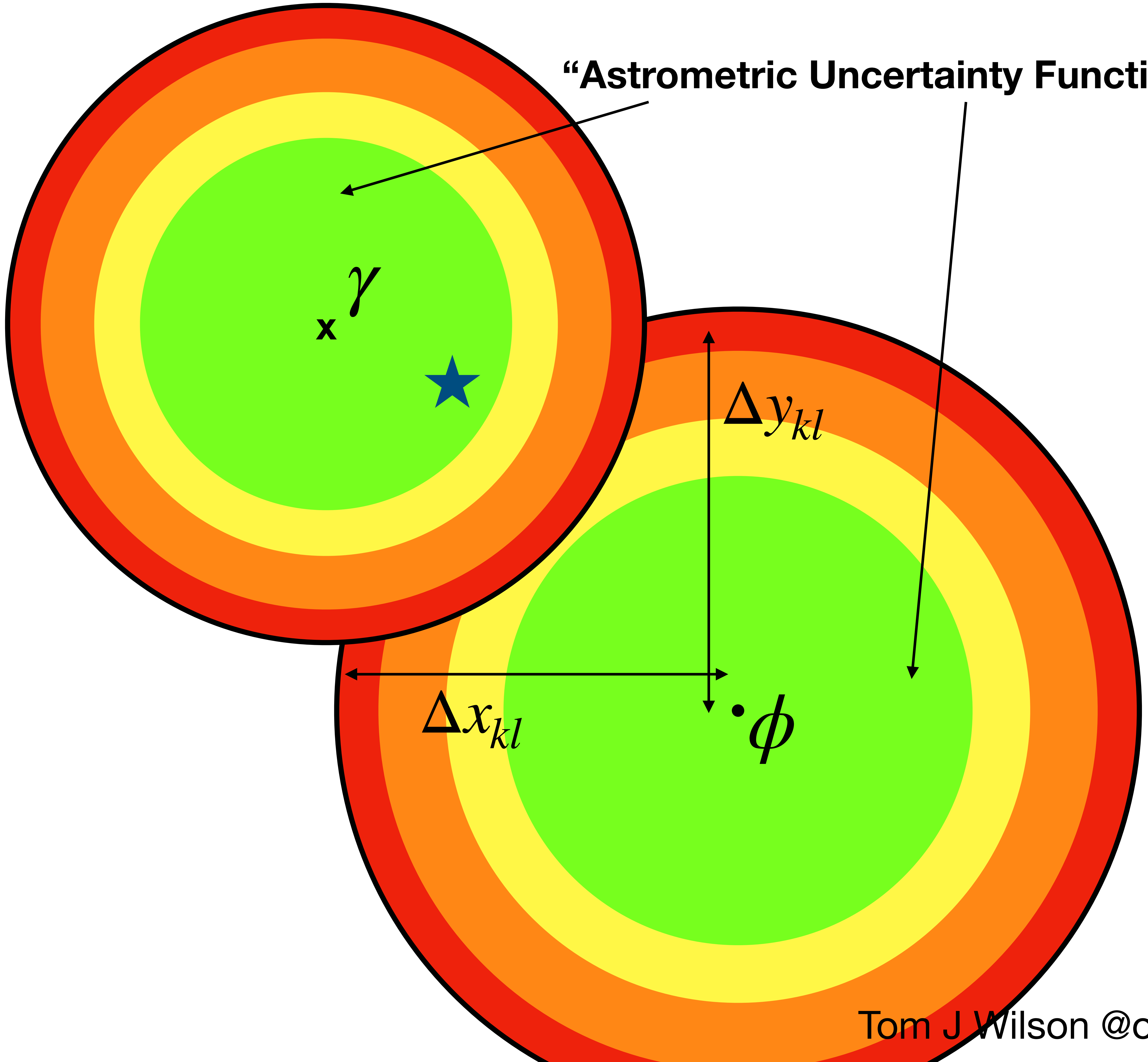

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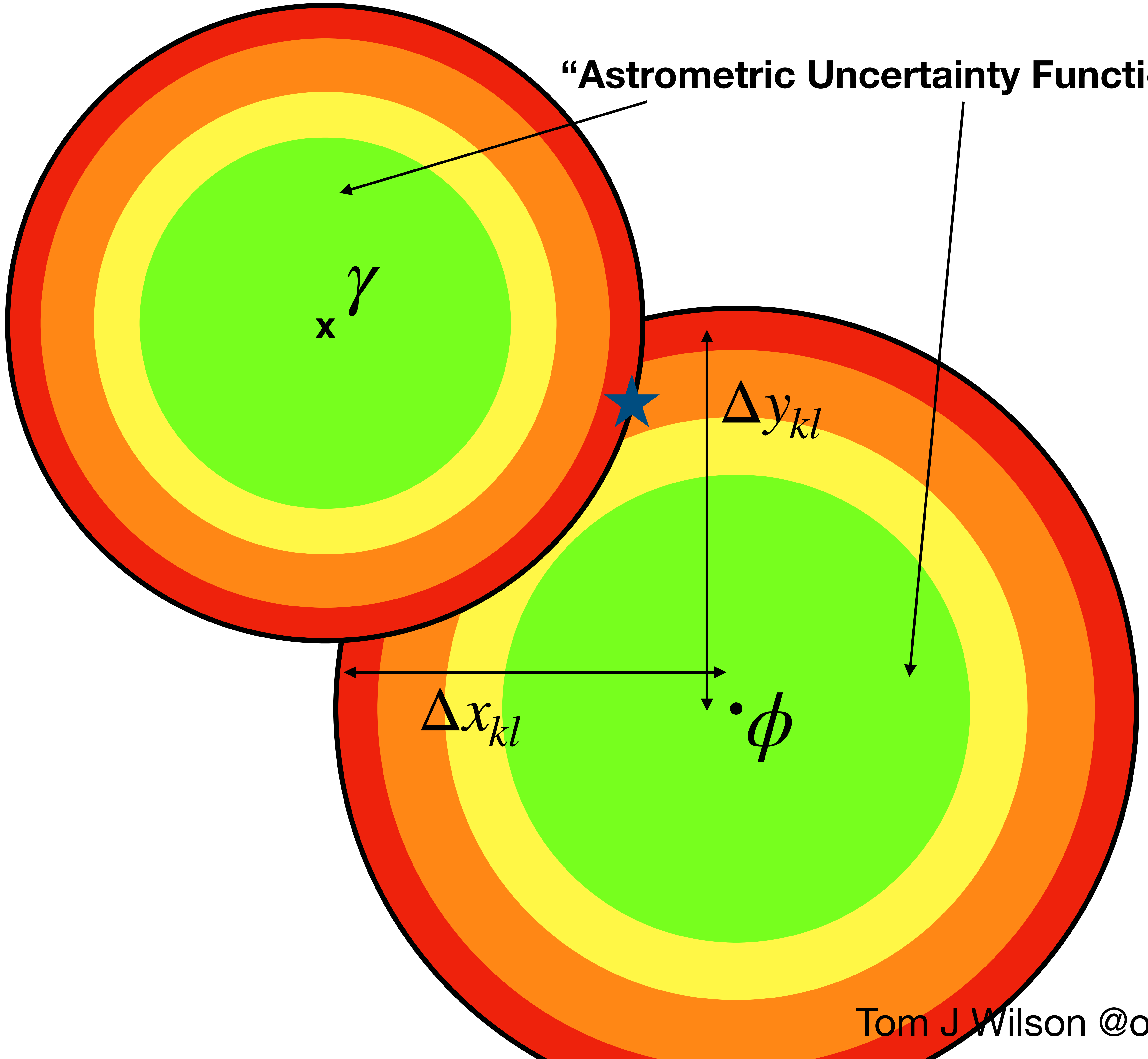

Wilson & Naylor (2018b)

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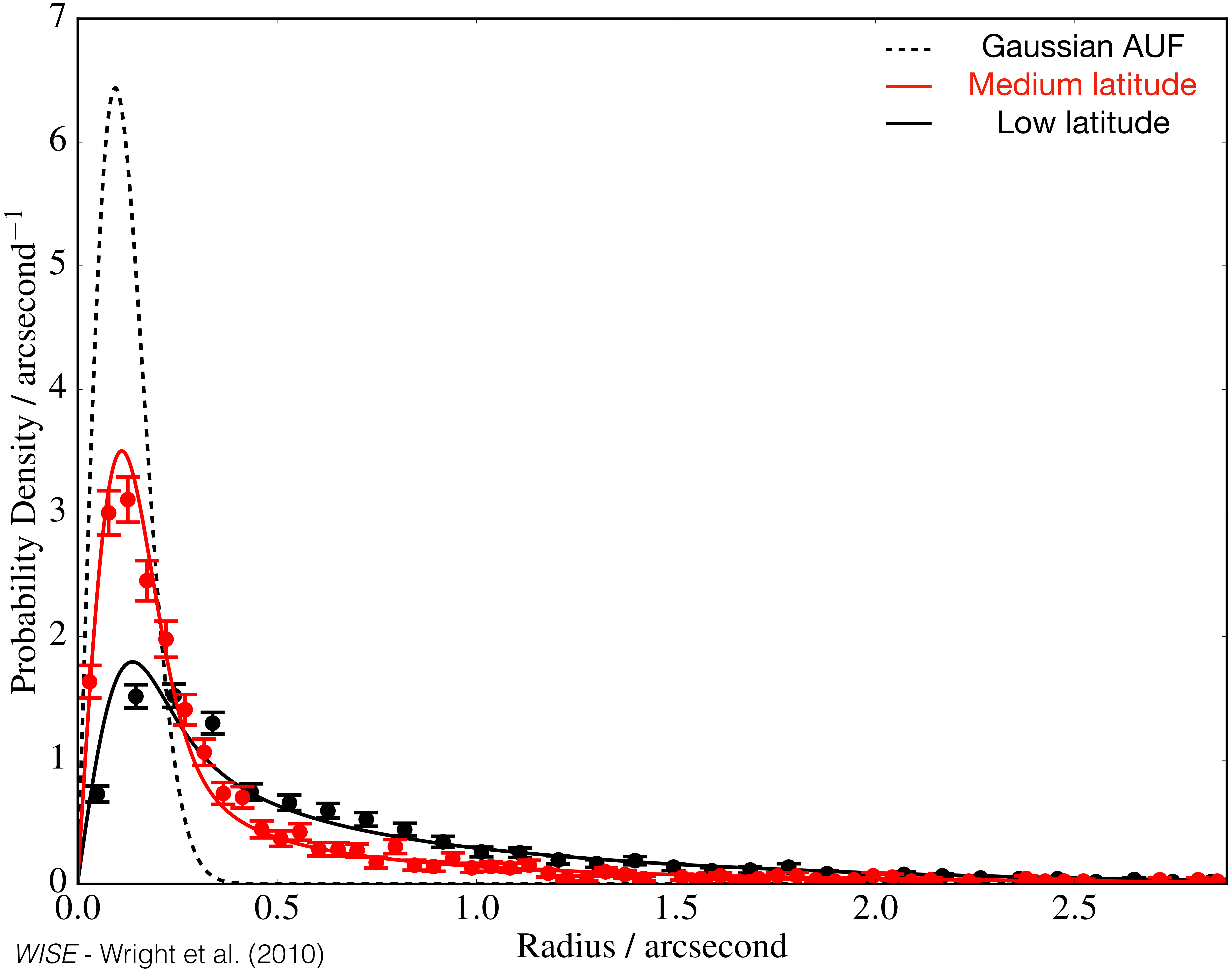
Additional Components of the AUF



WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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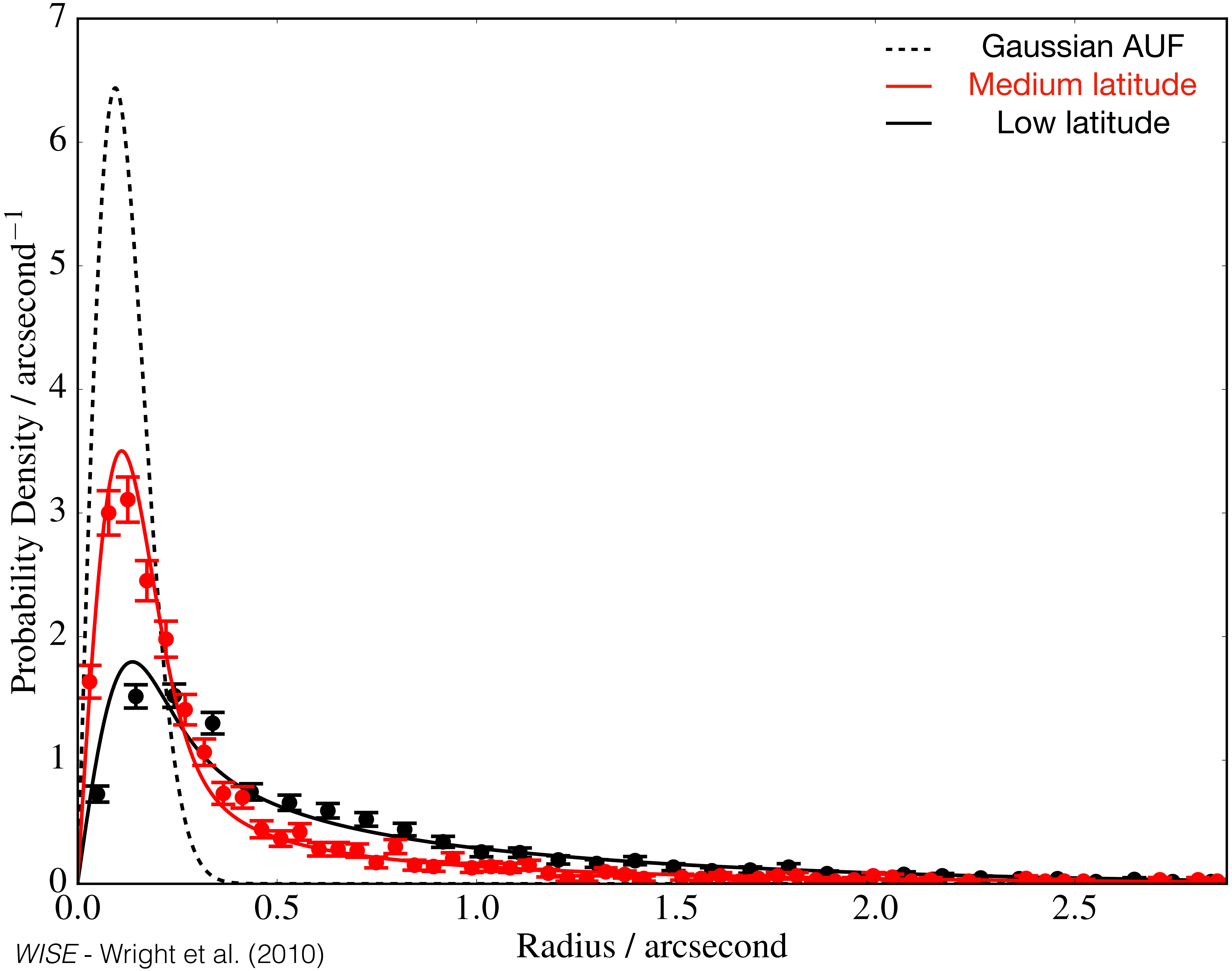

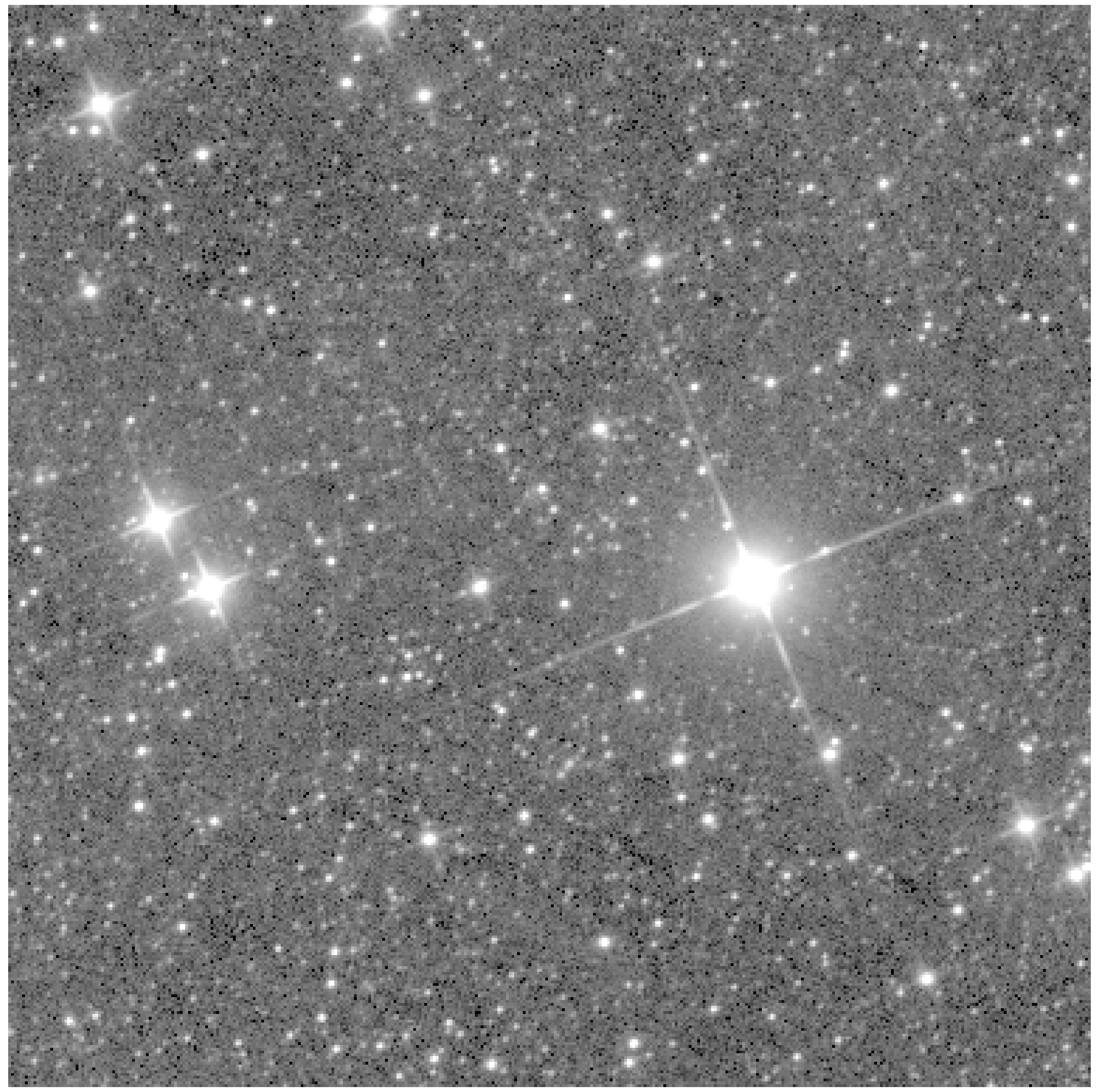

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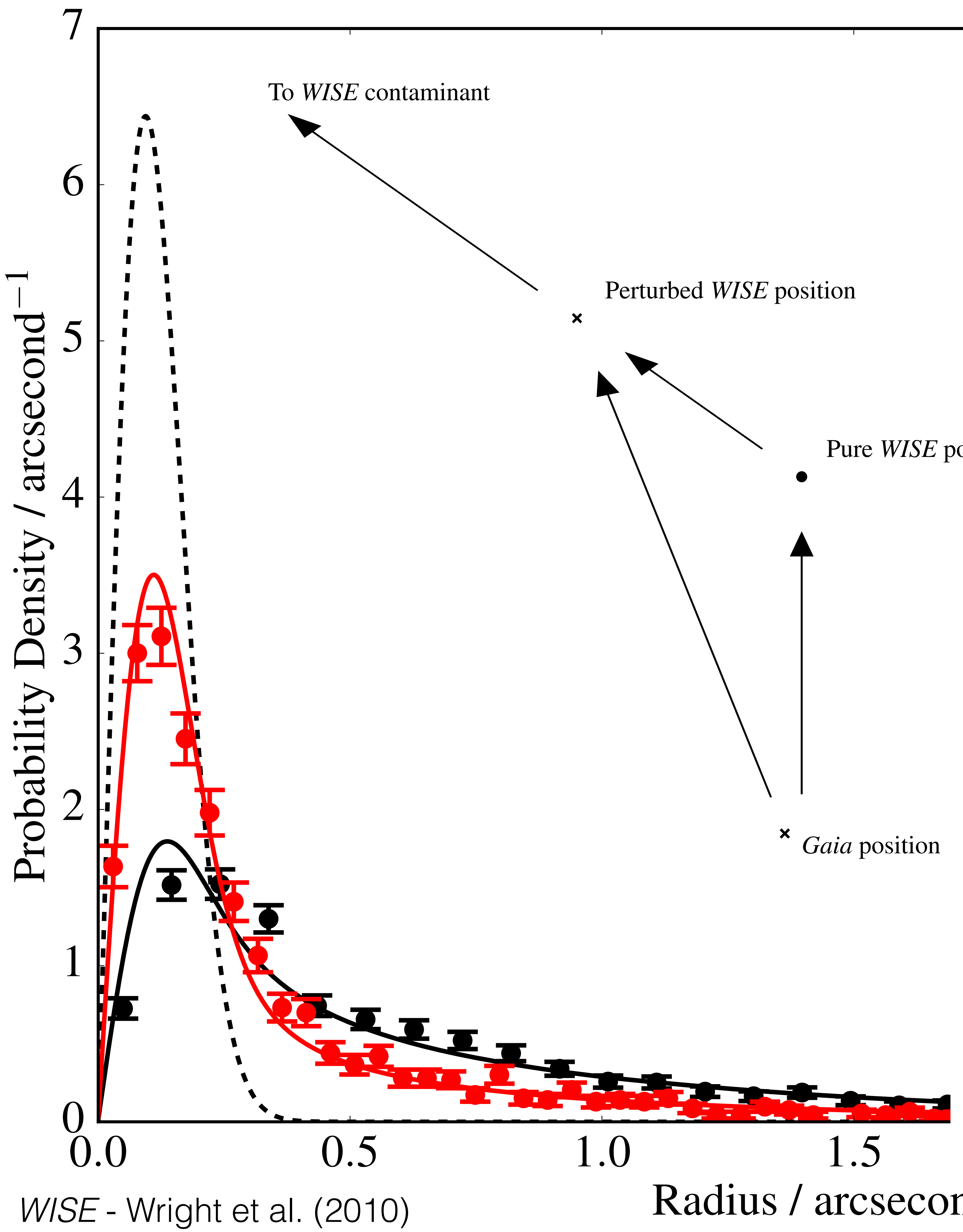

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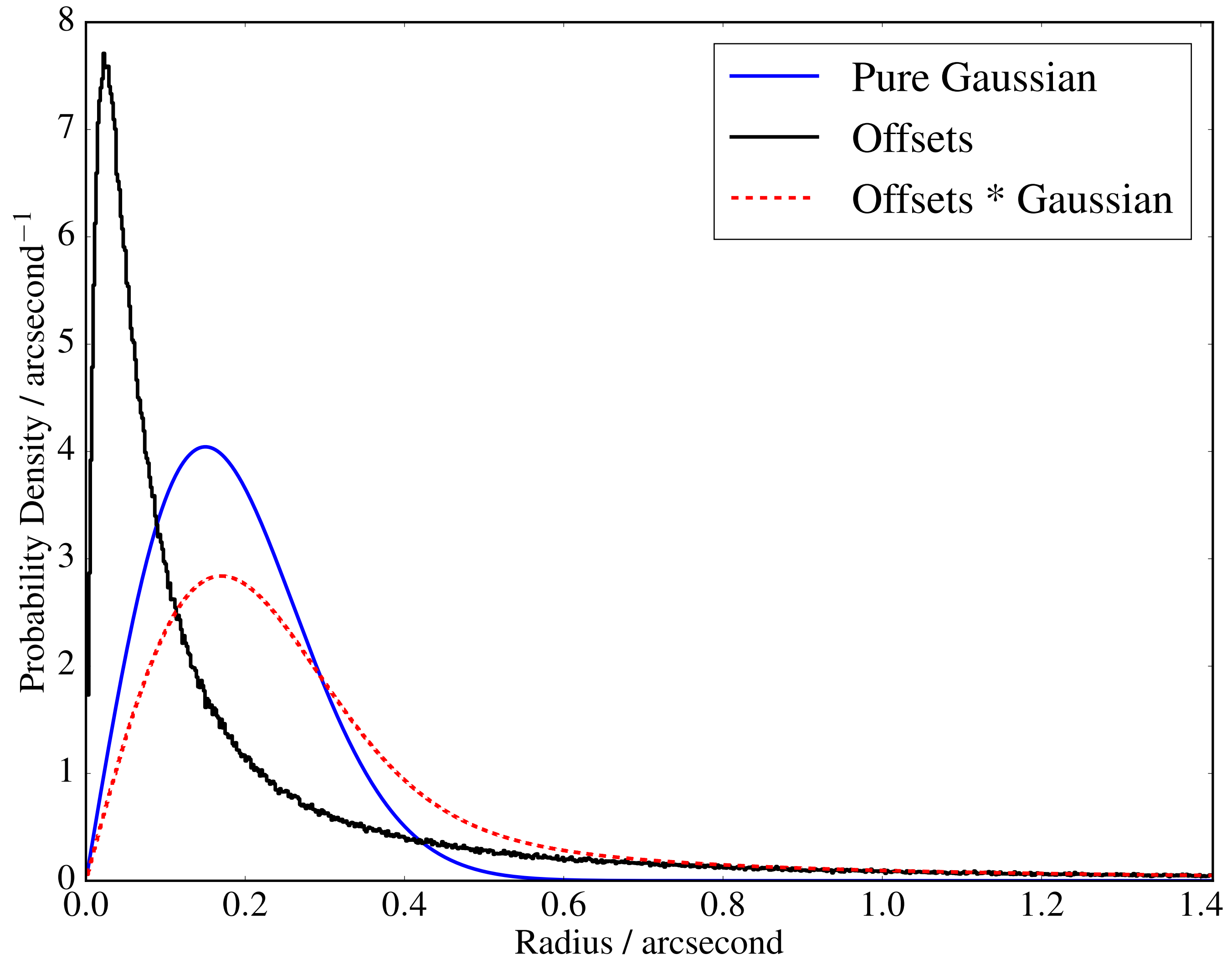
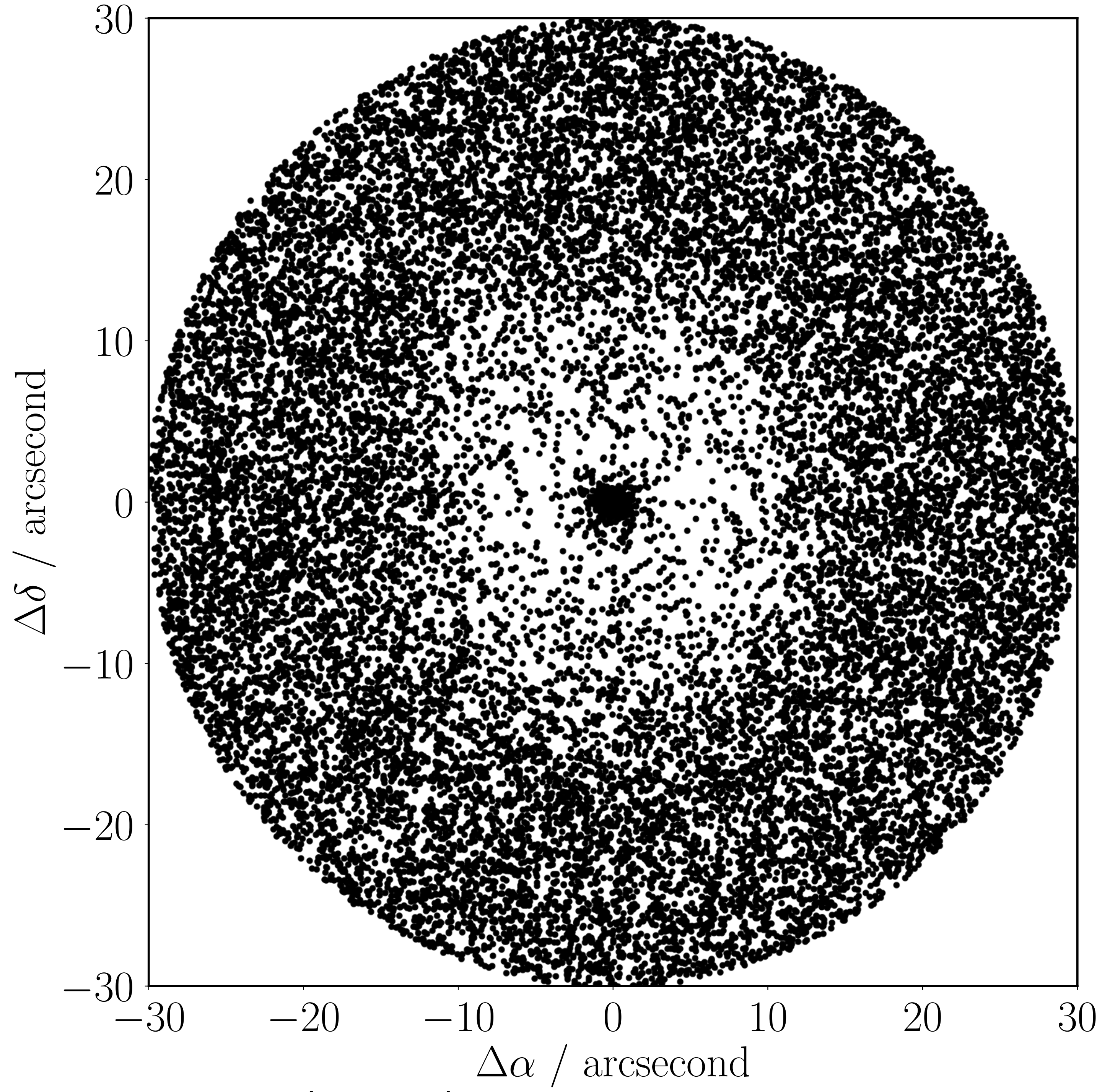


Wilson & Naylor (2018b)



Wilson & Naylor (2017)

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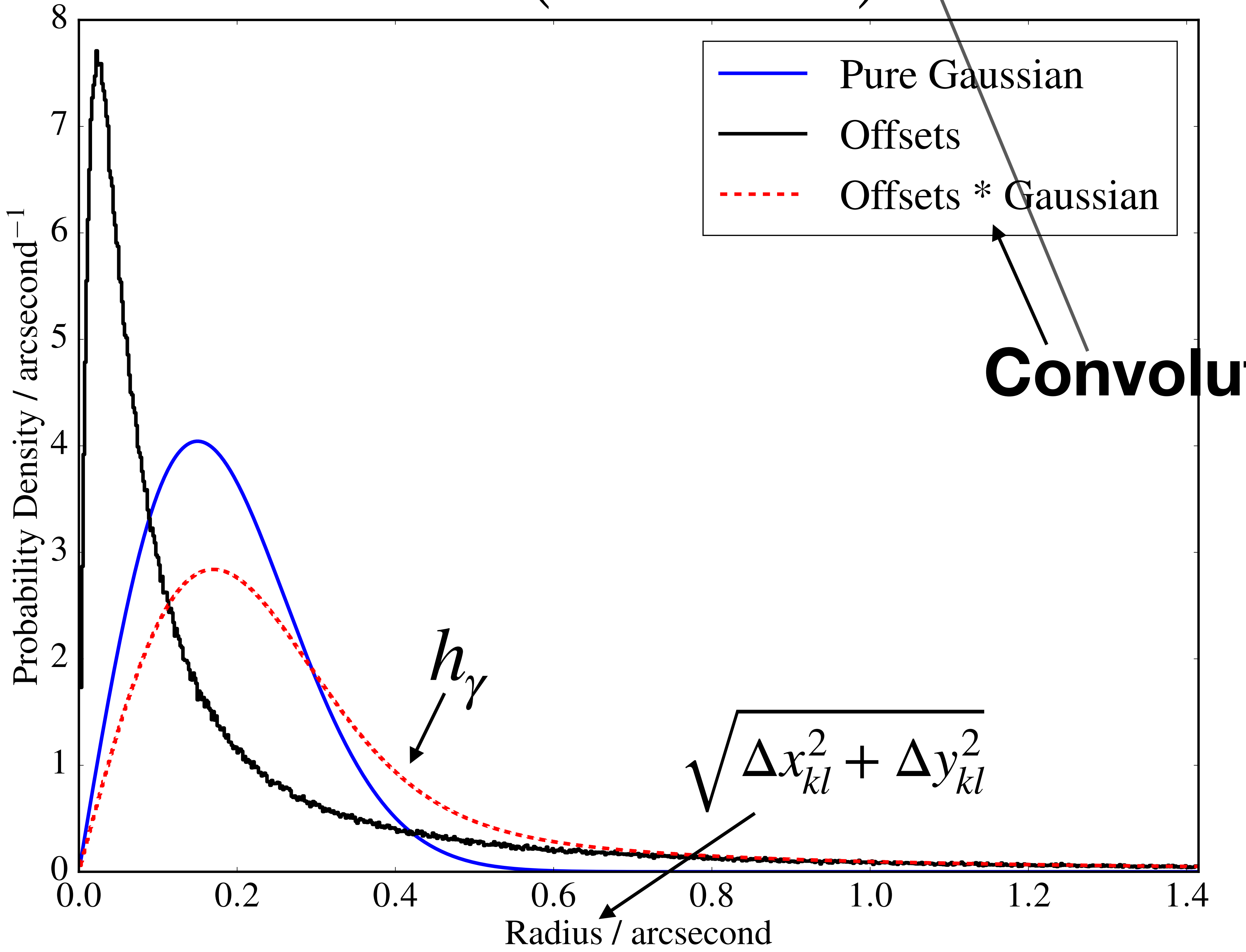

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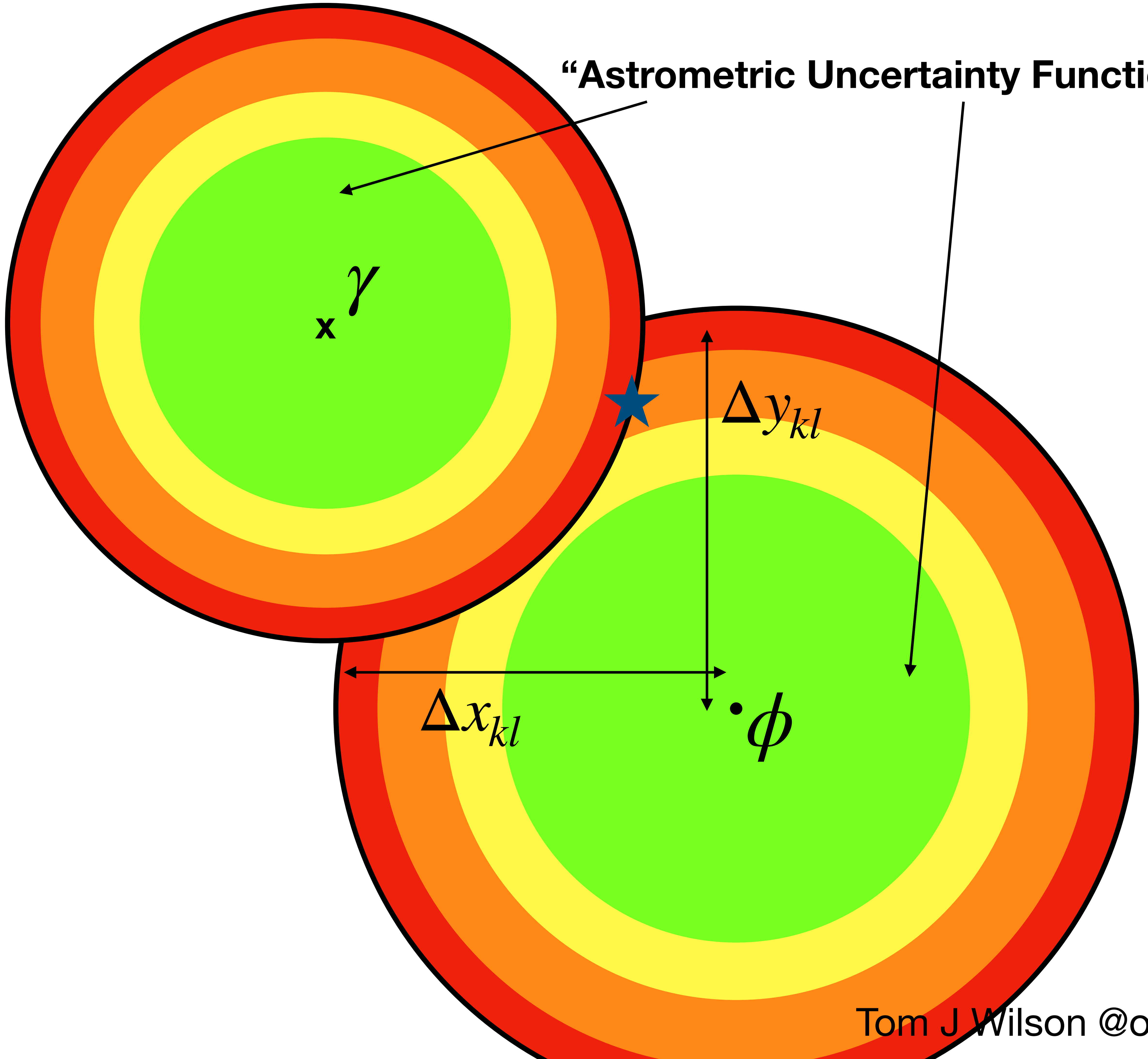

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Calculating the Astrometric Uncertainty Function

The issue: new components of the AUF are not analytic, and numerical convolutions are computationally expensive

$$\iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0$$

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The solution: do convolution in Fourier space

$$F(\rho) = \mathscr{F}(f(x))$$

$$(f * g)(x) = \mathscr{F}^{-1}(F \cdot G)$$

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The issue: Fourier transforms in two dimensions are still computationally difficult

$$\mathscr{F}(g(u,v)) = \iint_{-\infty}^{+\infty} g(x,y)\exp\left[-j2\pi(ux+vy)\right]\,\mathrm{d}x\,\mathrm{d}y$$

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The solution: assume circular symmetry and reduce to one-dimensional Hankel (Fourier-Bessel) transform

$$G(\rho, \phi) = G(\rho) = 2\pi \int_0^\infty r\, g(r)\, J_0(2\pi r \rho)\, \mathrm{d}r$$

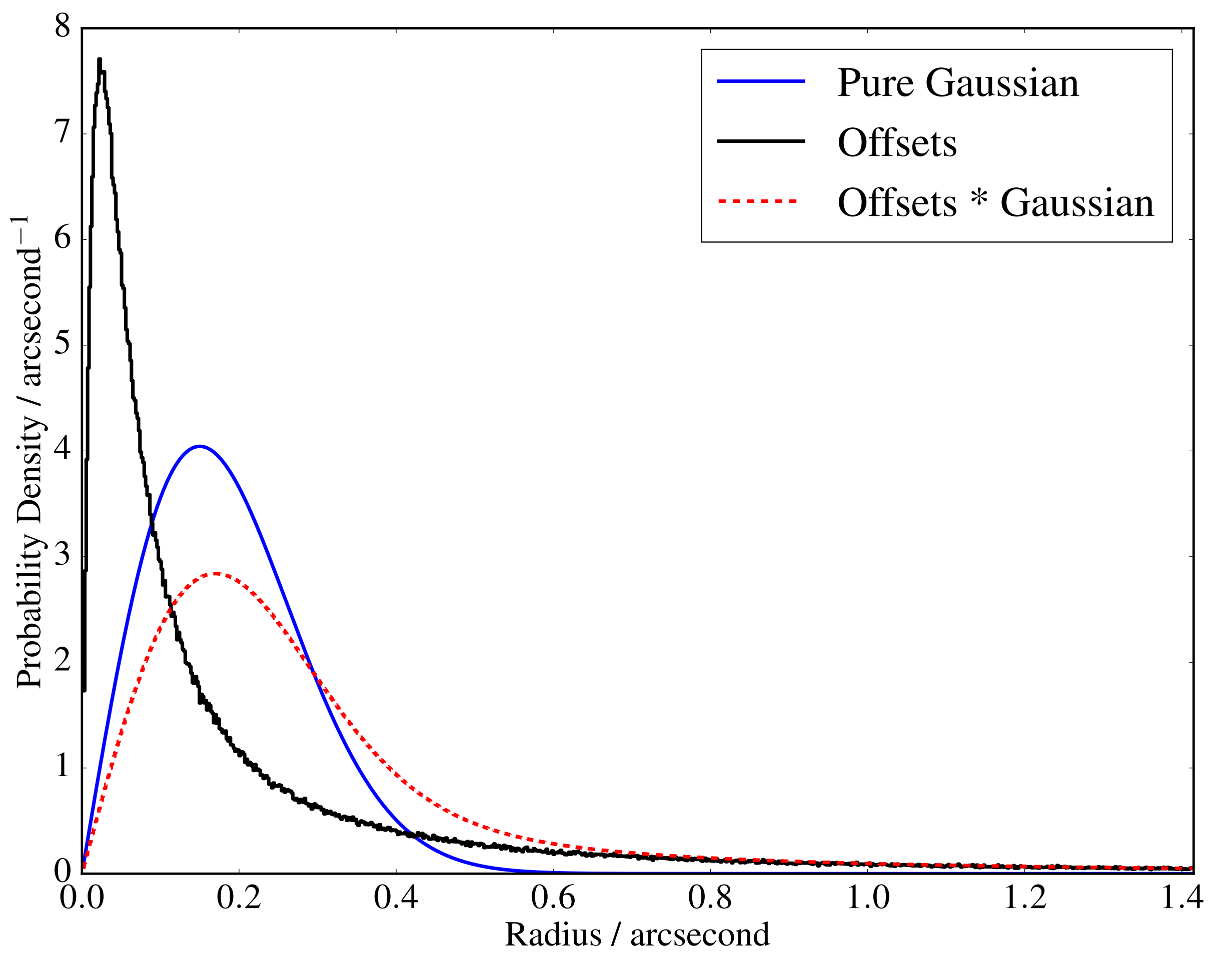
Introduction to Fourier Optics - J.W. Goodman

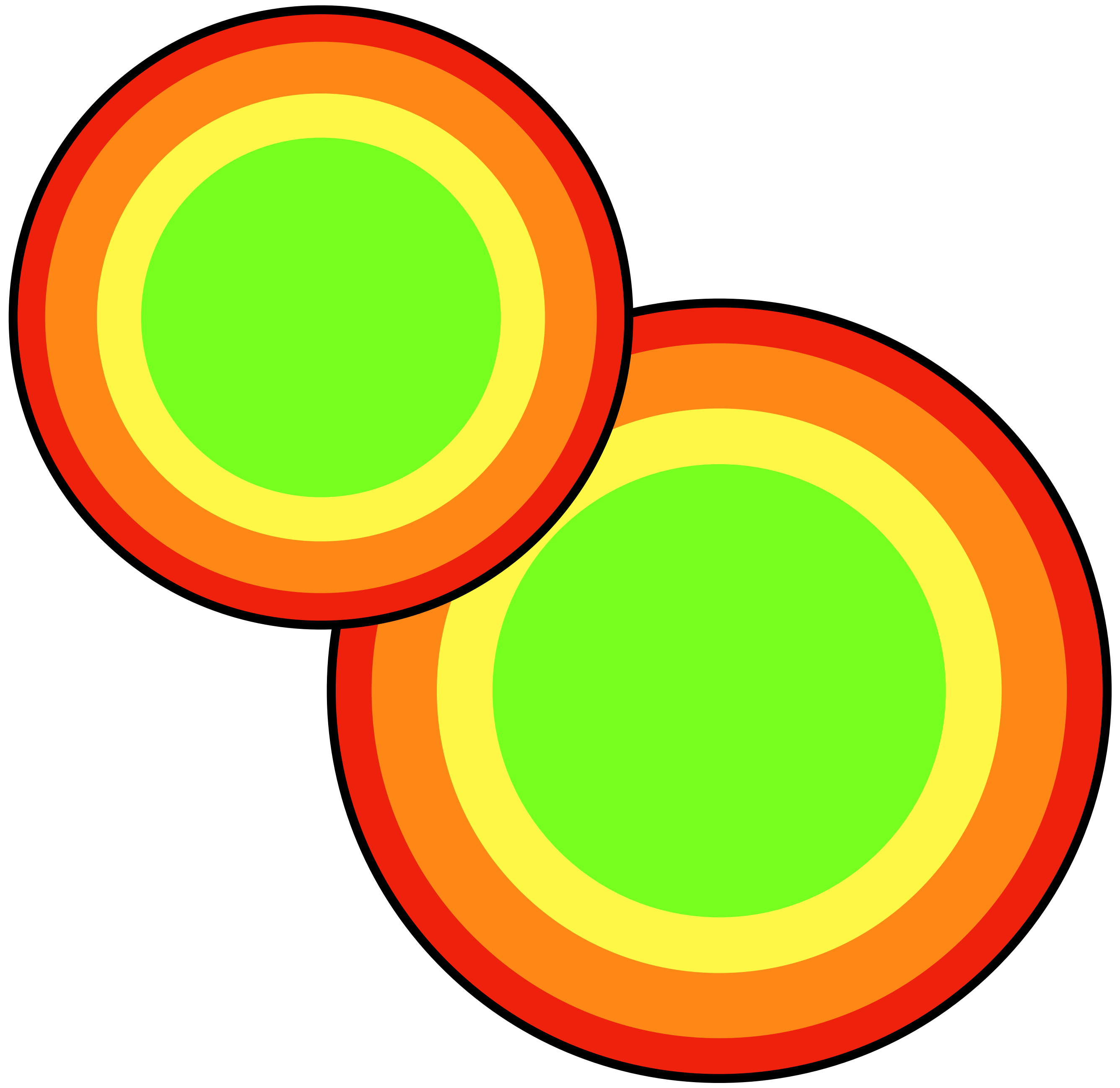
Tom J Wilson @onoddil

Counterpart Likelihoods In Practice

1) Gaussian, centroiding component of AUF has analytic Fourier-space expression
2) Simulate many PSFs, derive sample of perturbations due to blended sources
3) Fourier transform distribution of perturbations
4) Convolution becomes 1-D multiplication and 1-D inverse Fourier transform

(Replace step 2 with your chosen extra sources of reasons why a source isn't measured at its "true" location!)





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Calculating the Astrometric Uncertainty Function

Advantages: ✓

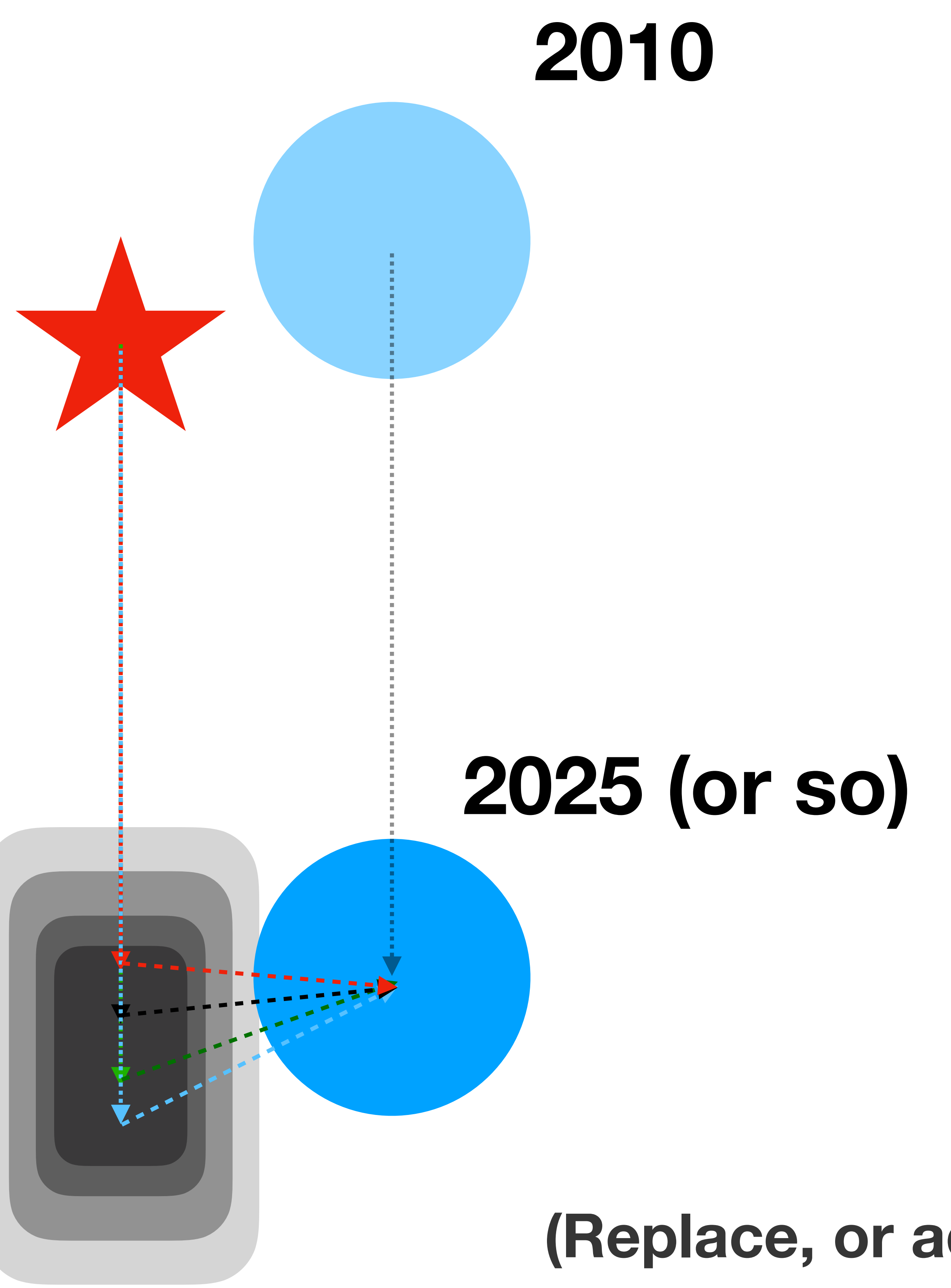
- **Allows for the generalisation of the AUF**
- Non-centroid AUF components crucial for crowded or faint fields
- Hankel Transform speeds up 2-D calculation

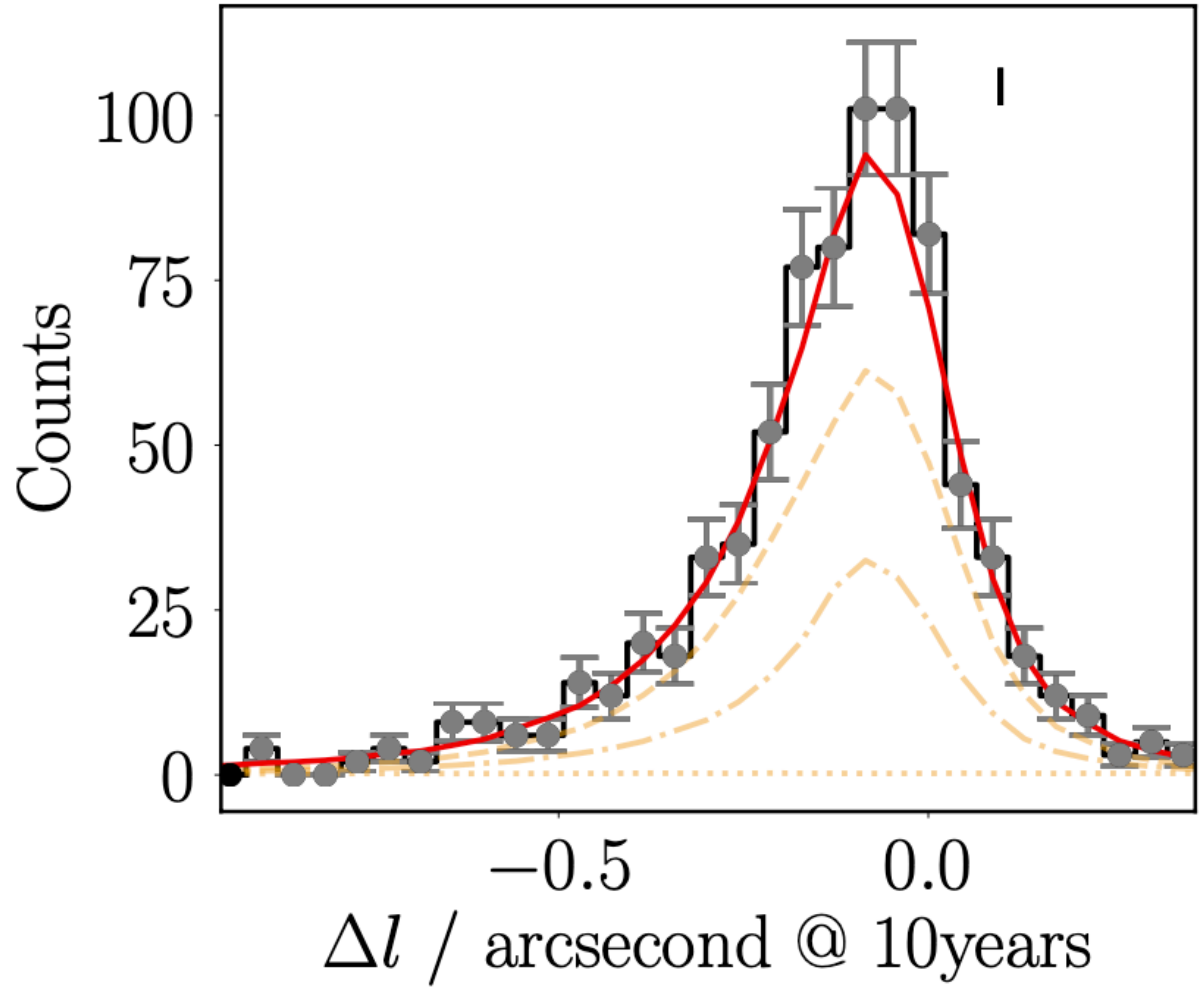
Disadvantages: ✗

- Numerical precision needs sufficient integral resolution
- Requires all components of the AUF to be circularly symmetric

Including Unknown Proper Motions

Using a model for the distribution of potential proper motions, and hence astrometric drifts, of a source of a given sky position and brightness we can include "fast forwarding" of sources through time across different catalogues when individual proper motions are not known





Because this function works in *separation*, rather than pure *position*, space, we apply the distribution after the convolution to calculate G, but the same basic numerical framework applies.

$$G' = G * h'_{\mathrm{pm}} \qquad G = h_\gamma * h_\phi$$

(Replace, or add, your deviation from "true" position here!)

$$h_\gamma = h_{\gamma,\mathrm{centroiding}} * h_{\gamma,\mathrm{perturbation}}$$

Wilson & Naylor (in prep.)
Gaia eDR3 - Gaia Collaboration, Brown A. G. A., et al. (2021)

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Conclusions

- **PDF describing positions of sources measured in photometric catalogues assumed Gaussian**
- **However, other components of the AUF are non-Gaussian — will be crucial for LSST (as with *WISE*)**
- **These might not be analytic, and hence require numerical methods to derive; simplifying assumption of circular symmetry enables convolutions to be performed in Fourier space with reasonable computation**
- **Models for contributions to offsets between sources for crowded field blending perturbations, and unknown proper motions — but mathematical framework flexible to all unknown kinds of separations**

- **Upcoming LSST:UK cross-match service macauff**

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Wilson & Naylor, 2017, MNRAS, 468, 2517
Wilson & Naylor, 2018a, MNRAS, 473, 5570
Wilson & Naylor, 2018b, MNRAS, 481, 2148
https://github.com/Onoddil/macauff

Macauff