Beyond the centroid: the need for a more complete model of LSST astrometric uncertainties

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Science and Technology Facilities Council

University of Exeter

LSST:UK Consortium

VERA C. RUBIN OBSERVATORY

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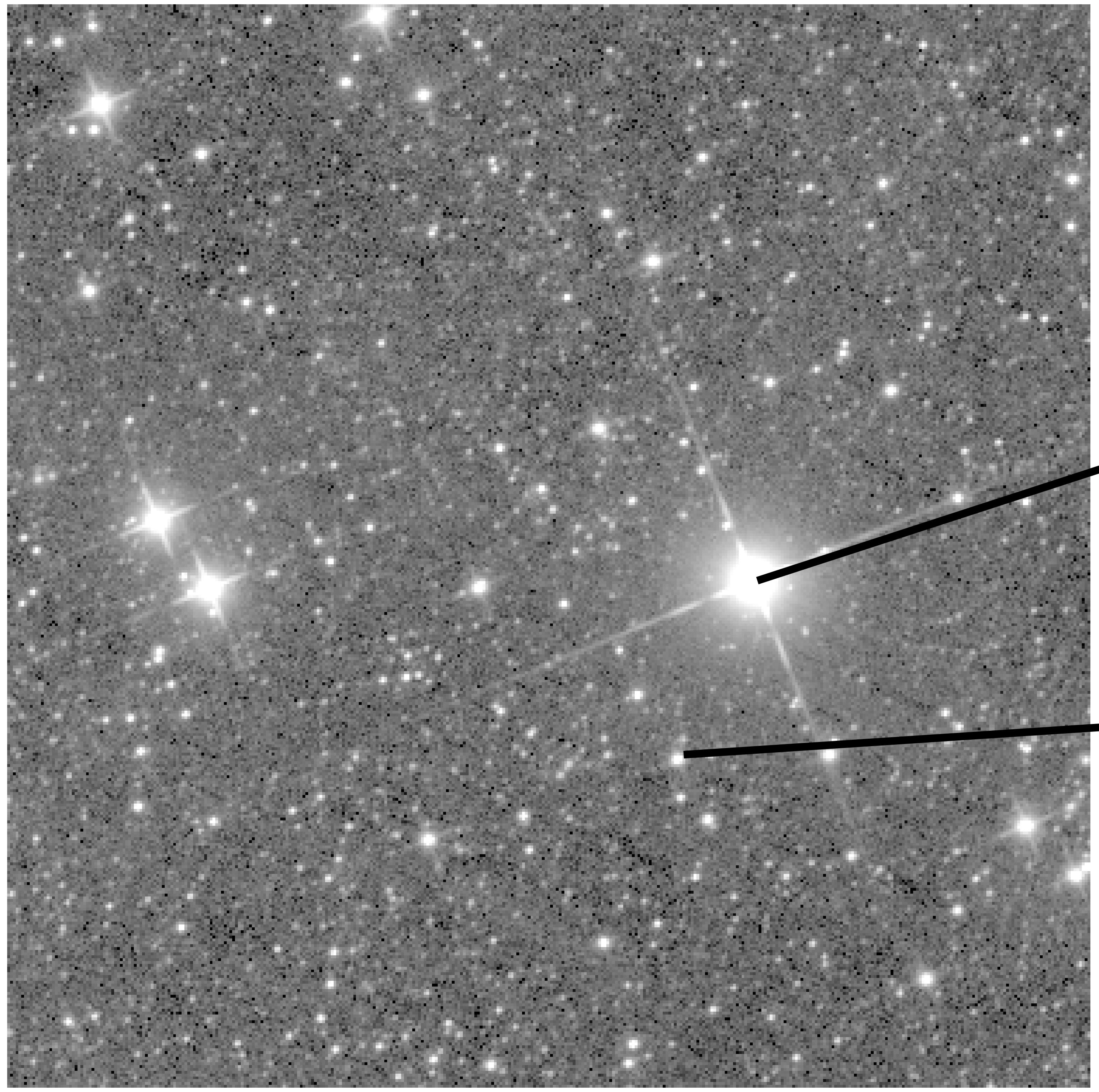
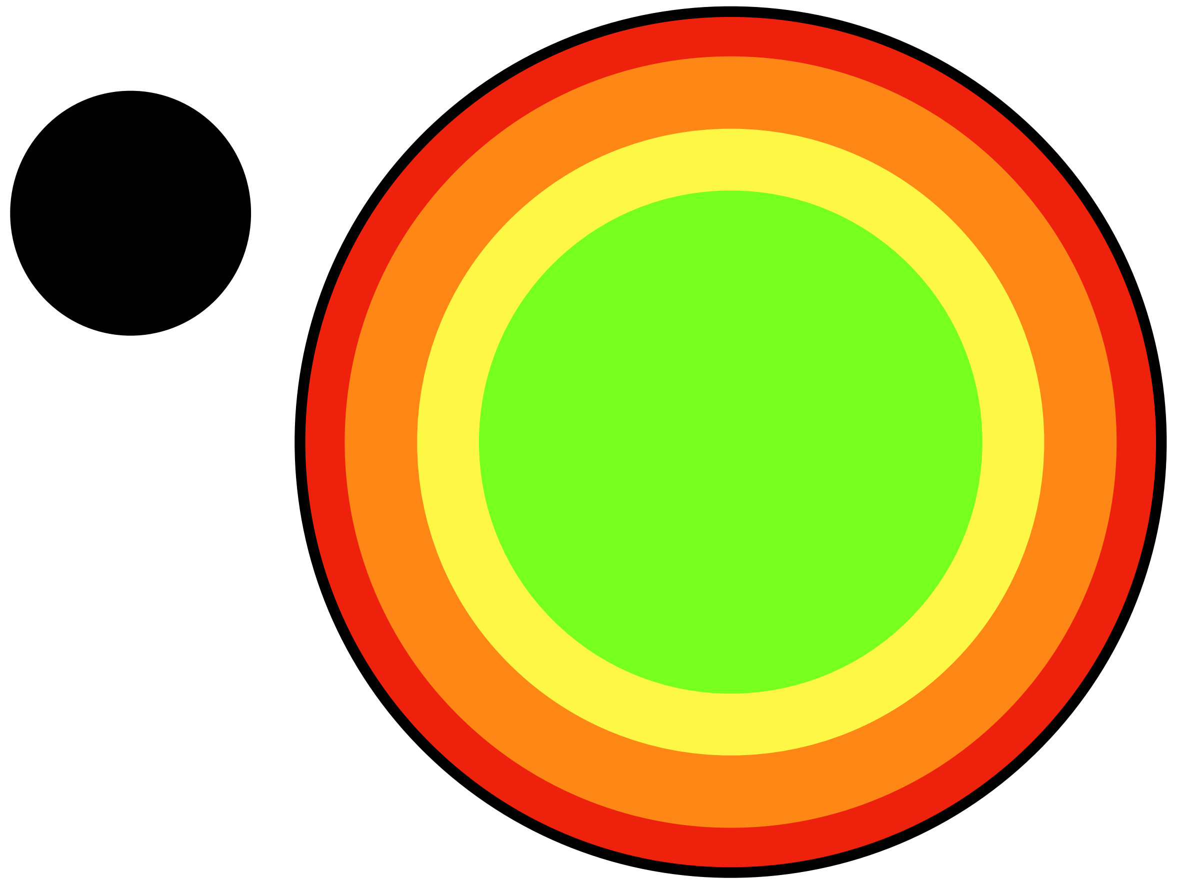
What's In A Photometric Catalogue?

(Ironically, it's half astrometry!)



WISE - Wright et al. (2010)

Source ID	Position (deg)	Uncertainty (arcsecond)	Brightness (mag)	Uncertainty (mag)
1	218.4763	0.073	14.94	0.04
2	218.3951	0.217	20.32	0.15



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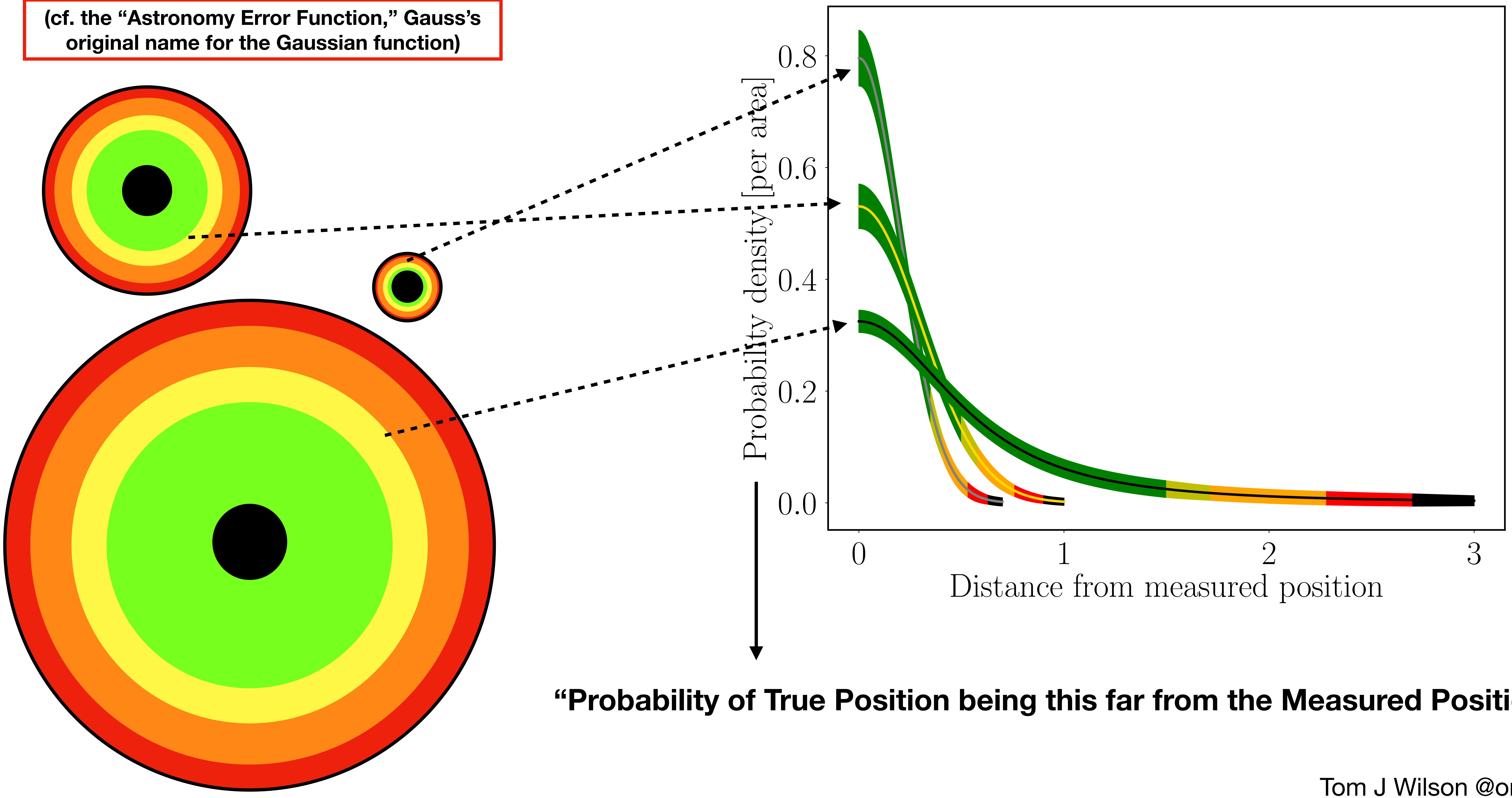
The “Astrometric Uncertainty Function”

(cf. the “Astronomy Error Function,” Gauss’s original name for the Gaussian function)



“Probability of True Position being this far from the Measured Position”

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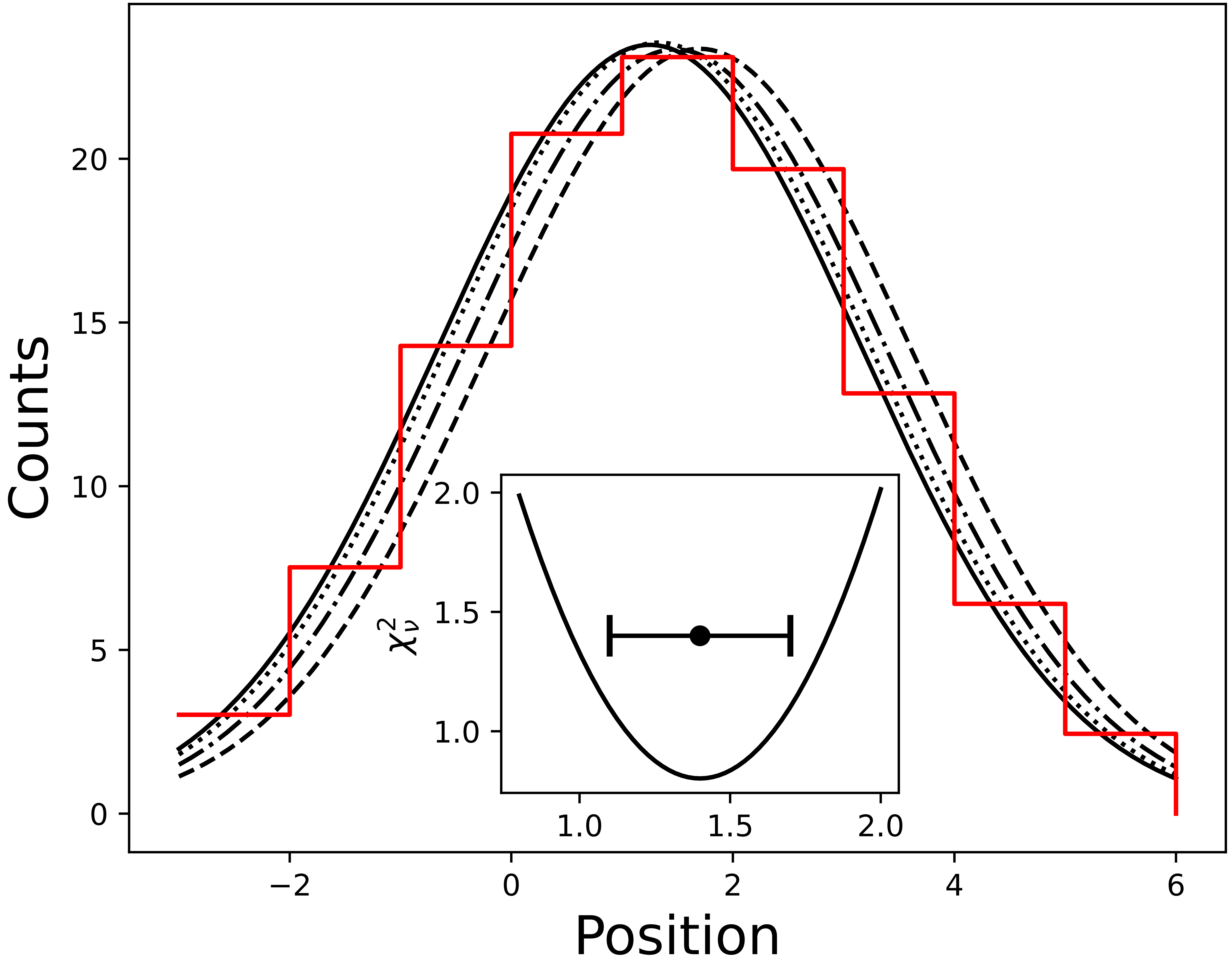
The "Astrometric Uncertainty Function"

(cf. the "Astronomy Error Function," Gauss's original name for the Gaussian function)

The Centroid Component of the AUF



$$p(t\,|\,m) \propto \frac{\exp\left(-\frac{1}{2}\left(\mathbf{x}-\boldsymbol{\mu}\right)^T \boldsymbol{\Sigma}^{-1}\left(\mathbf{x}-\boldsymbol{\mu}\right)\right)}{\sqrt{(2\pi)^k\,|\,\boldsymbol{\Sigma}\,|}}$$

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$$

The “Astrometric Uncertainty Function”

(cf. the “Astronomy Error Function,” Gauss’s original name for the Gaussian function)

Basically all literature to date assumes that positional errors of sources are *purely* Gaussian!

$$P(i) = \frac{\frac{Xc(m_i)\,\boxed{g(\Delta x_i, \Delta y_i)}}{Nf(m_i)}}{1 - X + \sum_j \frac{Xc(m_j)\,g(\Delta x_j, \Delta y_j)}{Nf(m_j)}}$$

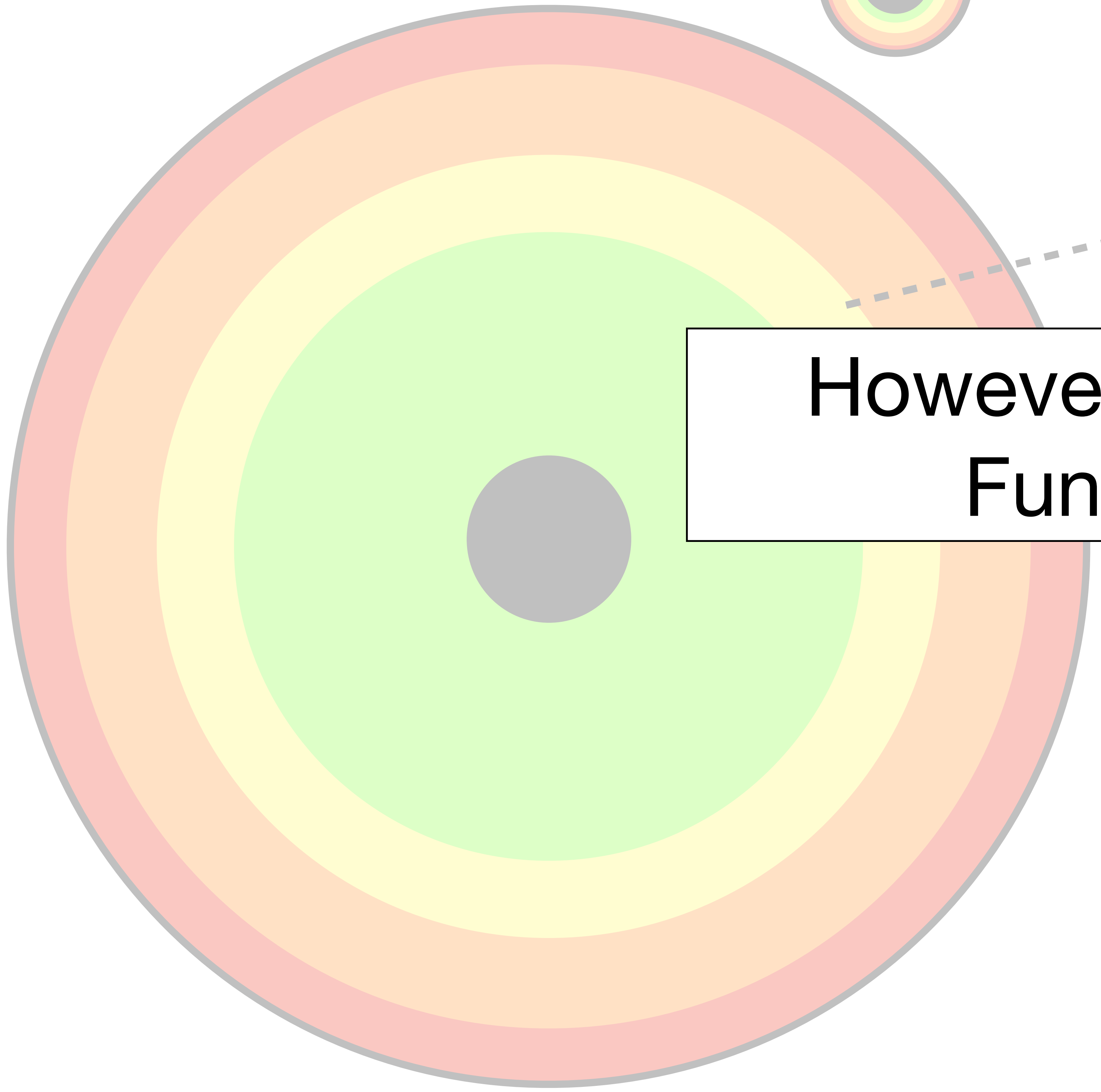
Naylor, Broos, & Feigelson (2013)

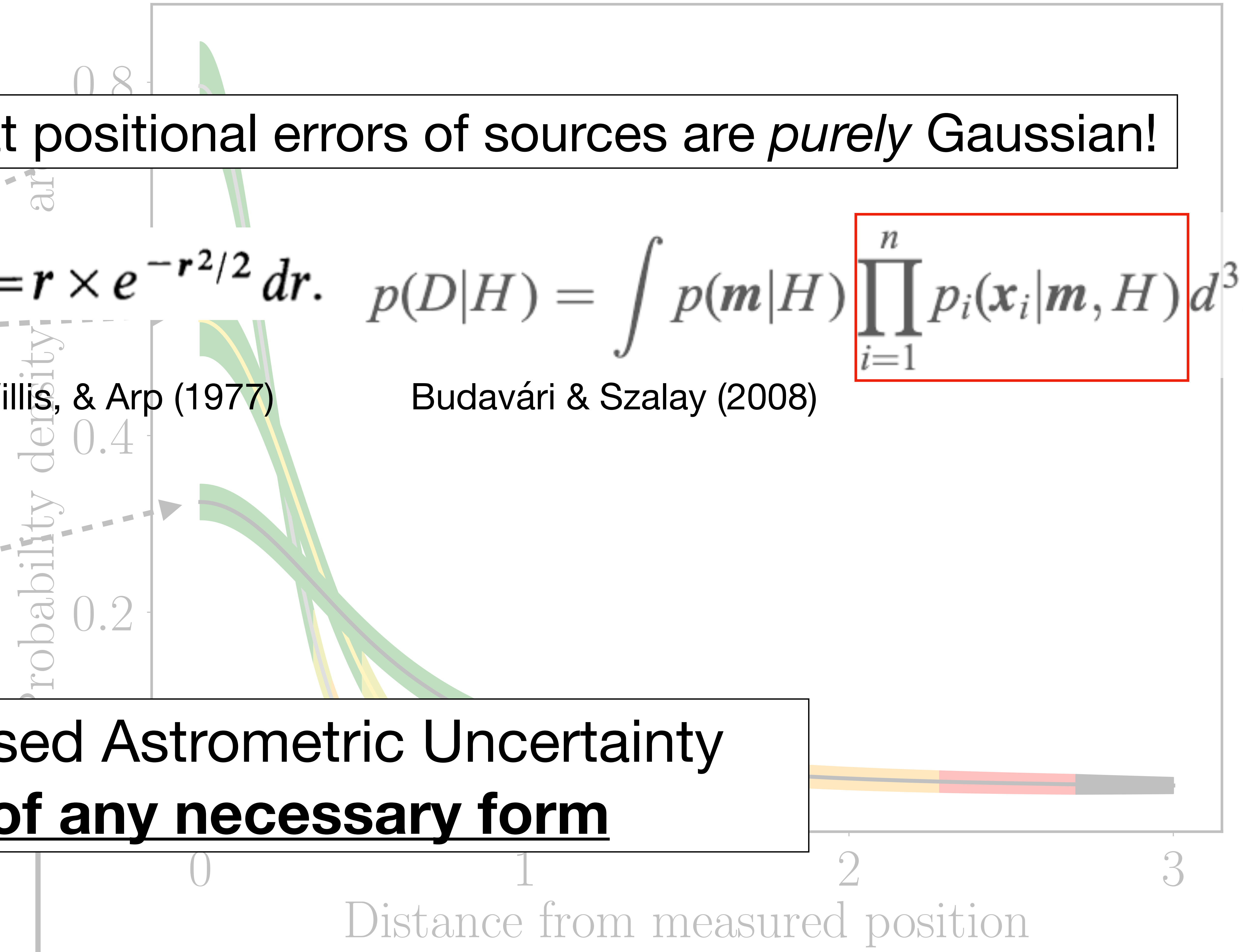
$$dp(r|id) = r \times e^{-r^2/2}\,dr.$$

de Ruiter, Willis, & Arp (1977)

$$p(D|H) = \int p(\boldsymbol{m}|H) \boxed{\prod_{i=1}^{n} p_i(\boldsymbol{x}_i|\boldsymbol{m}, H)}\, d^3 m$$

Budavári & Szalay (2008)



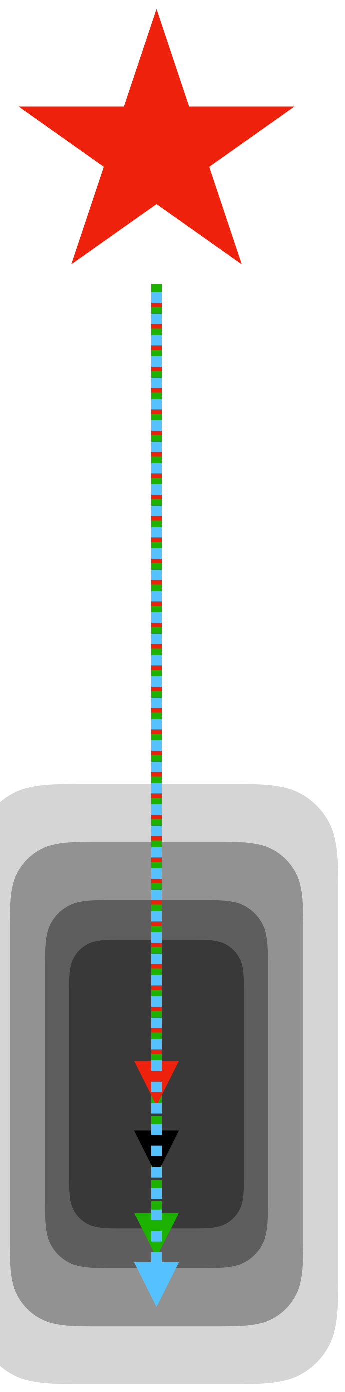


However, the generalised Astrometric Uncertainty Function **<u>can be of any necessary form</u>**

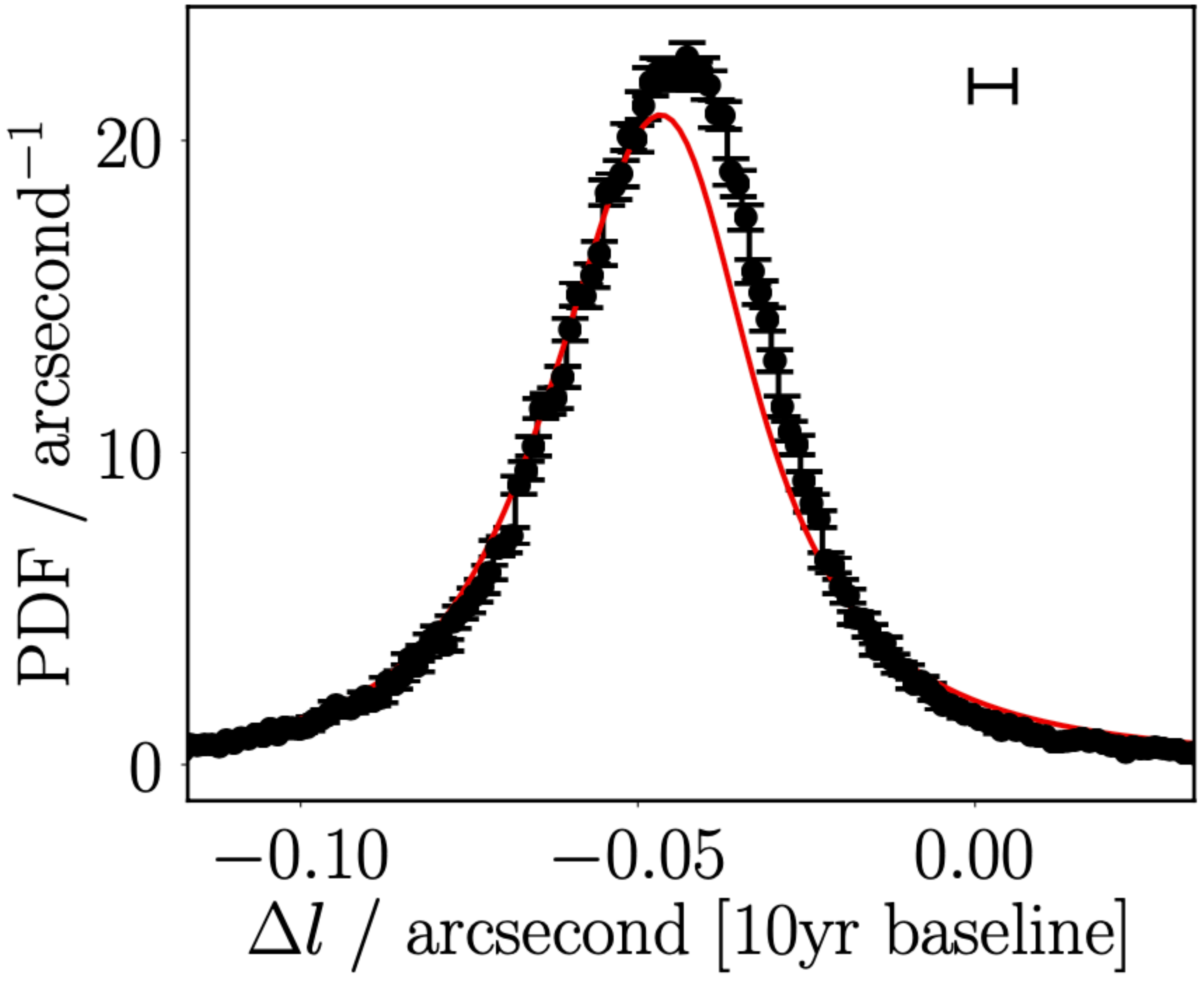
“Probability of True Position being this far from the Measured Position”

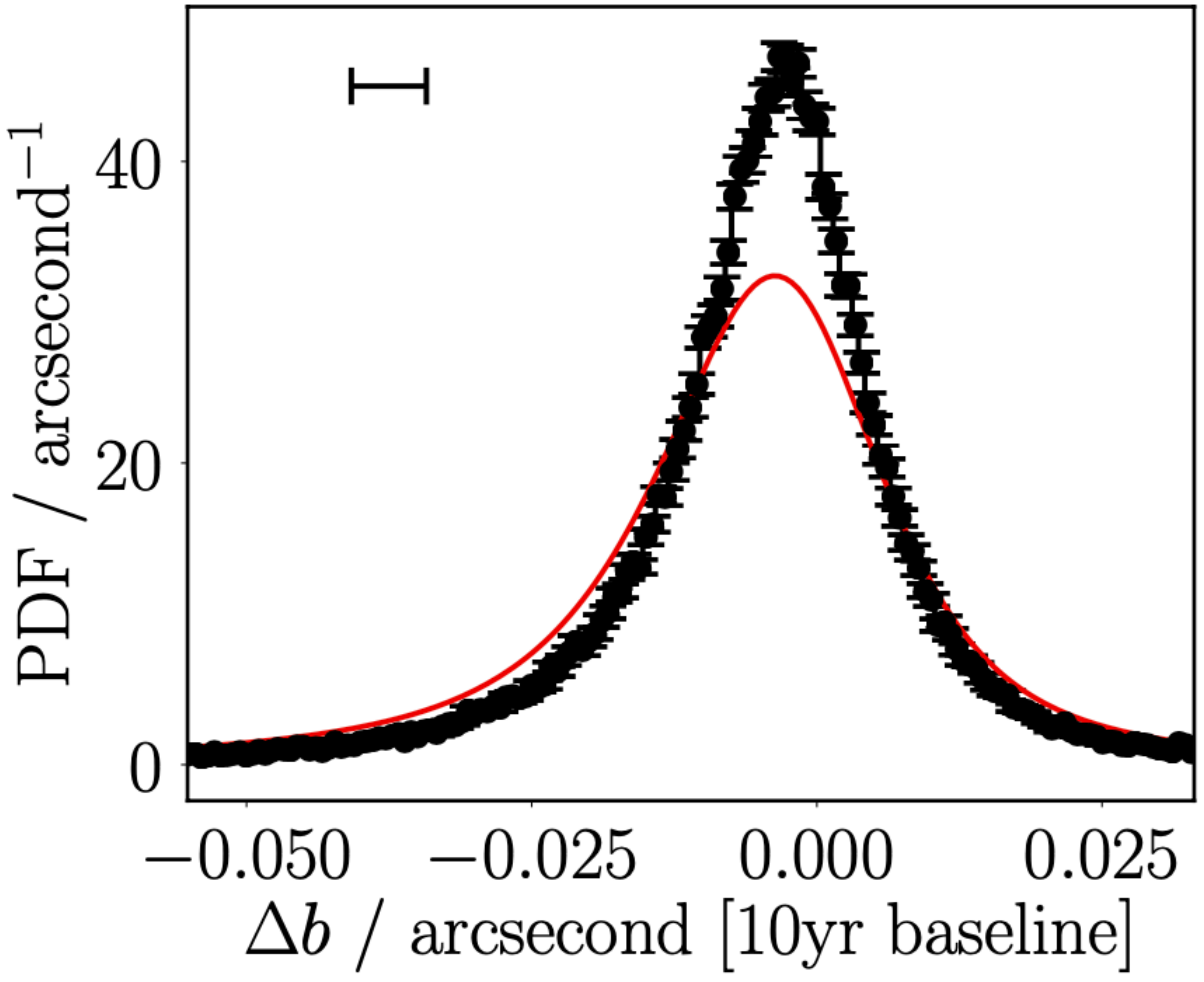
AUF Components: Unknown Proper Motions

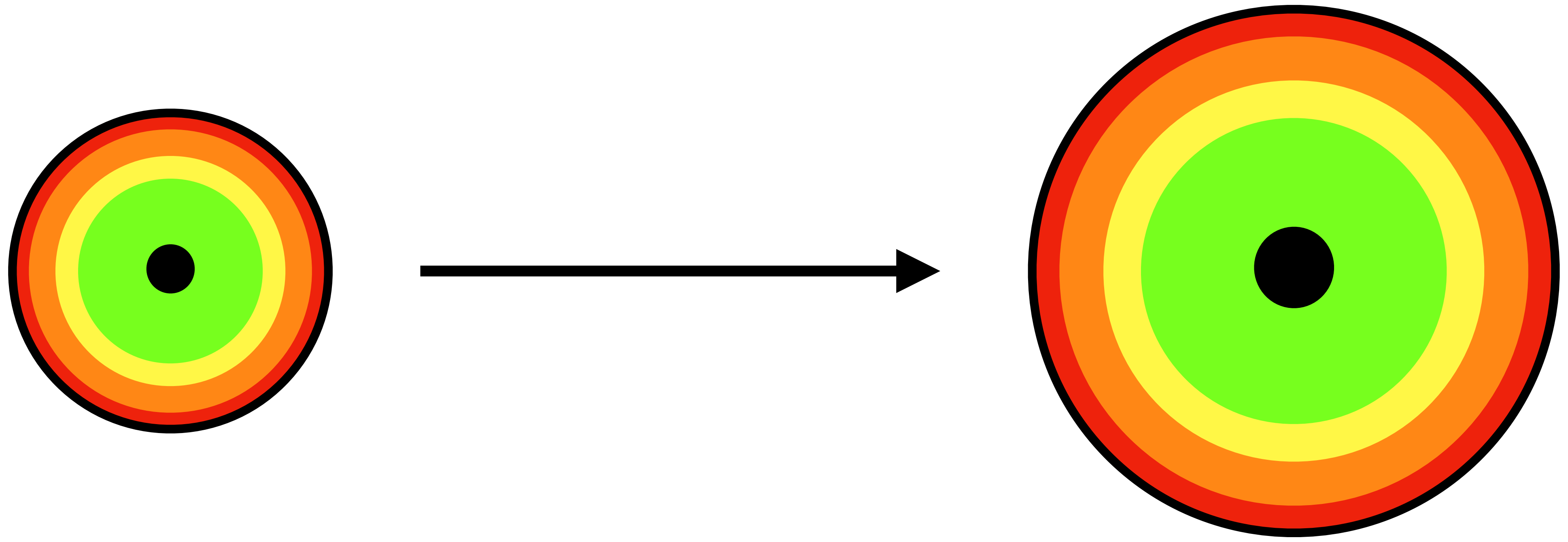
Object in 2015



Projected to 2025







(This also applies to *uncertain* proper motions, where we can incorporate the covariance matrix of weakly-constrained proper motions, e.g. just above the single-visit LSST limit)

Wilson (2023, RASTI, 2, 1)
Gaia eDR3 - Gaia Collaboration et al. (2021, A&A, 649, A1)

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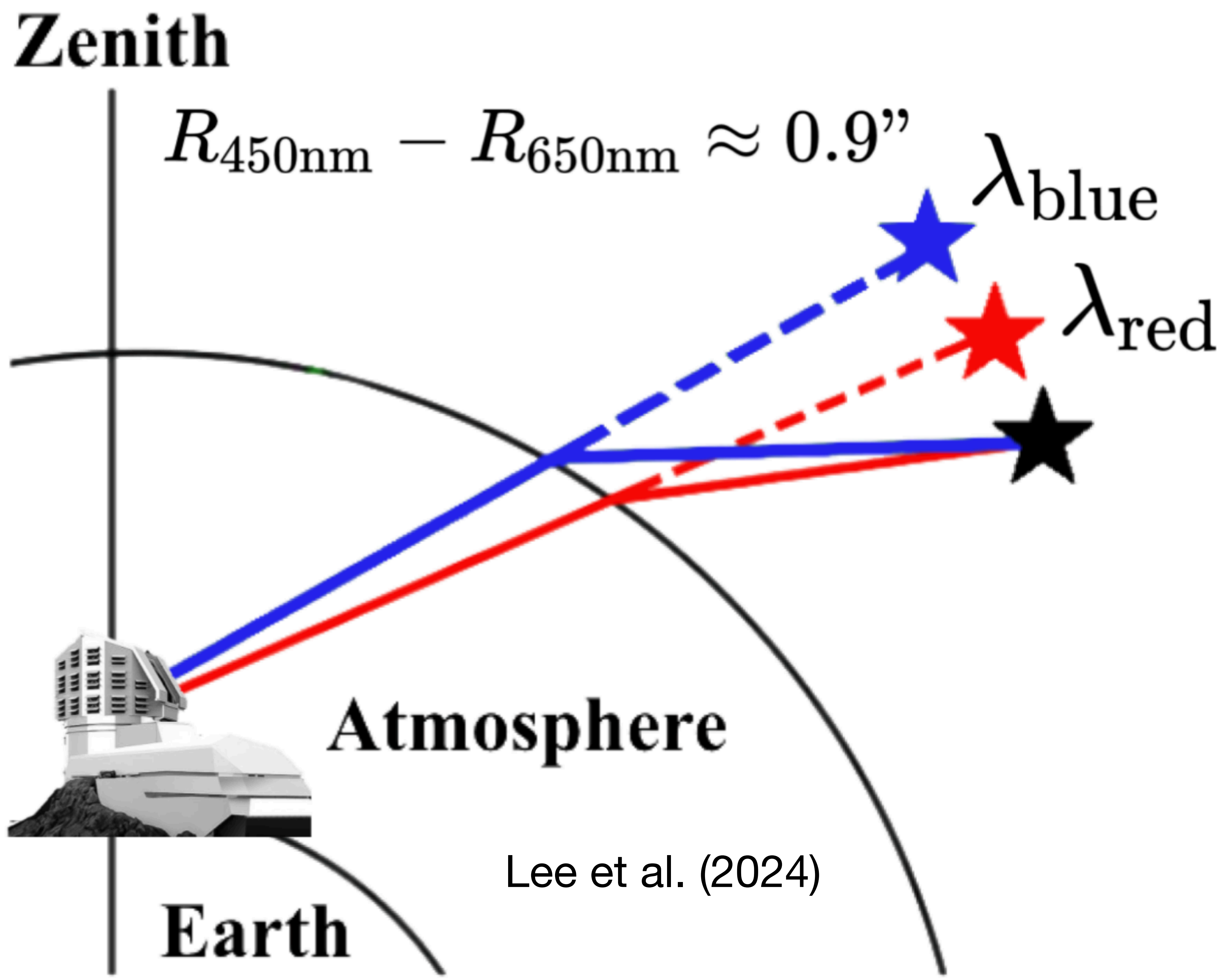
AUF Components: DCR



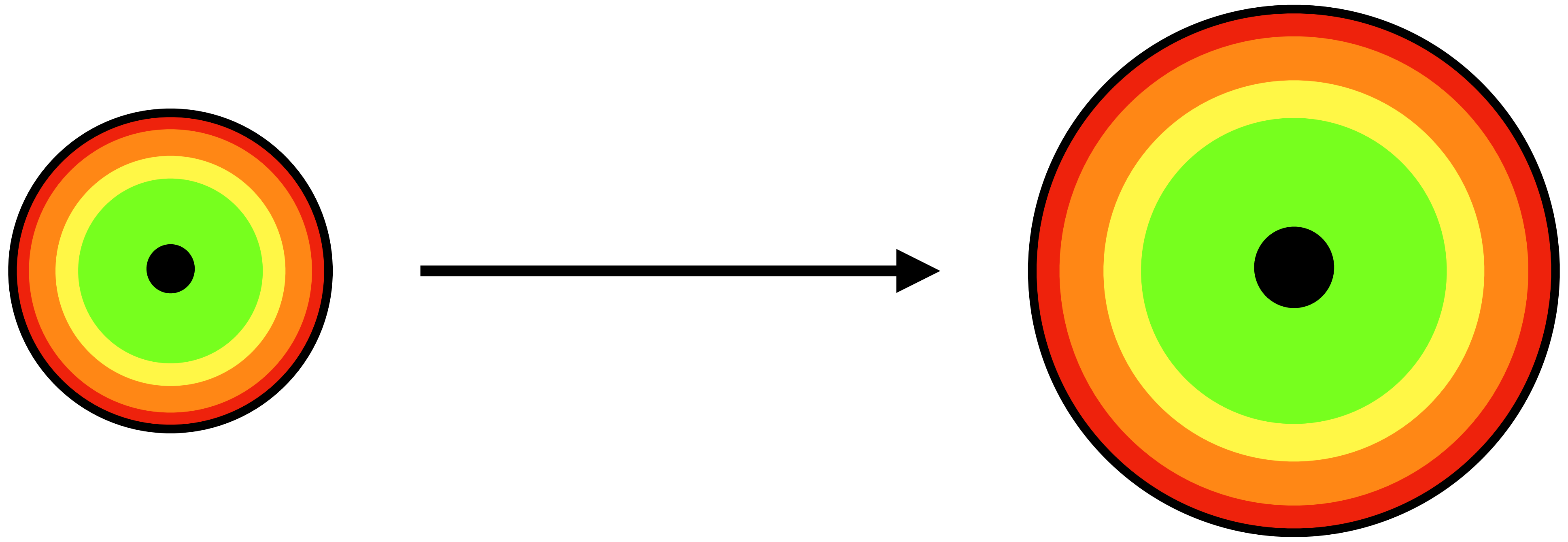
Lee et al. (2024)

e.g. gbdes, Bernstein et al. (2017)

$$\Delta \mathbf{x}^w = K_b c \tan z \, \hat{\mathbf{p}}$$

Unknown/uncertain per-band (b) scaling factor

Unknown/uncertain photometric colour c



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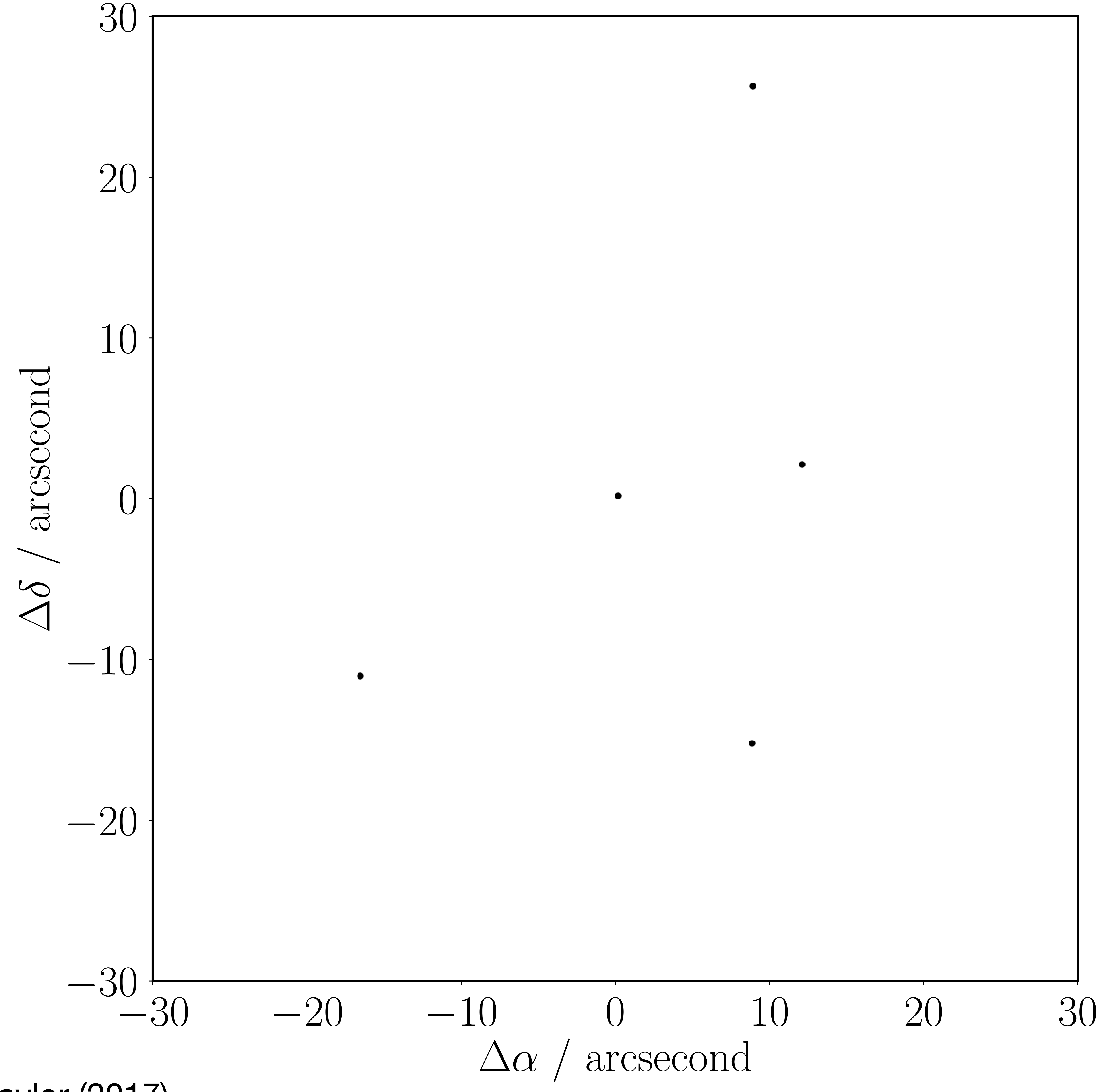
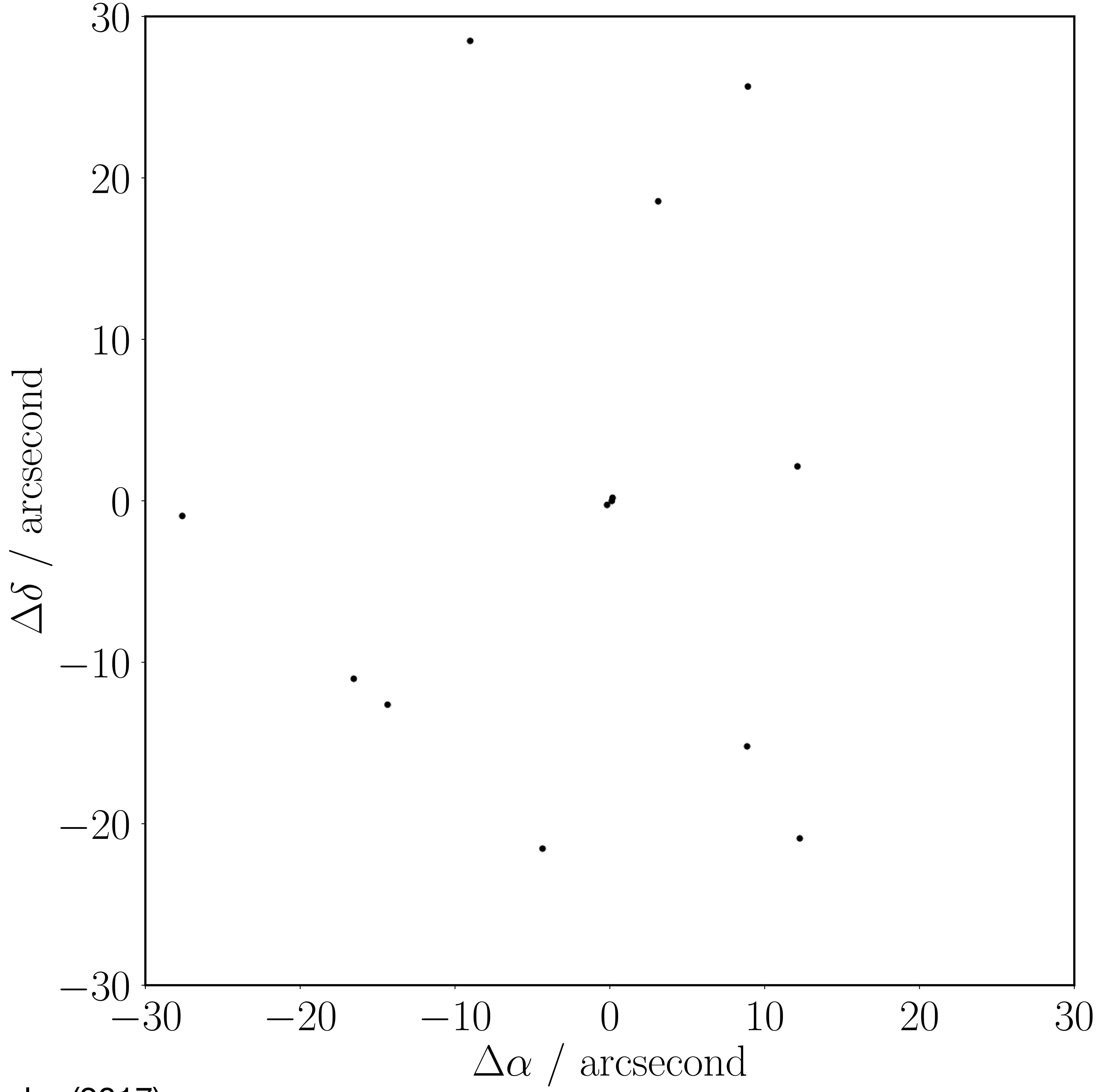
Crowded Fields and the Problem With LSST



Wilson & Naylor (2017)

Crowded Fields and the Problem With LSST



Wilson & Naylor (2017)

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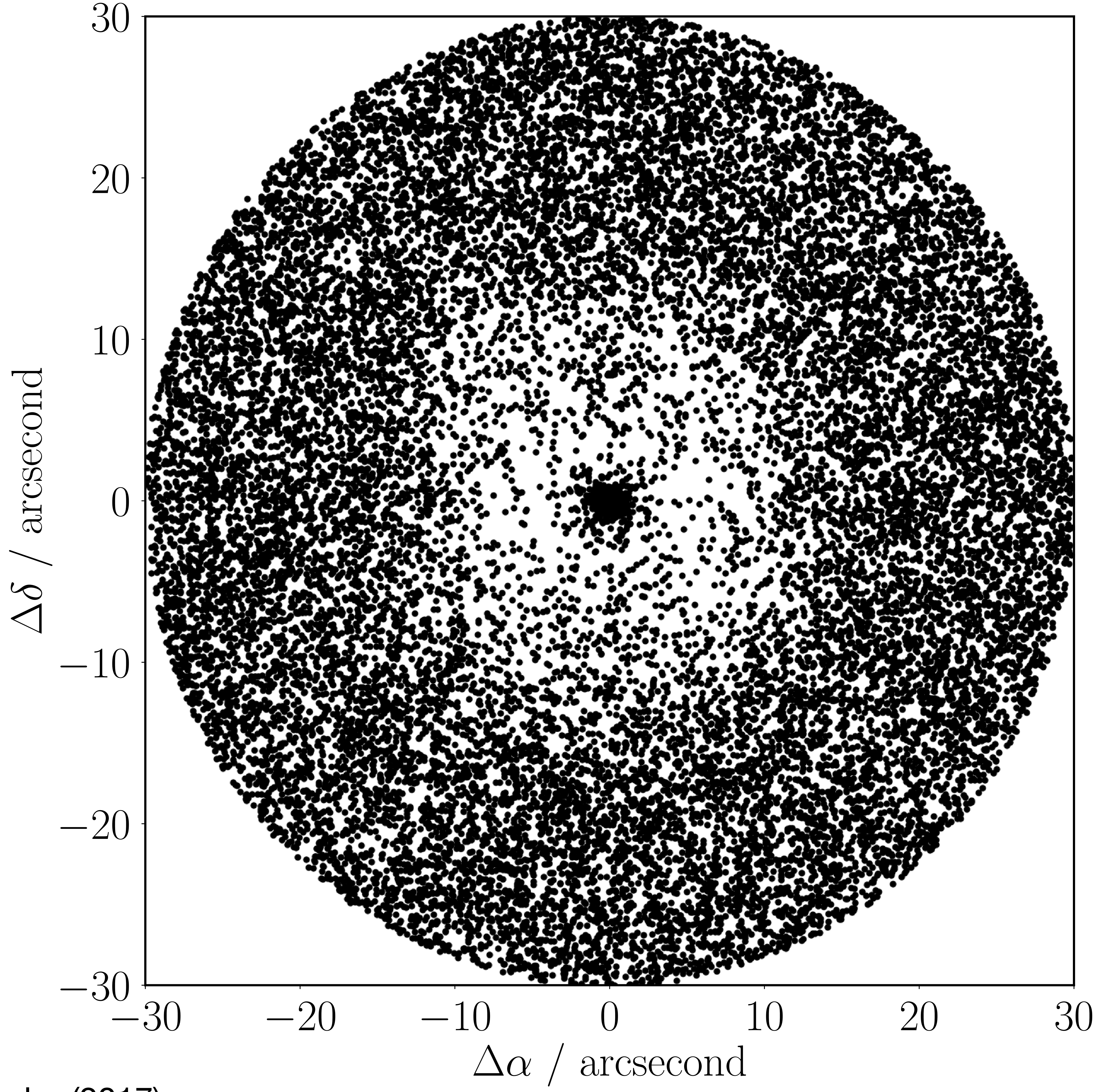
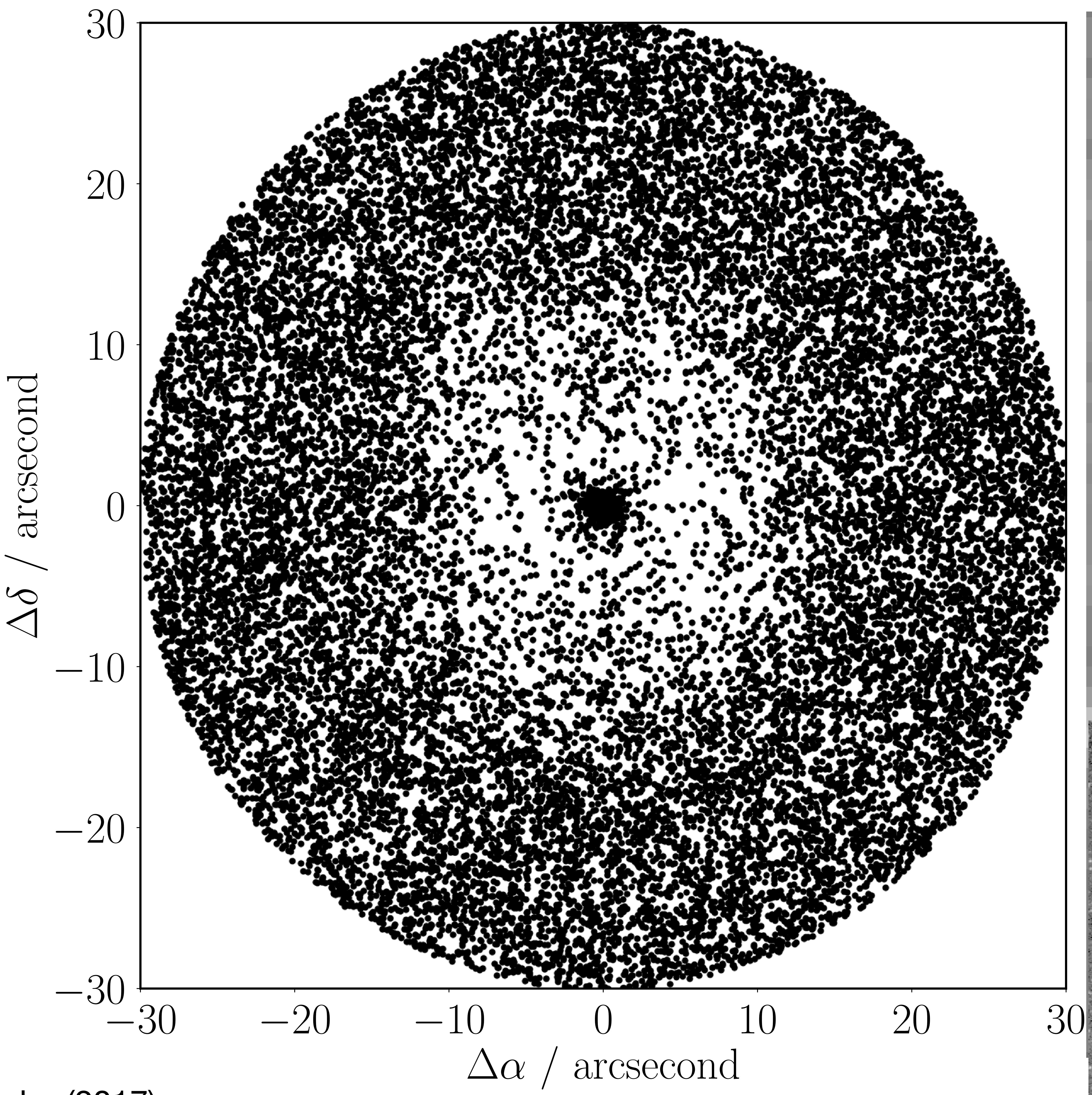
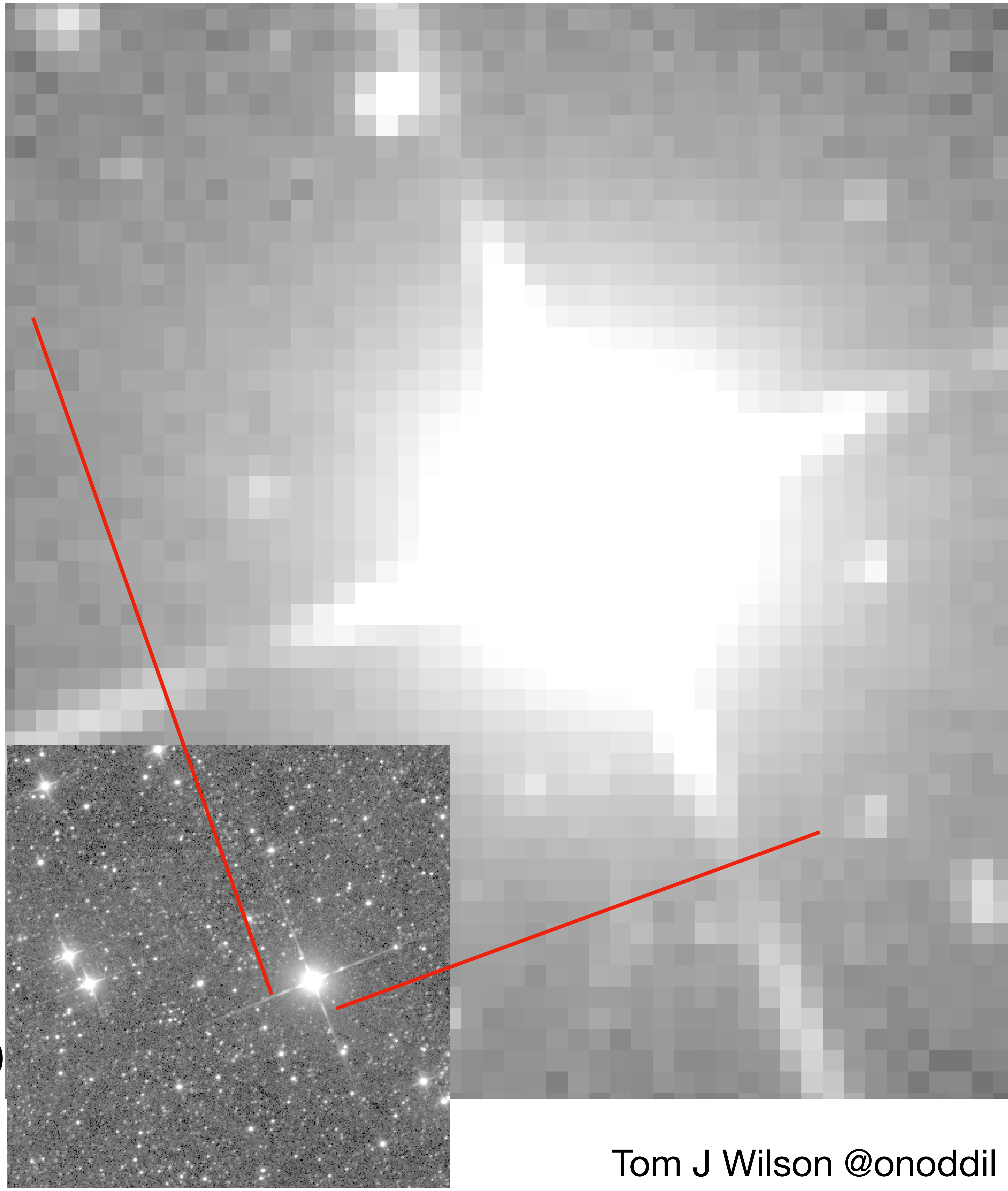


Wilson & Naylor (2017)

Crowded Fields and the Problem With LSST



Wilson & Naylor (2017)



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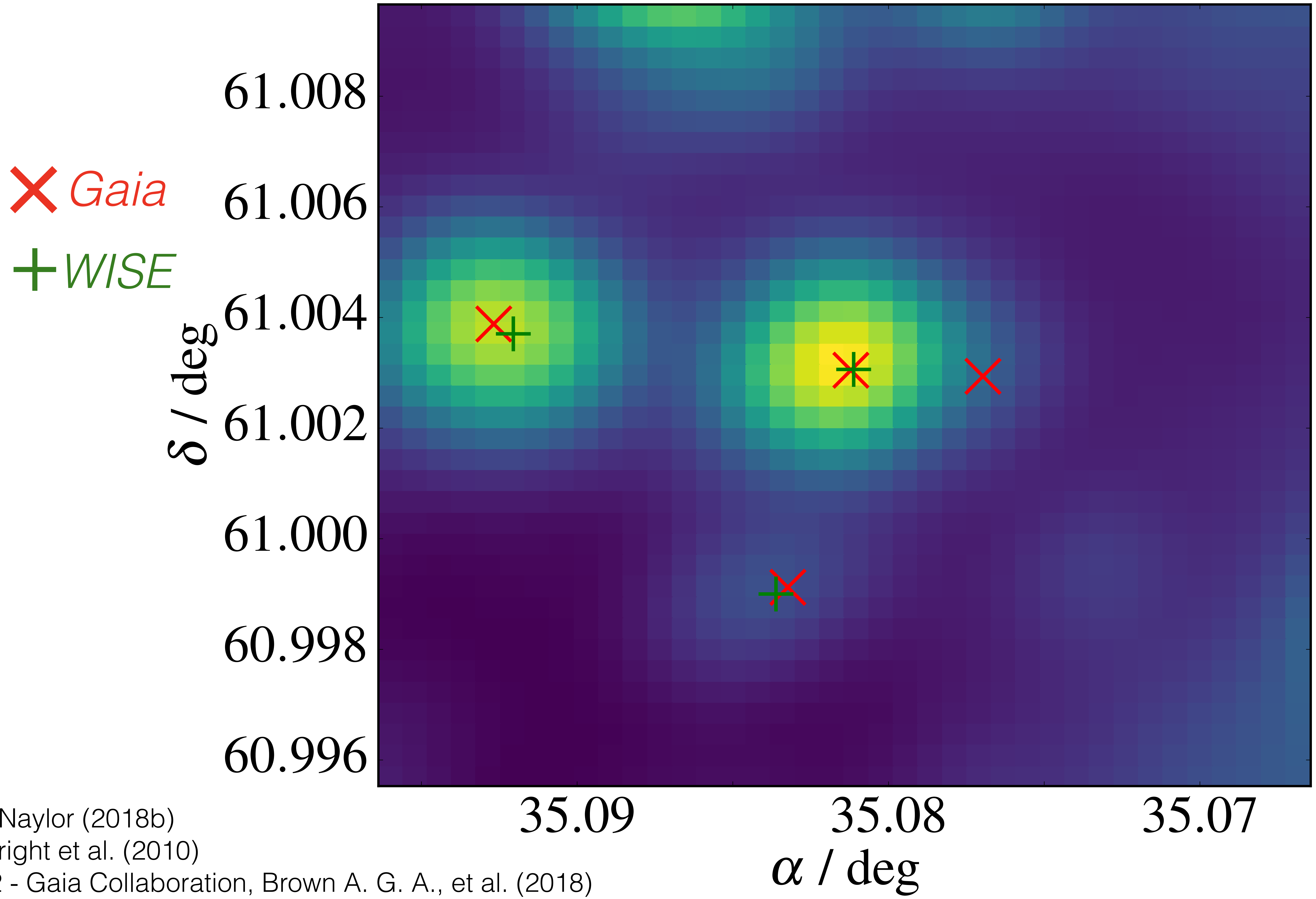
Unresolved, Hidden Contaminant Objects



Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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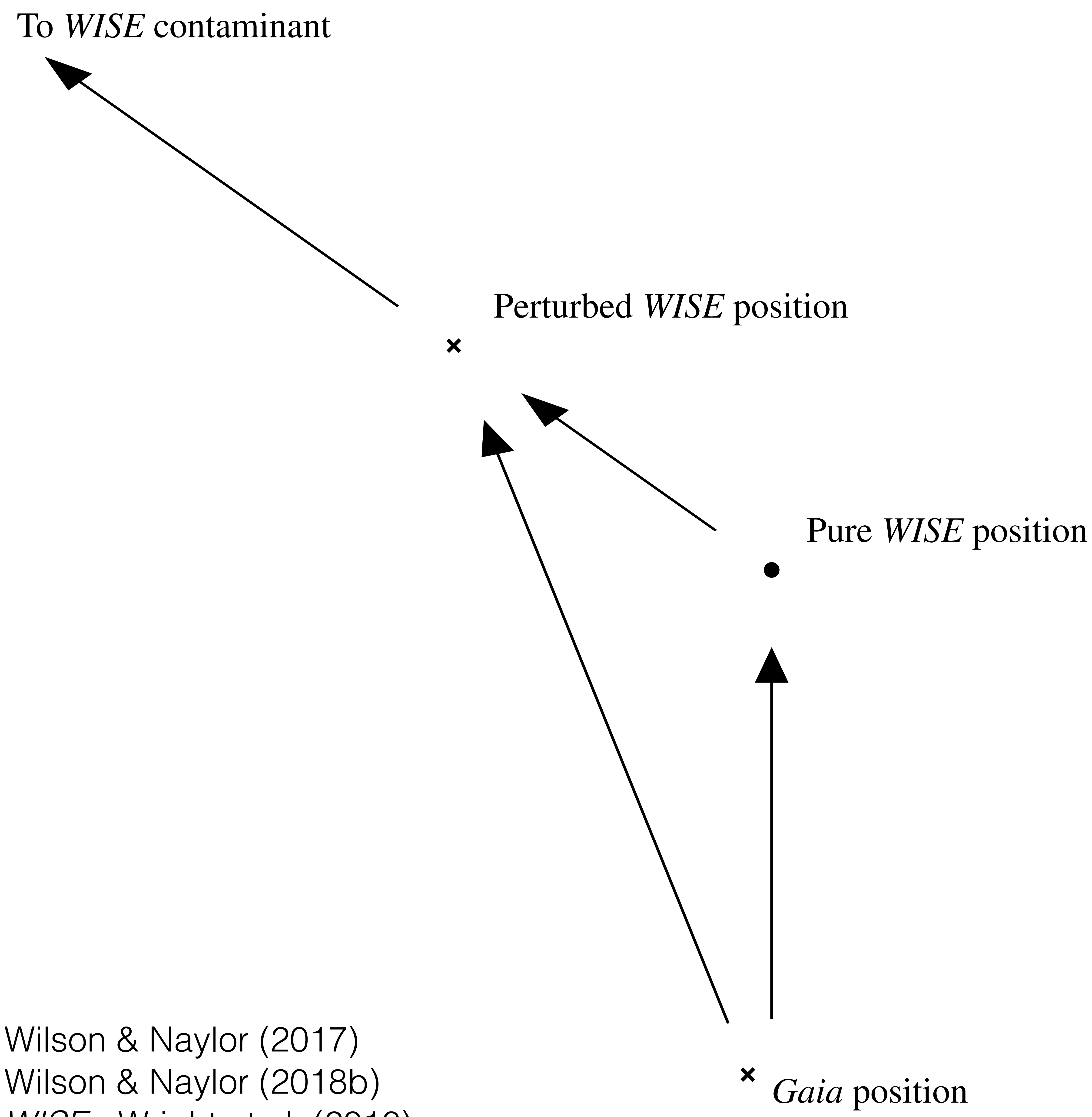
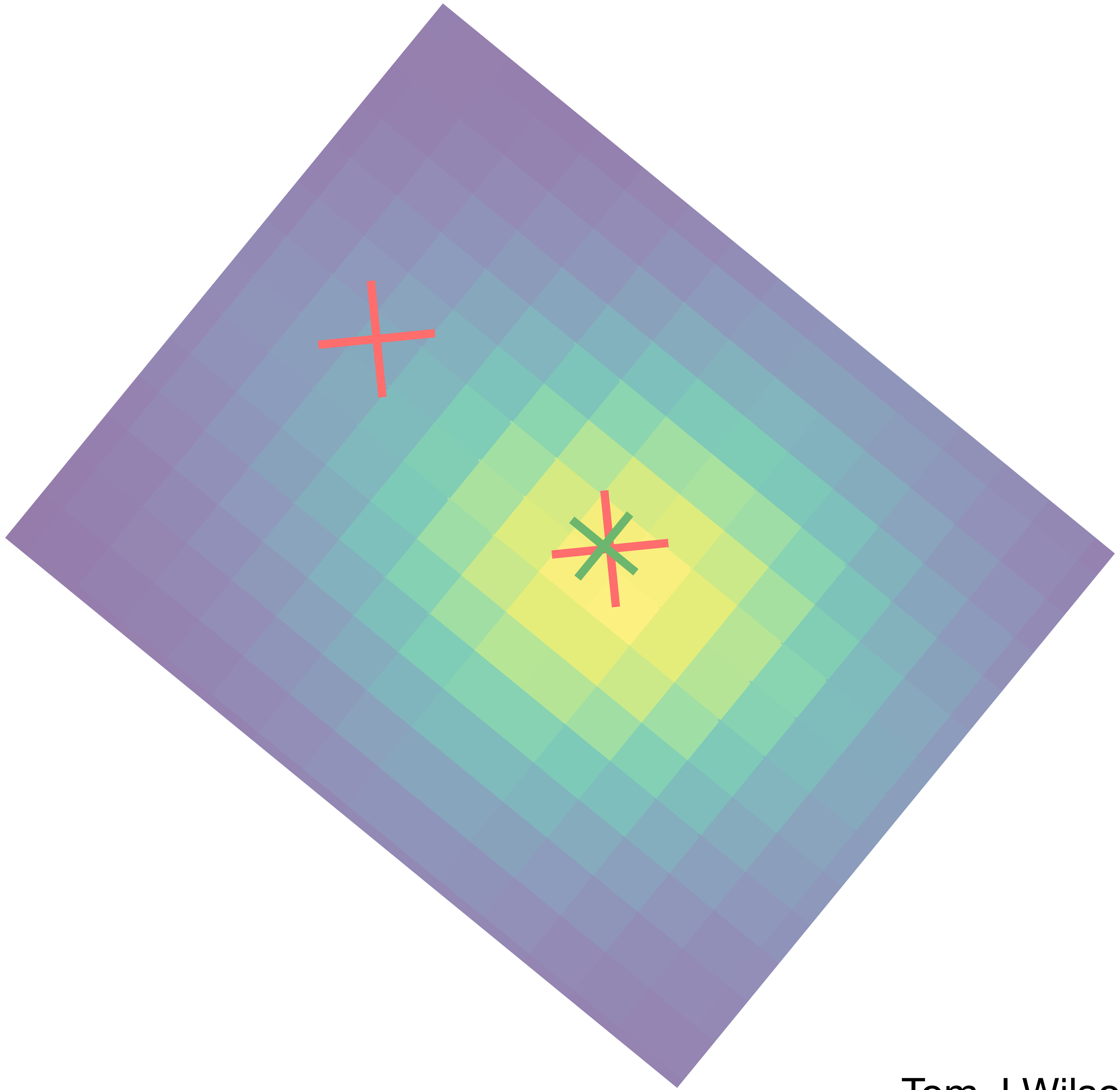
AUF Components: Perturbation





Wilson & Naylor (2017)
Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
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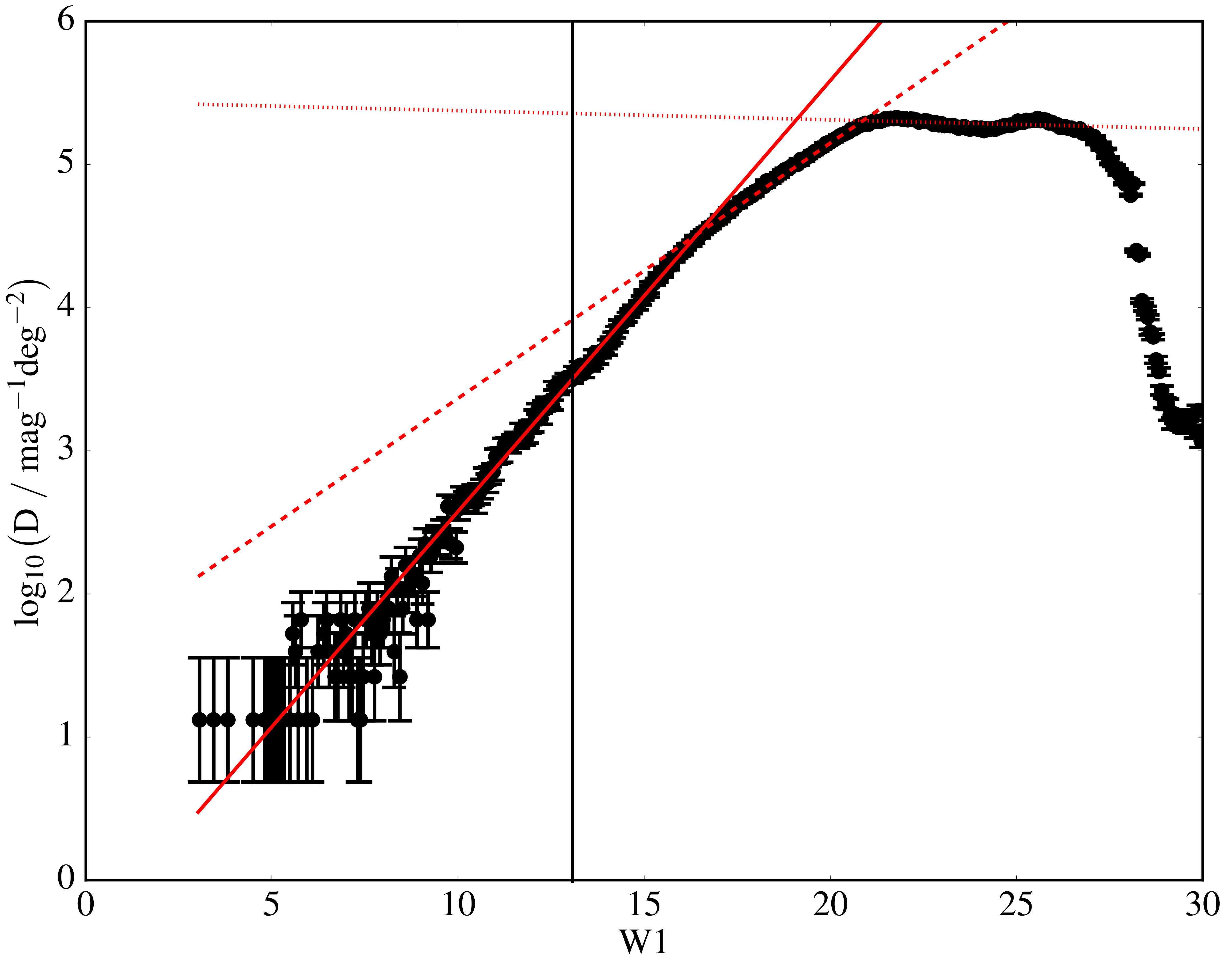
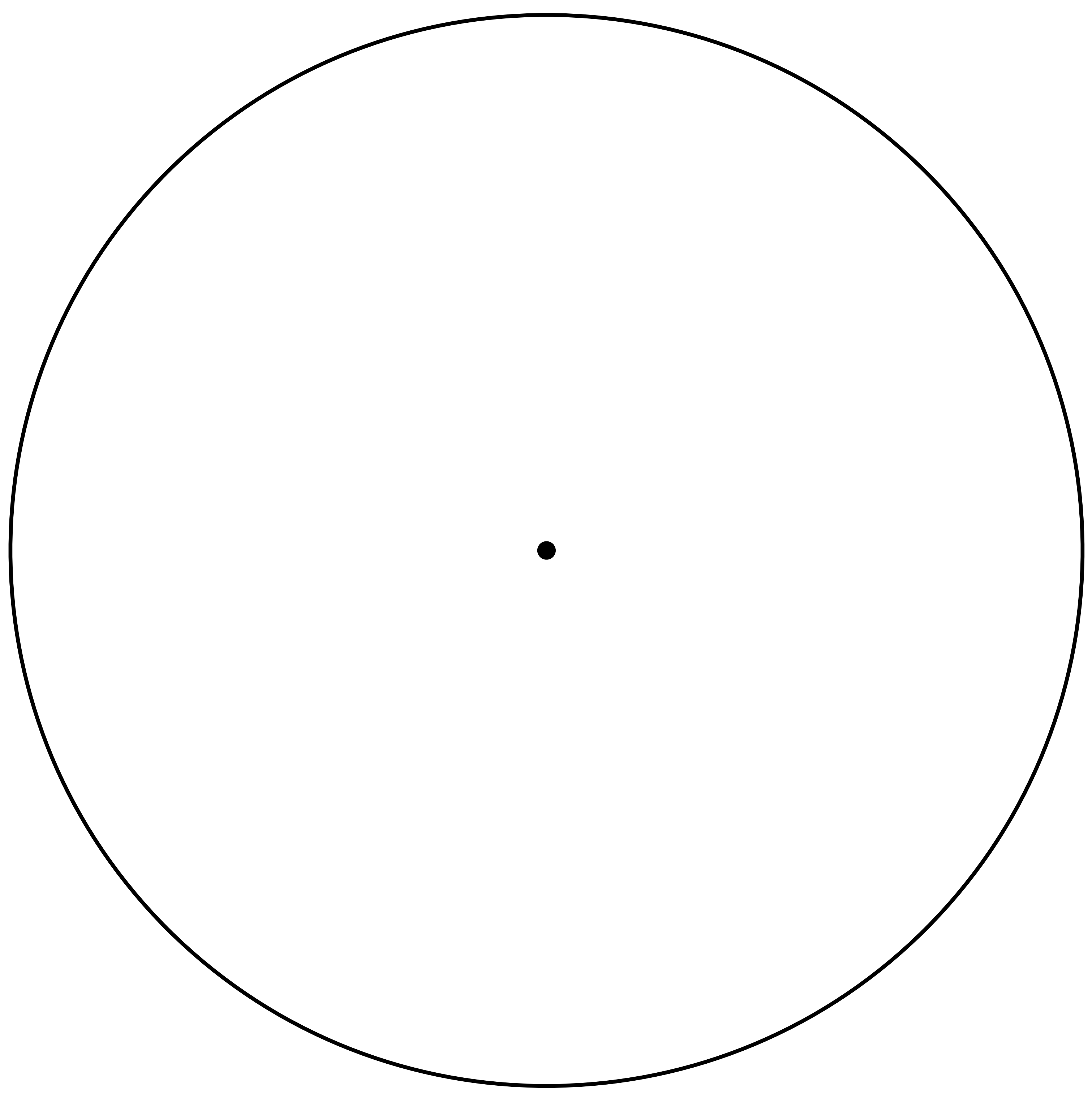
Building Empirical AUFs: Perturbation



(sources per PSF circle ~ 10^-6 sources per mag per sq deg)

PSF radius ~ 1.2 FWHM (Rayleigh criterion)



Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)

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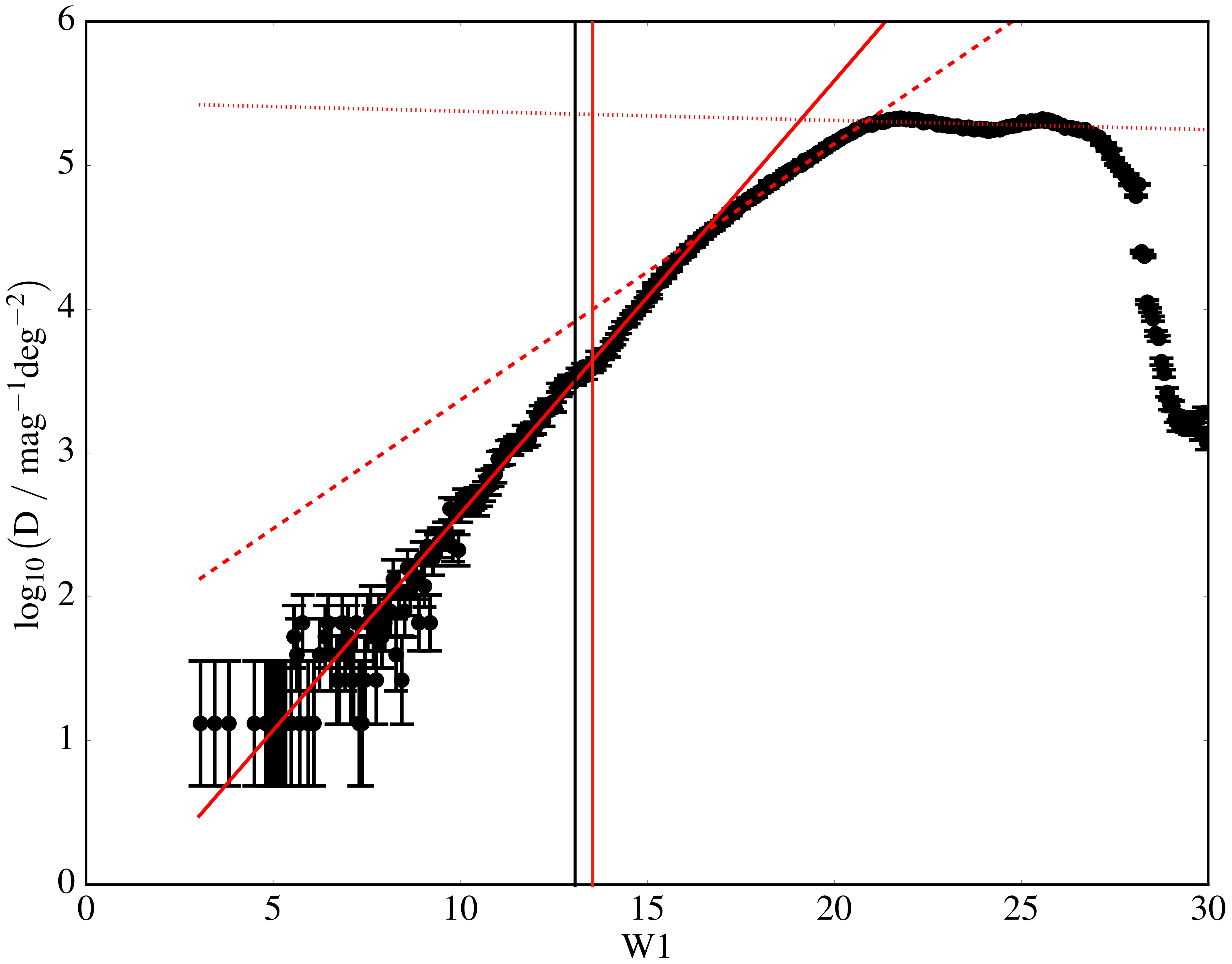
Building Empirical AUFs: Perturbation



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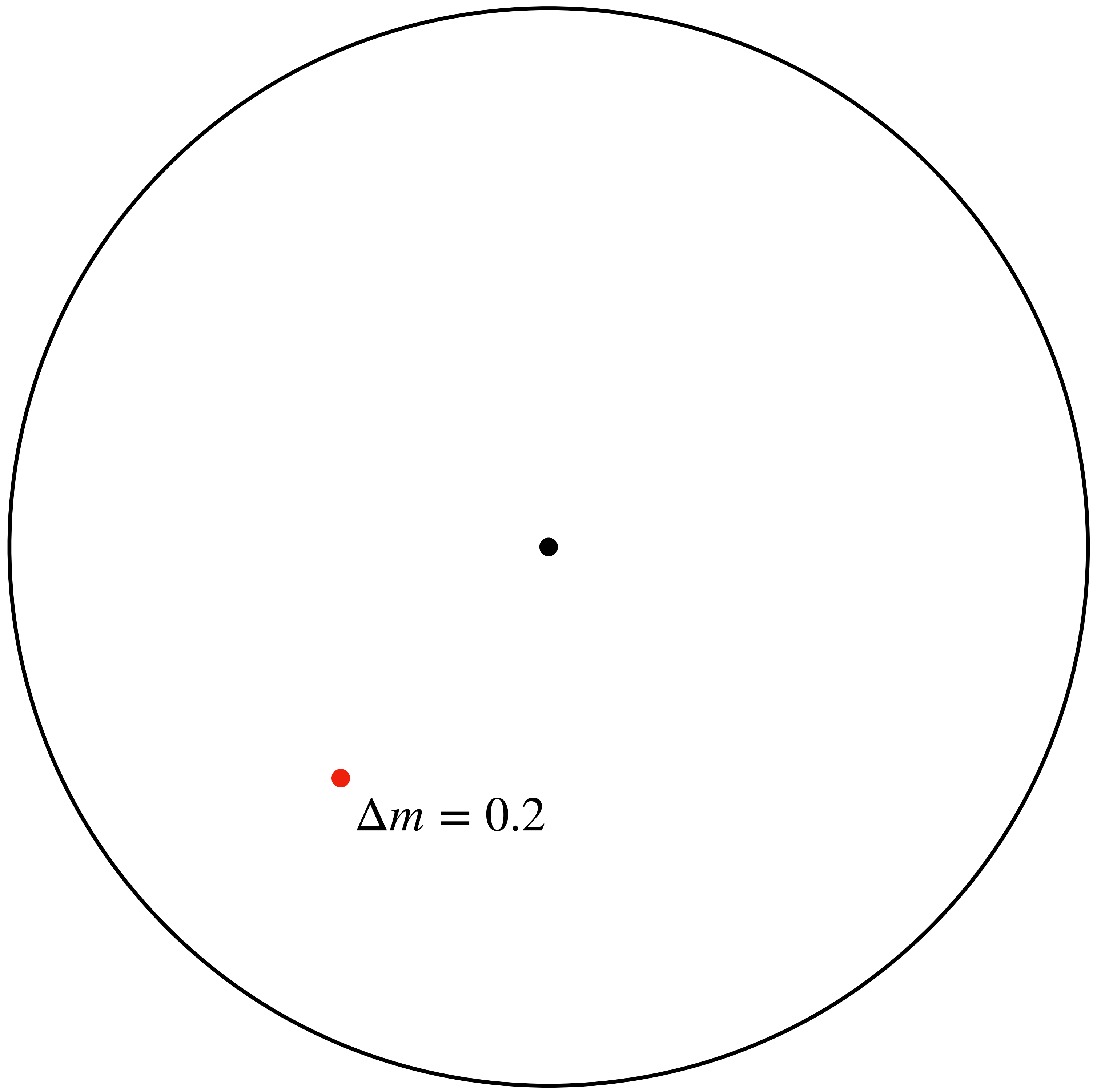

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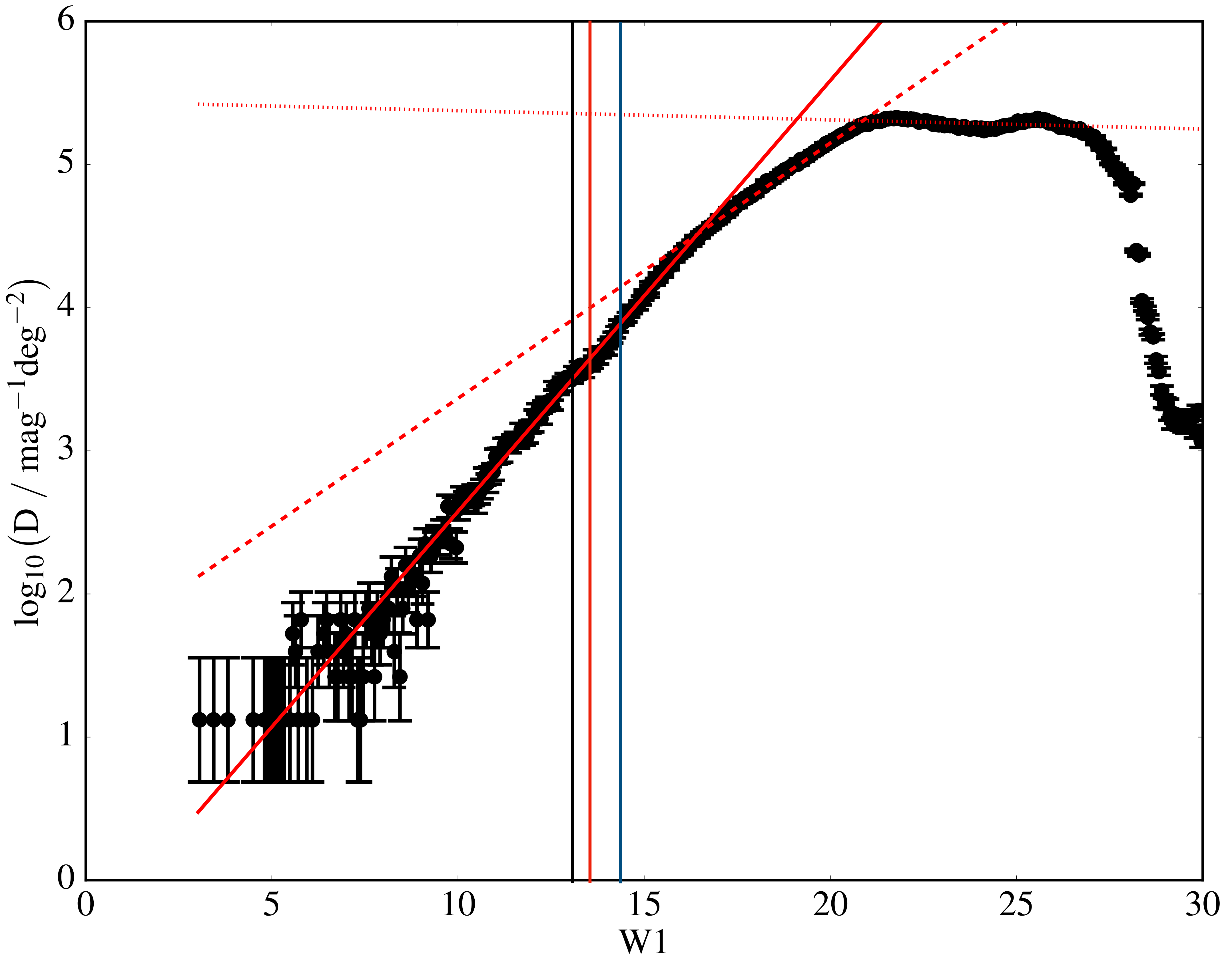

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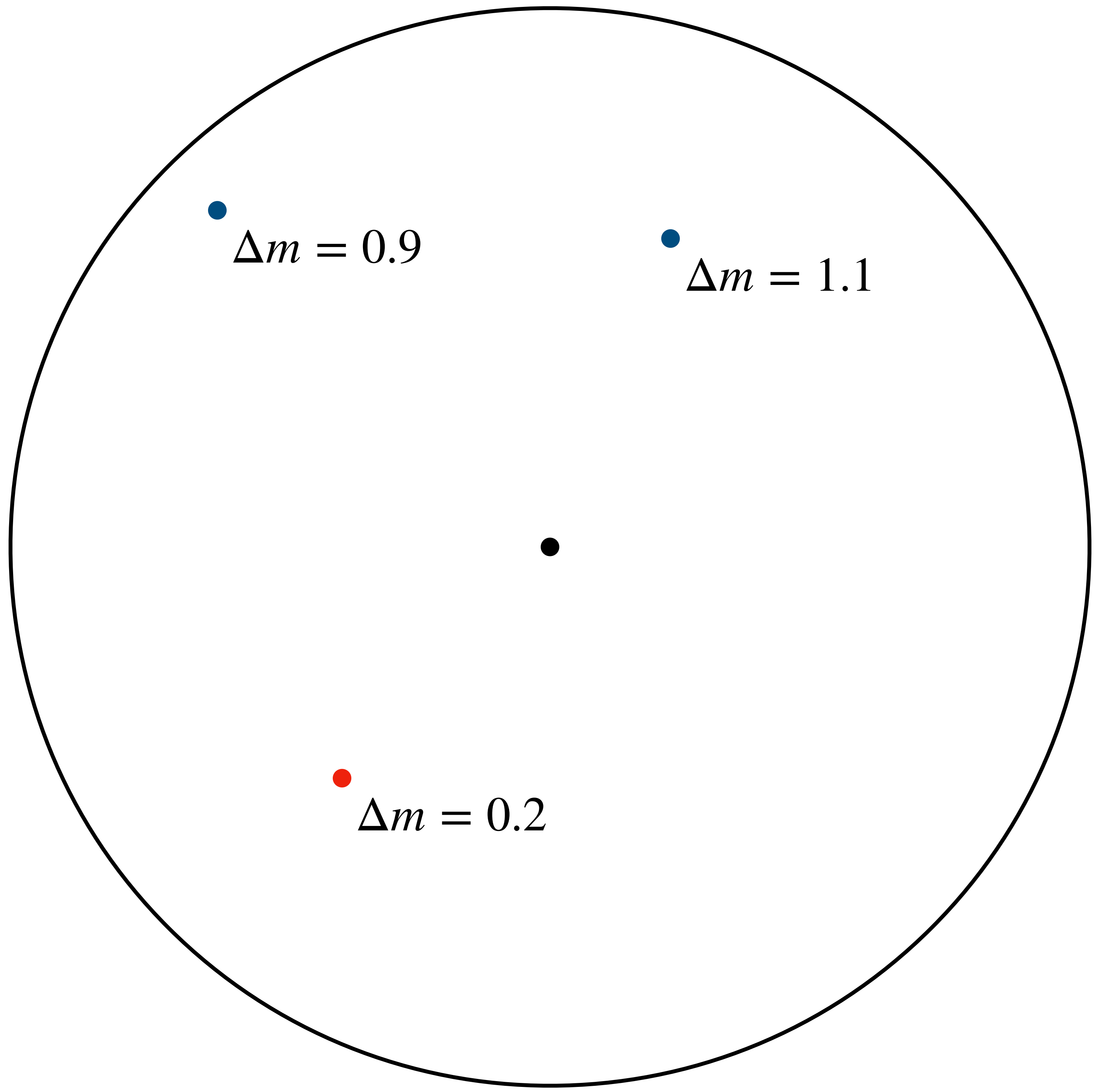

Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)

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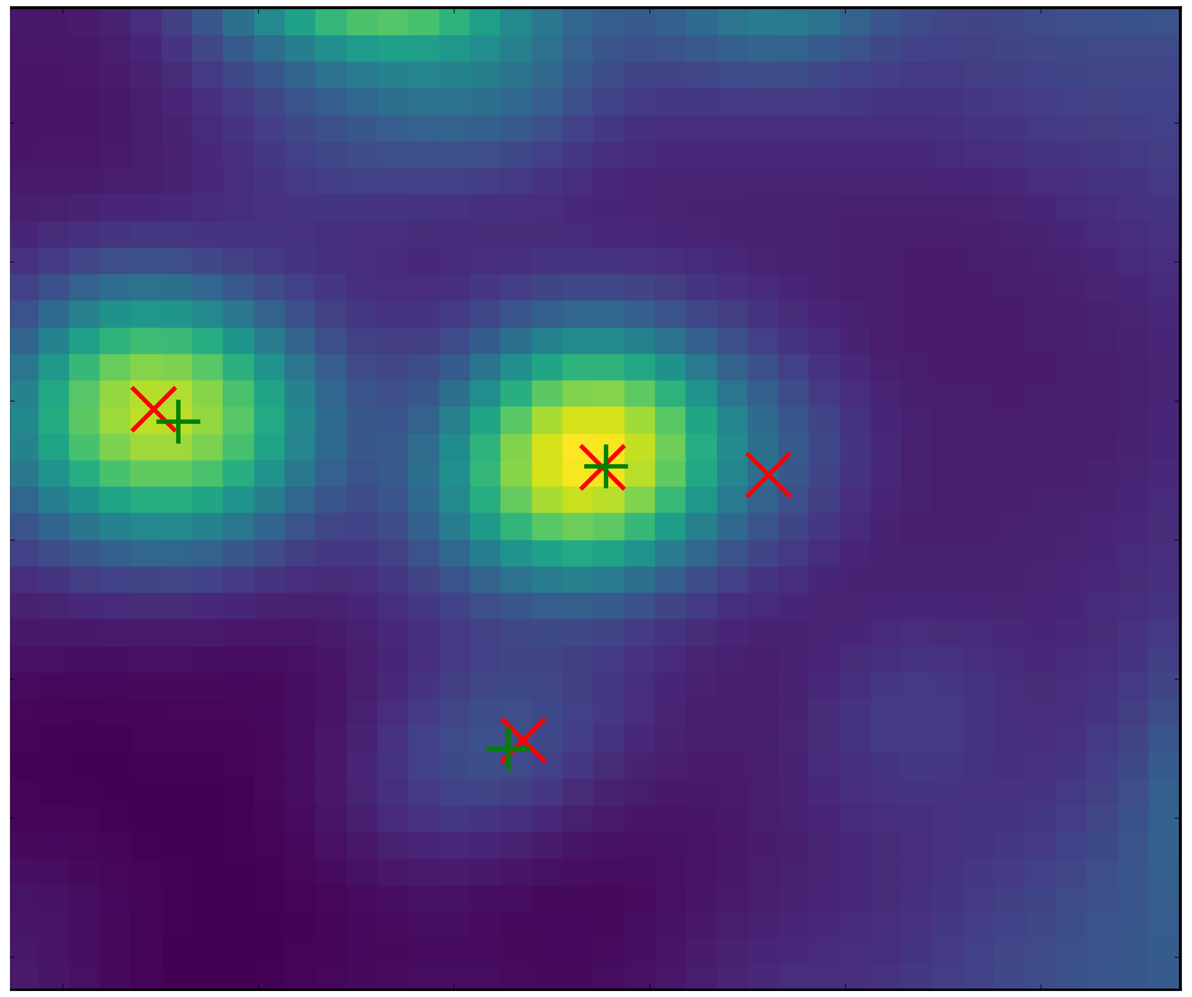
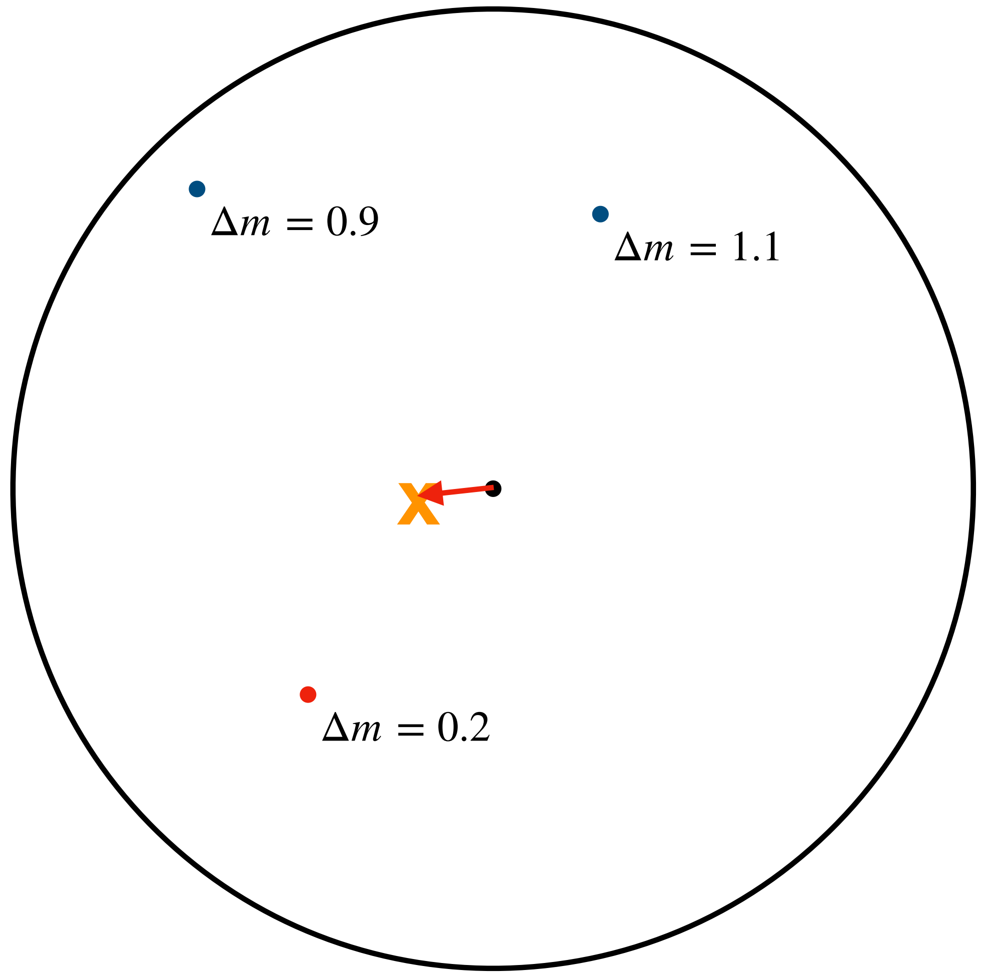
Building Empirical AUFs: Perturbation

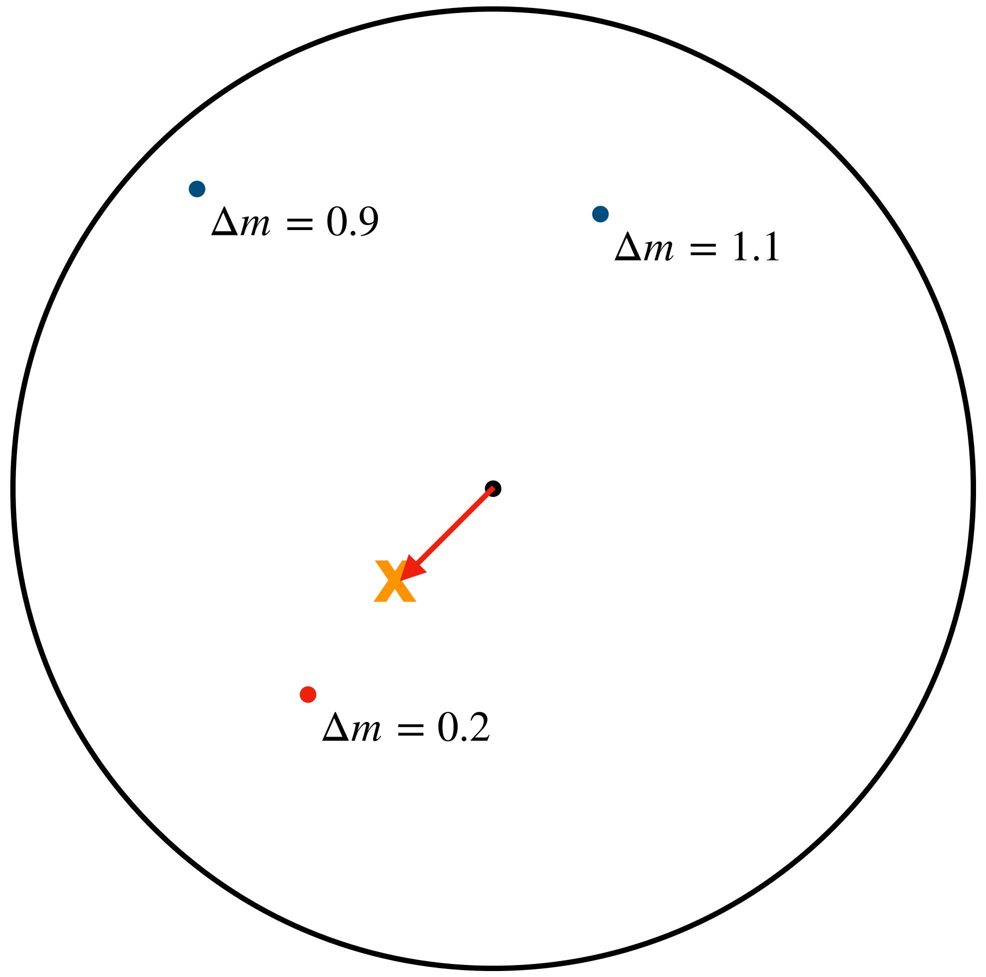
High SNR PSF or Aperture Photometry

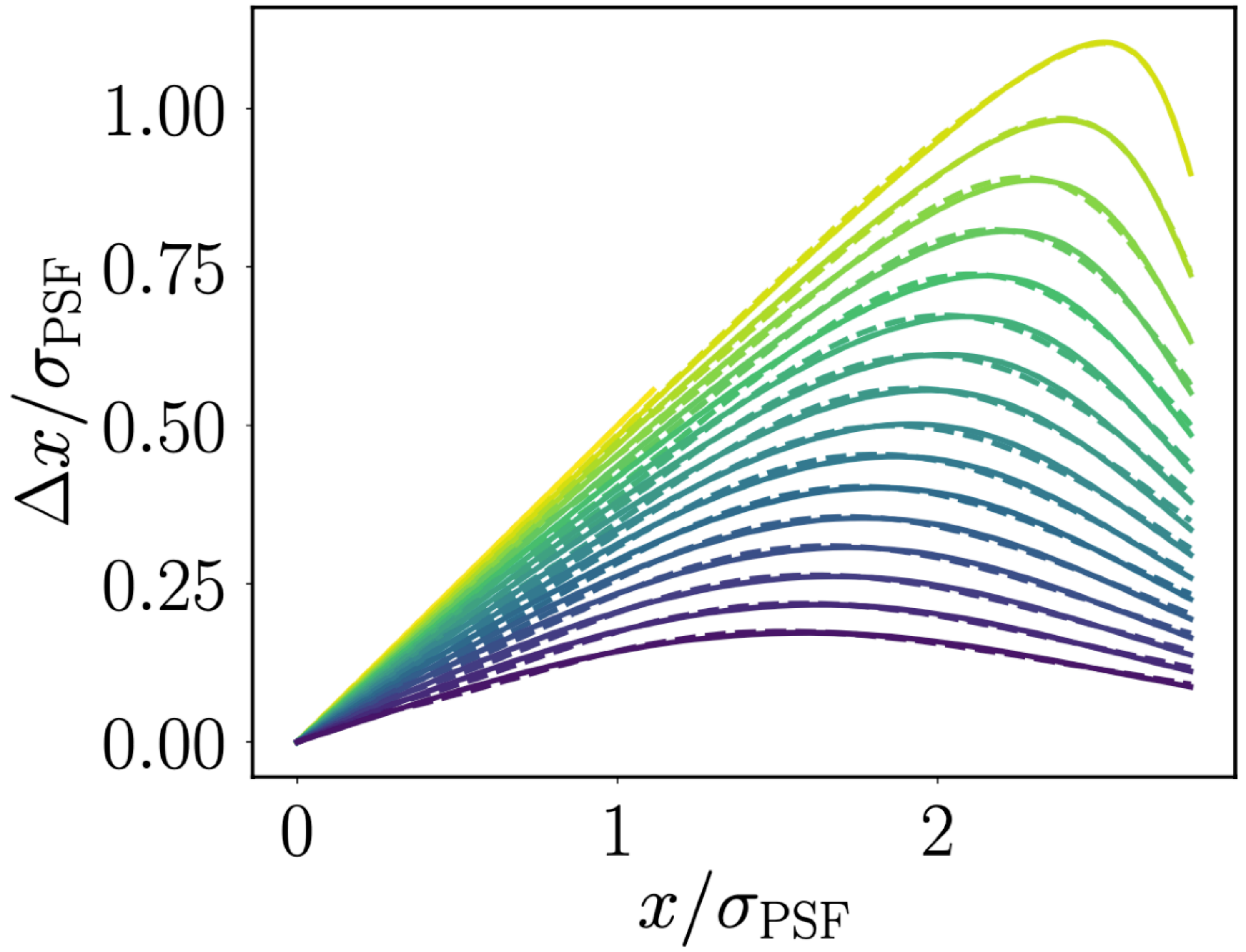




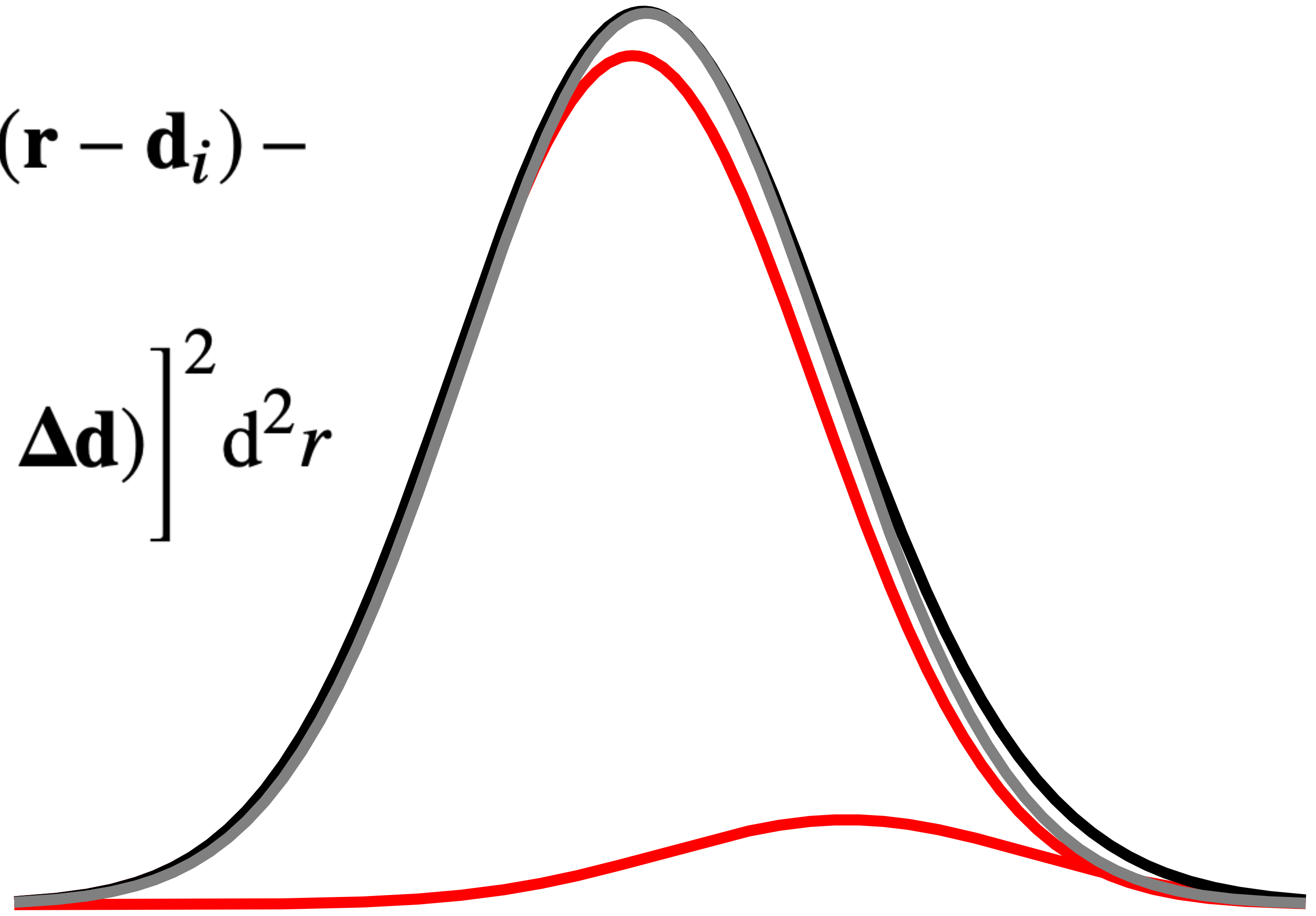
$$\Delta x = \frac{\sum_i f_i x_i}{1 + \sum_i f_i}$$

Low SNR PSF Photometry





$$\log \mathcal{L} = -\frac{1}{2} \times L \int_{-\infty}^{\infty} \left[\phi(\mathbf{r}) + \sum_i f_i \phi(\mathbf{r} - \mathbf{d}_i) - (1 + \Delta f)\phi(\mathbf{r} - \boldsymbol{\Delta}\mathbf{d}) \right]^2 \mathrm{d}^2 r$$



Wilson & Naylor (2018b, in prep.)
Plewa & Sari (2018)

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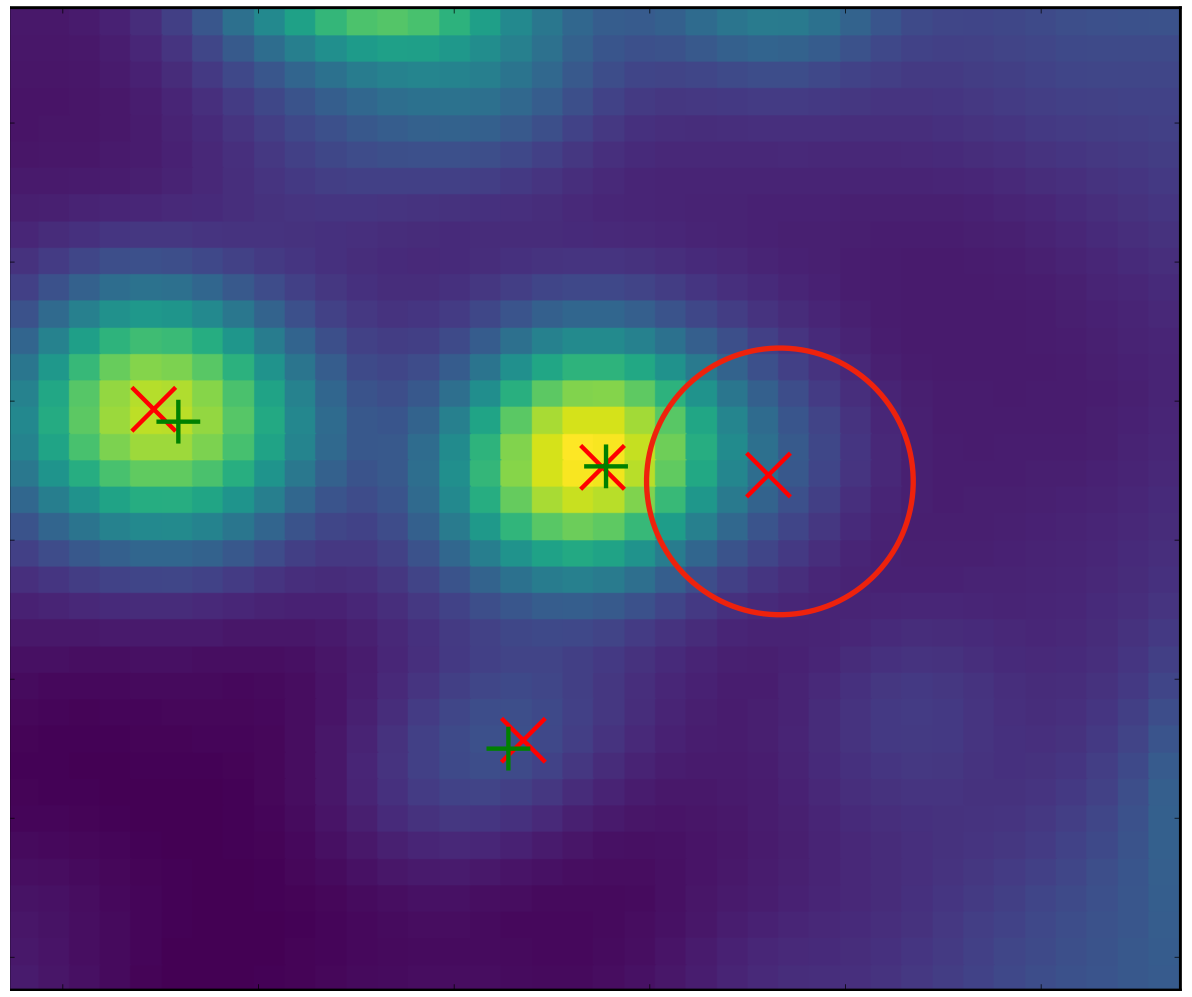
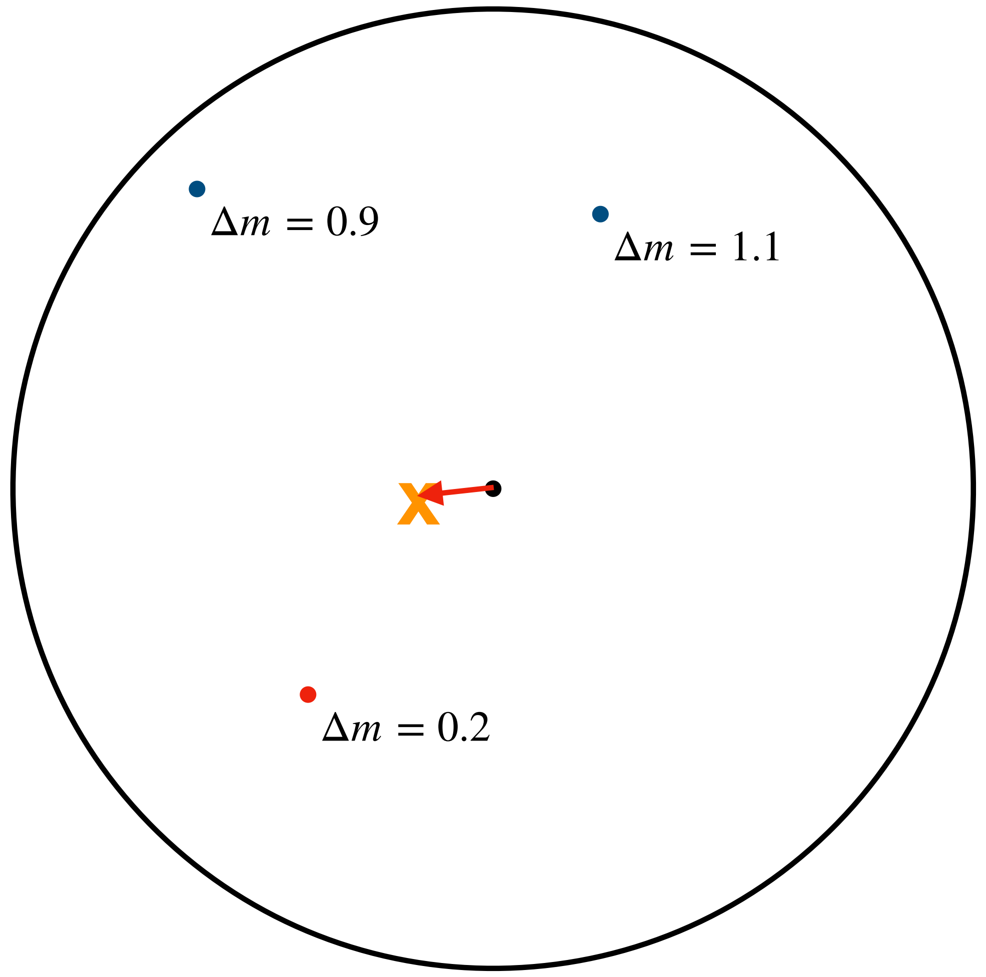
Perturbation AUF Component: Flux Brightening

High SNR PSF or Aperture Photometry



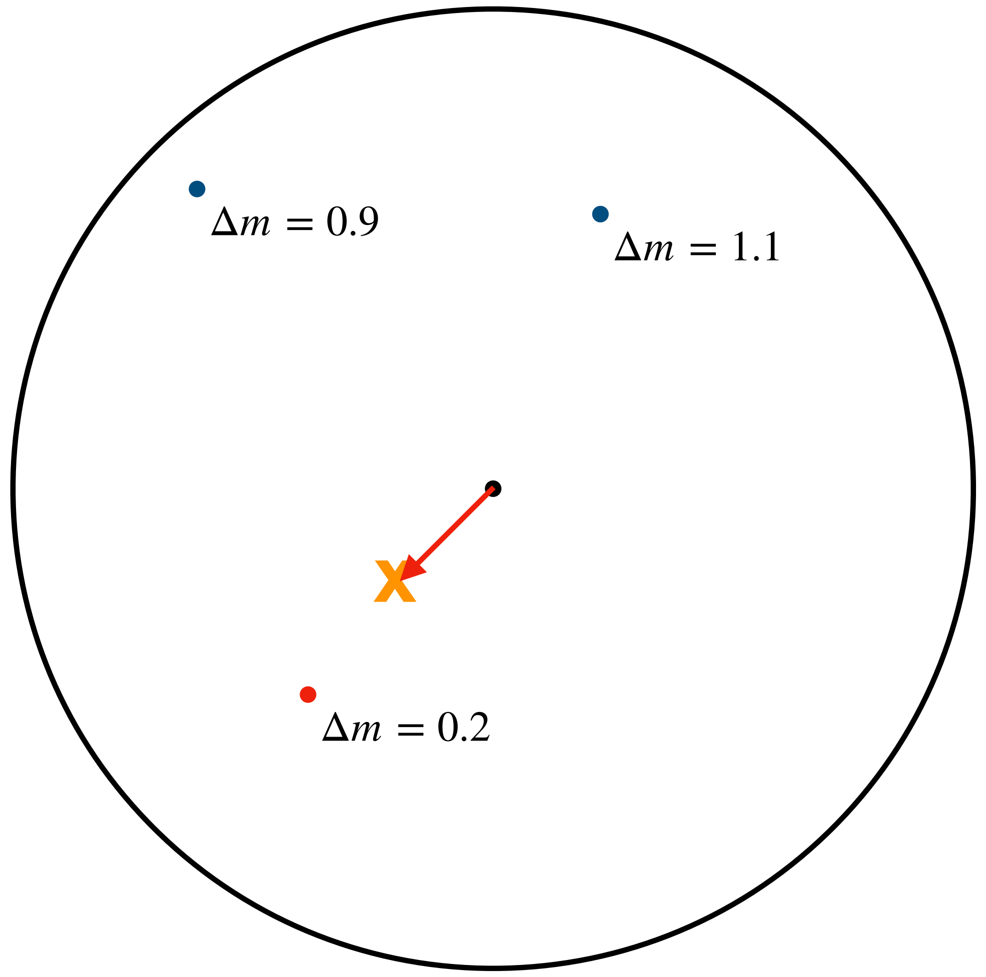


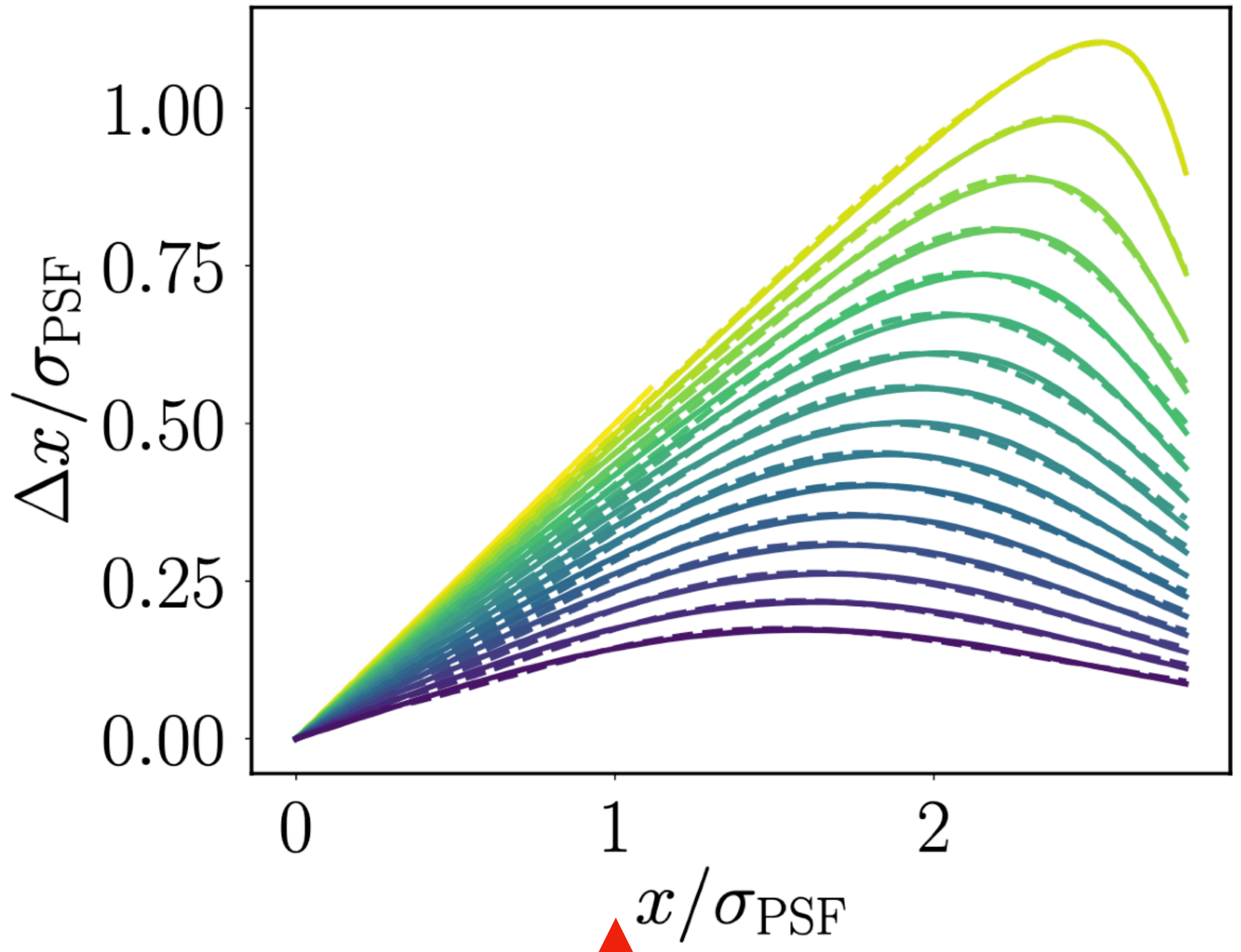
$$\Delta x = \frac{\sum_i f_i x_i}{1 + \boxed{\sum_i f_i}}$$

$$\Delta f = \sum_i f_i$$

(This raises questions about the validity of quoting photometric statistical precisions if objects are systematically biased)

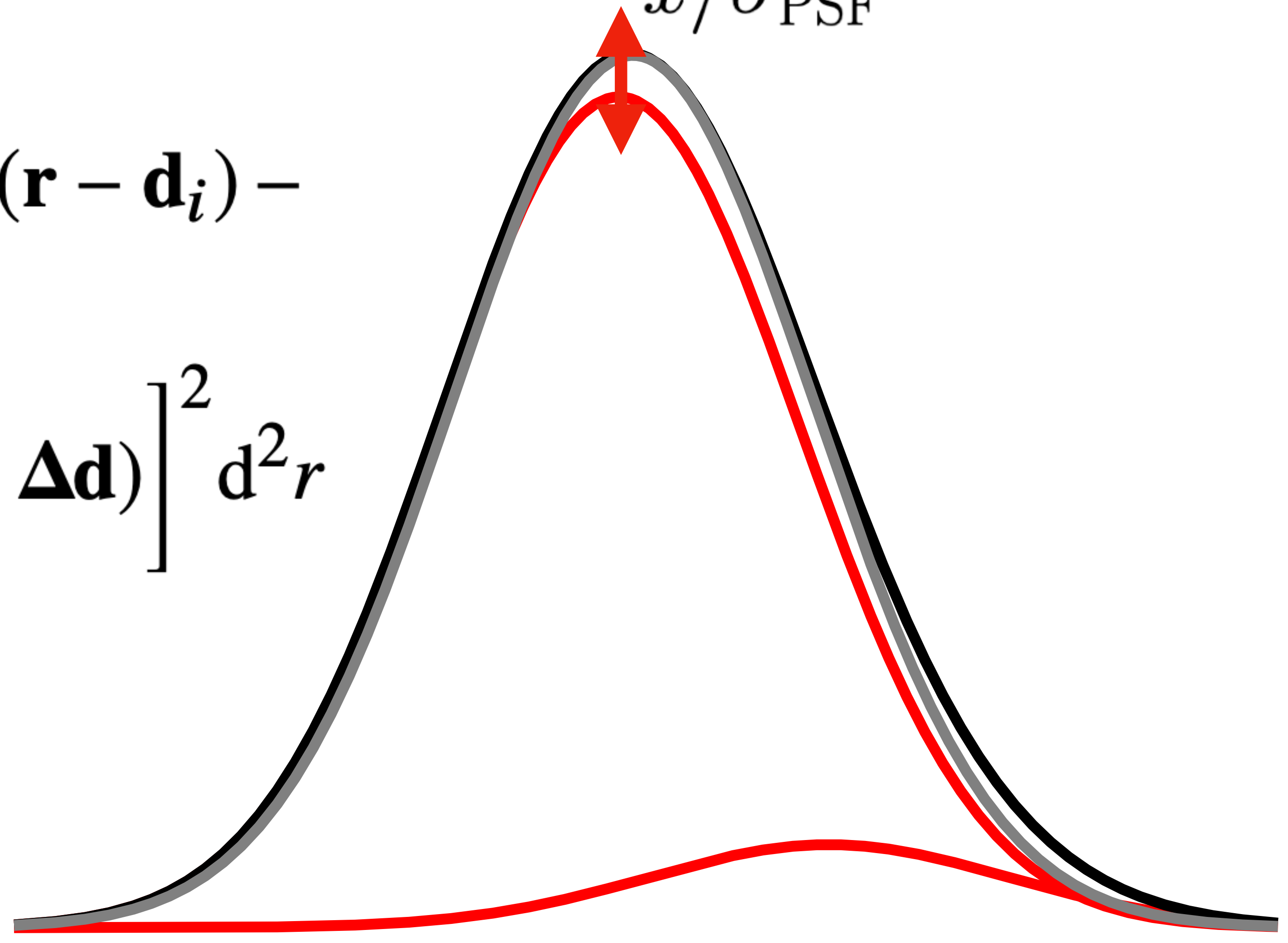
Low SNR PSF Photometry





$$\log \mathcal{L} = -\frac{1}{2} \times L \int_{-\infty}^{\infty} \left[\phi(\mathbf{r}) + \sum_i f_i \phi(\mathbf{r} - \mathbf{d}_i) - \boxed{(1 + \Delta f)}\phi(\mathbf{r} - \boldsymbol{\Delta}\mathbf{d}) \right]^2 \mathrm{d}^2 r$$

$$\Delta f = \psi'(\boldsymbol{\Delta}\mathbf{d}) - 1 + \sum_i f_i \psi'(\mathbf{d}_i - \boldsymbol{\Delta}$$



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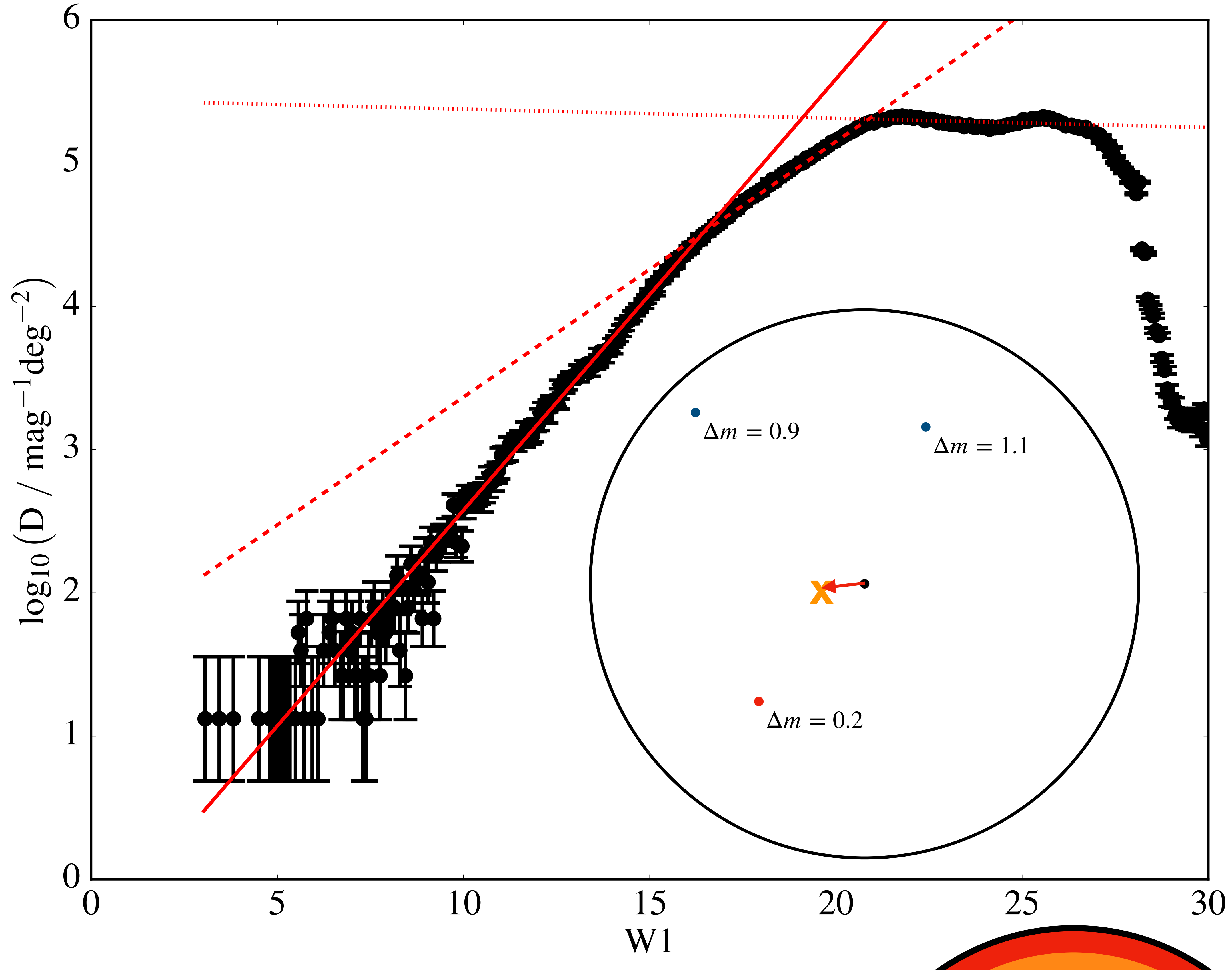
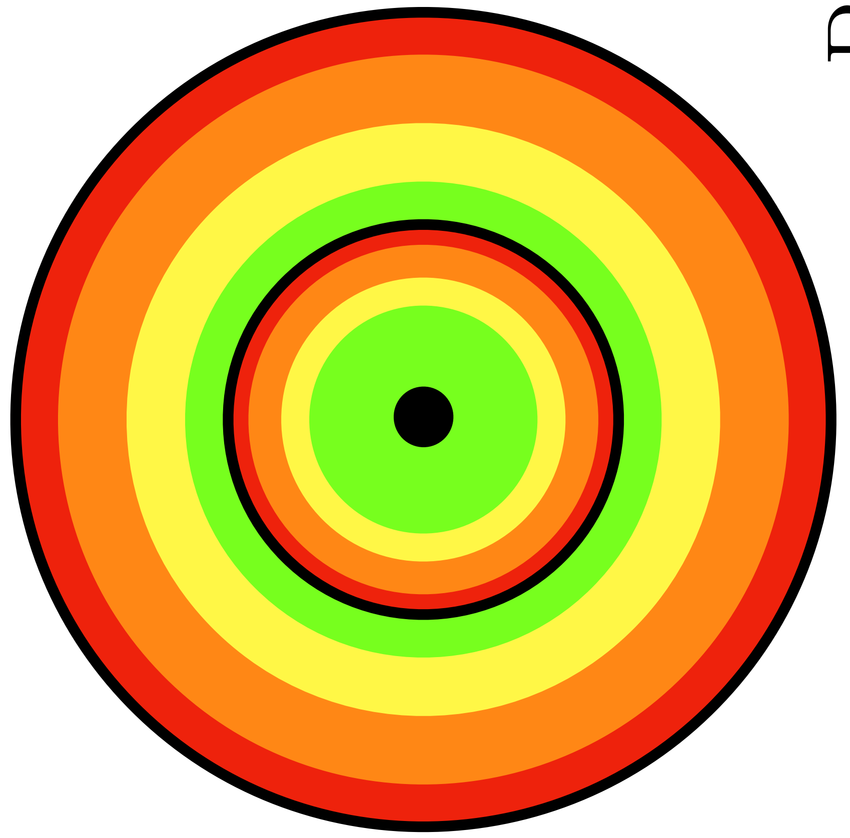
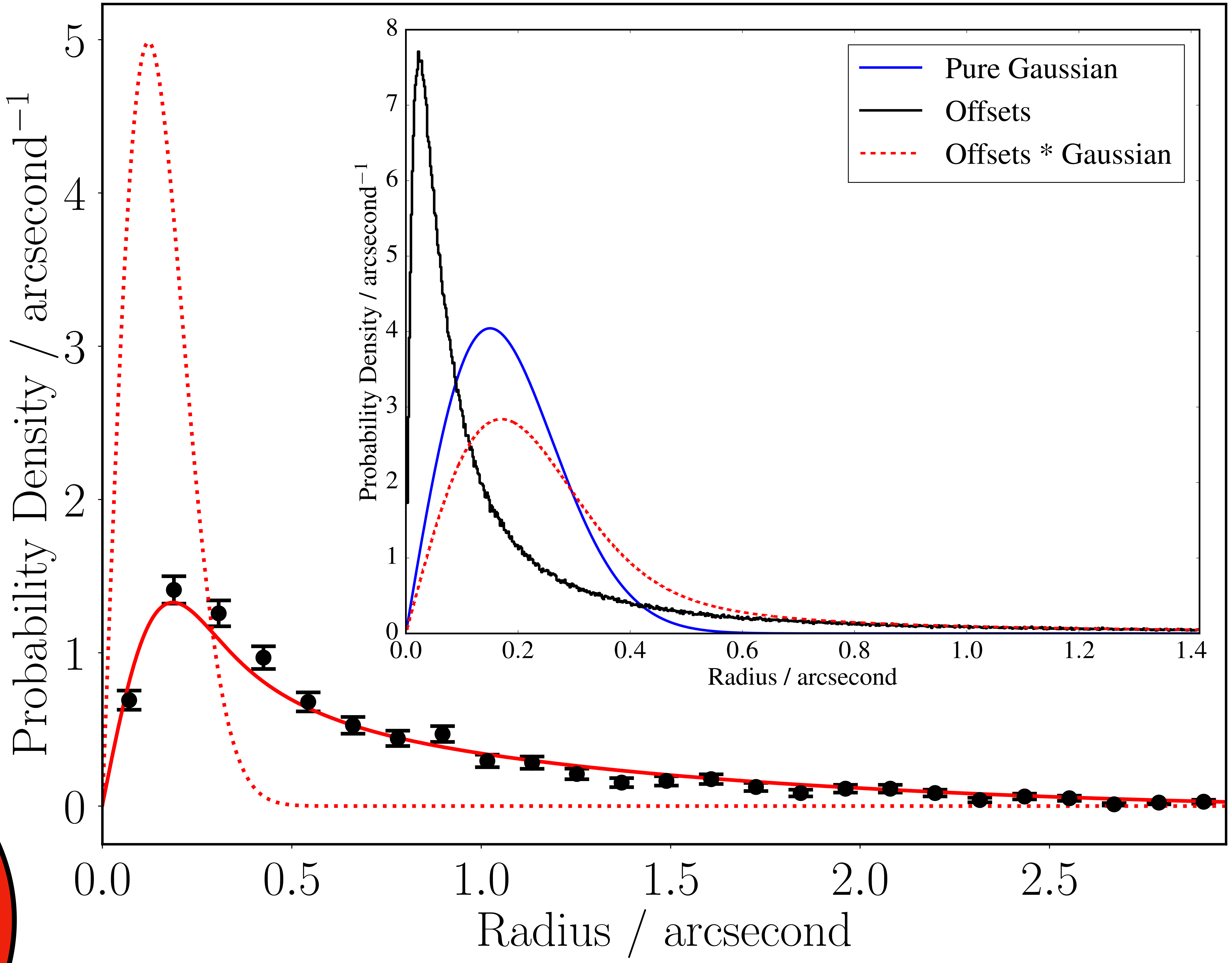
Building Empirical AUFs: Perturbation







Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)
WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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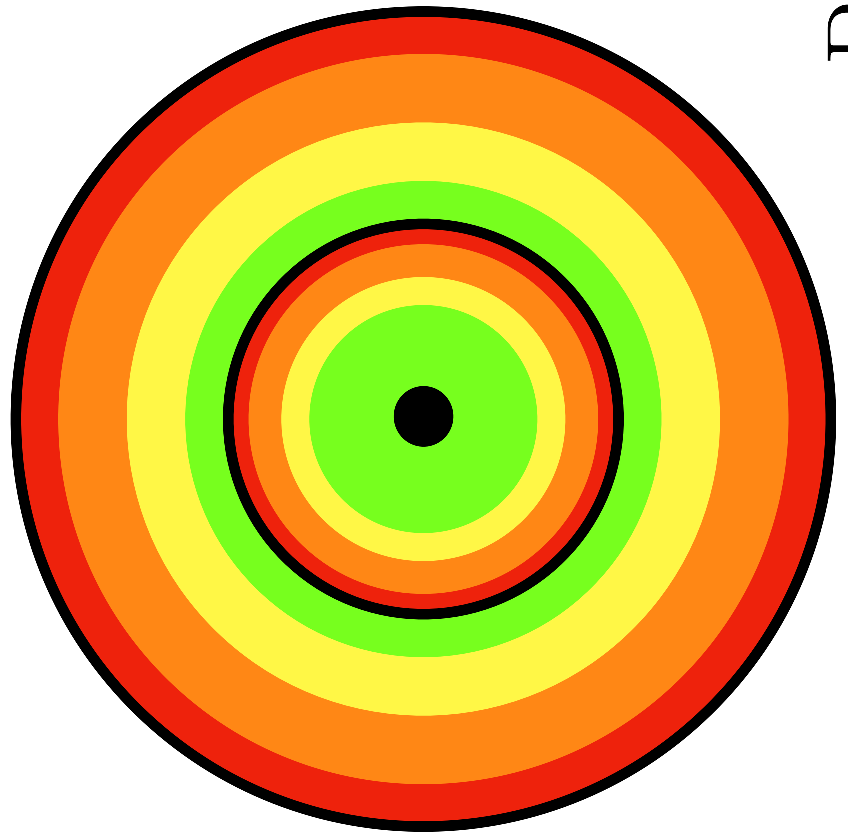
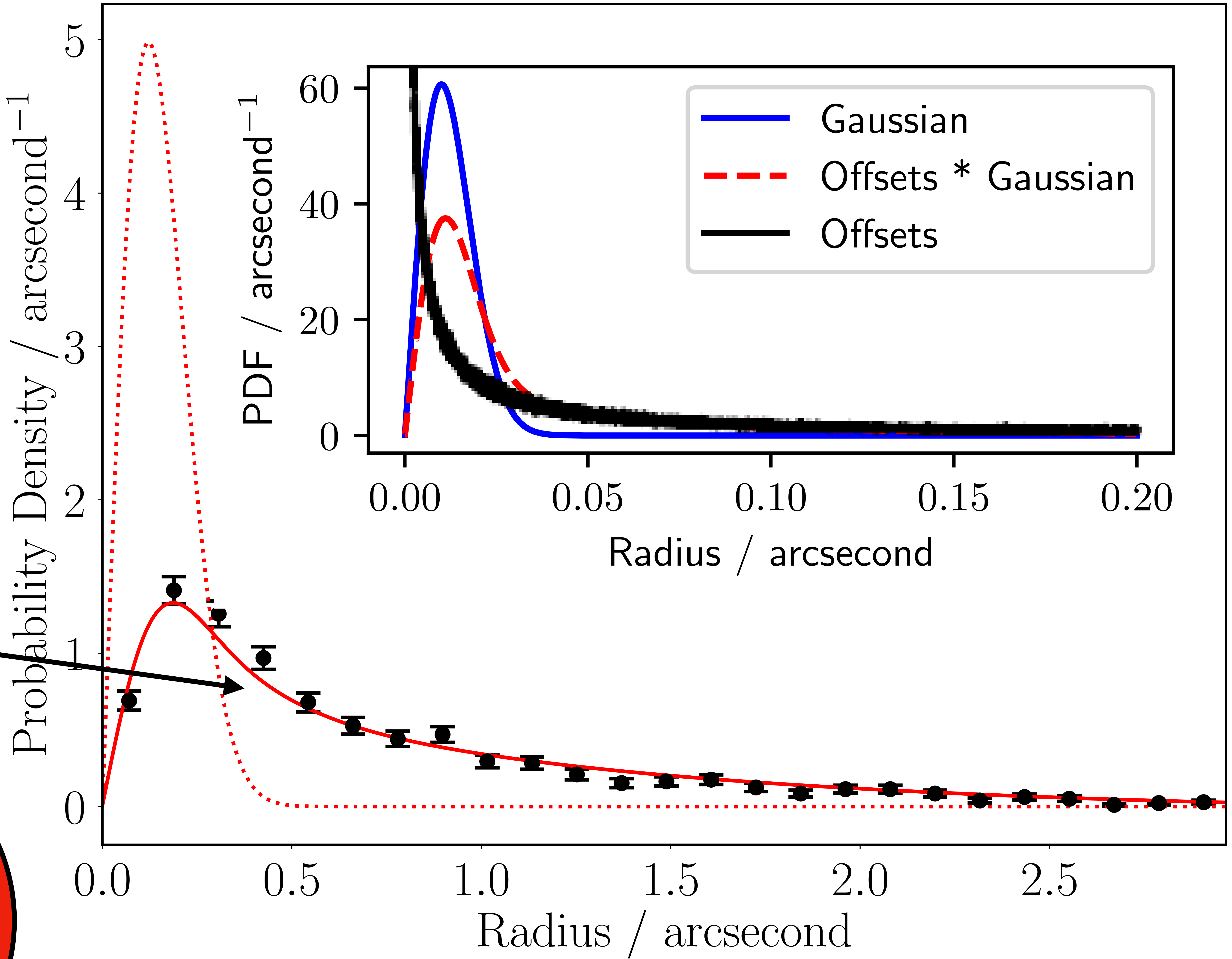
Building Empirical AUFs: Perturbation

LSST will suffer approximately the same number of unresolved contaminants per PSF area as *WISE*! The Perturbation component of the AUF will overwhelm the Centroid component for most of the Galactic Plane.





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Conclusions

- A generalised approach to the Astrometric Uncertainty Function, extending source position uncertainty from just noise-based centroid precision, allows for inclusion of a multitude of effects, such as perturbation due to blended sources, uncorrected DCR, or unknown proper motions.

- Unknown or poorly-constrained proper motion modelling is possibly needed near and below LSST single-visit magnitudes.
- DCR corrections as a component of the AUF may be useful for some cases, but ideally this is handled at the pipeline level from first principles! However, we are able to, and plan to, test this.
- However, the component of the AUF due to perturbation from unresolved, blended contaminant objects will be crucial for correctly understanding LSST astrometry — and hence any cross-matches to ancillary surveys — as it will be have impacts significantly larger than measured centroid precisions. Ironically, we suffer from having too precise a dataset!

- Will include additional information on the crowding of sources, allowing for selection of uncontaminated objects, or modelling of excess flux — crucial for removal of red excess in SEDs
  - LSST will suffer ~10% flux brightening, which could be confused with extinction, distance, …

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Wilson & Naylor, 2017, MNRAS, 468, 2517
Wilson & Naylor, 2018a, MNRAS, 473, 5570
Wilson & Naylor, 2018b, MNRAS, 481, 2148
Wilson, 2022, RNAAS, 6, 60
Wilson, 2023, RASTI, 2, 1
Wilson & Naylor (in prep.) — stay tuned for more AUF-related improvements!