Generalising the Astrometric Uncertainty Function in the Era of LSST

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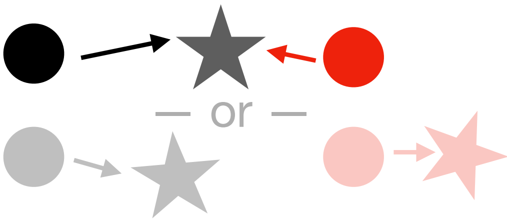
Crucial to the Bayesian cross-matching of photometric catalogues — the identification of sources detected in both (or all) datasets — is the maths describing the “counterpart likelihood”.



The probability that two sources in two catalogues are counterparts given the sky separation between them is the convolution of their respective *Astrometric Uncertainty Functions* (AUFs) (h).

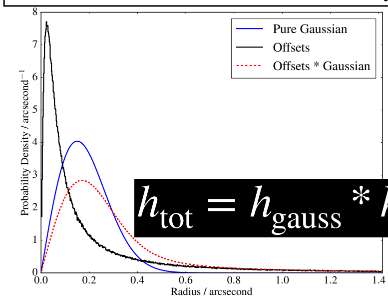
$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0 = N_\mathrm{c} \times (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl})$$

Wilson & Naylor (2018a)

This function is typically assumed to be Gaussian (e.g., Budavári & Szalay 2008; Naylor et al. 2013; Wolstencroft et al. 1986; Pineau et al. 2017), but it does not *need* to be.

$$B = \frac{2}{\sigma_1^2 + \sigma_2^2} \exp\left[-\frac{\psi^2}{2(\sigma_1^2 + \sigma_2^2)}\right] \qquad e^{-0.5(r/\sigma_{39})^2}$$

$$dp_\mathrm{id} = Qr \exp\left(\frac{-r^2}{2}\right) dr. \qquad \frac{\exp\left\{-\frac{1}{2}\sum_{i=1}^{n} Q_i(\boldsymbol{p})\right\}}{(2\pi)^n \prod_{i=1}^{n} \sqrt{\det \boldsymbol{V}_i}} \mathrm{d}\boldsymbol{p}$$



$$h_\mathrm{tot} = h_\mathrm{gauss} * h_\mathrm{perturb} * h_\mathrm{pm}$$

Removing the restriction of Gaussian AUFs allows for the inclusion of components other than the astrometric position centroiding in the AUF, such as perturbations due to blended sources, or unknown proper motions (Wilson & Naylor 2018b).

As these additional AUF terms may not be analytic, we have to turn to non-analytic solutions for the fast and accurate calculation of these counterpart likelihoods, using convolution theorem.

$$F(\rho) = \mathscr{F}(f(x))$$

$$(f * g)(x) = \mathscr{F}^{-1}(F \cdot G)$$

$$G(\rho, \phi) = G(\rho) = 2\pi \int_0^\infty r\, g(r)\, J_0(2\pi r \rho)\, \mathrm{d}r$$

To reduce a two-dimensional integral to a single dimension, we can use the Hankel (Fourier-Bessel) Transform.

Advantages:

- Allows for the generalisation of the AUF
- Non-centroid AUF components crucial for crowded or faint fields
- Hankel Transform speeds up 2-D calculation

Disadvantages:

- Numerical precision needs sufficient integral resolution
- Requires all components of the AUF to be circular symmetric

The generalised AUF will be crucial for the faint, crowded LSST sky, much as it is for the *WISE* catalogue, suffering similar crowding at its completeness limit (Wilson & Naylor, 2017).