Solving the Catalogue Cross-Match Problem in the Era of Gaia: The Effect of Unresolved Contaminant Objects on Photometric Catalogues

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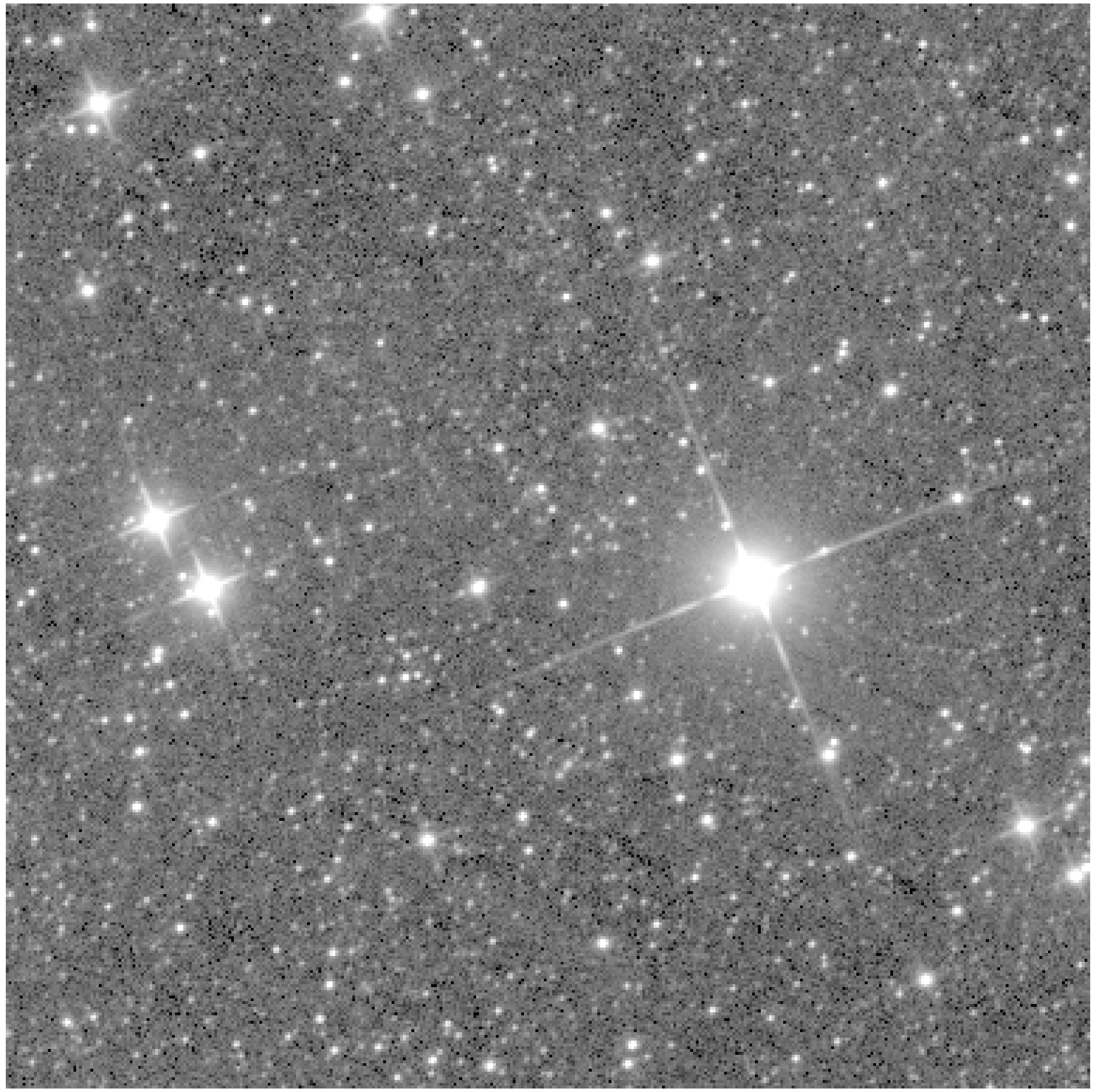
Photometric Observations



WISE - Wright et al. (2010)

WISE W1

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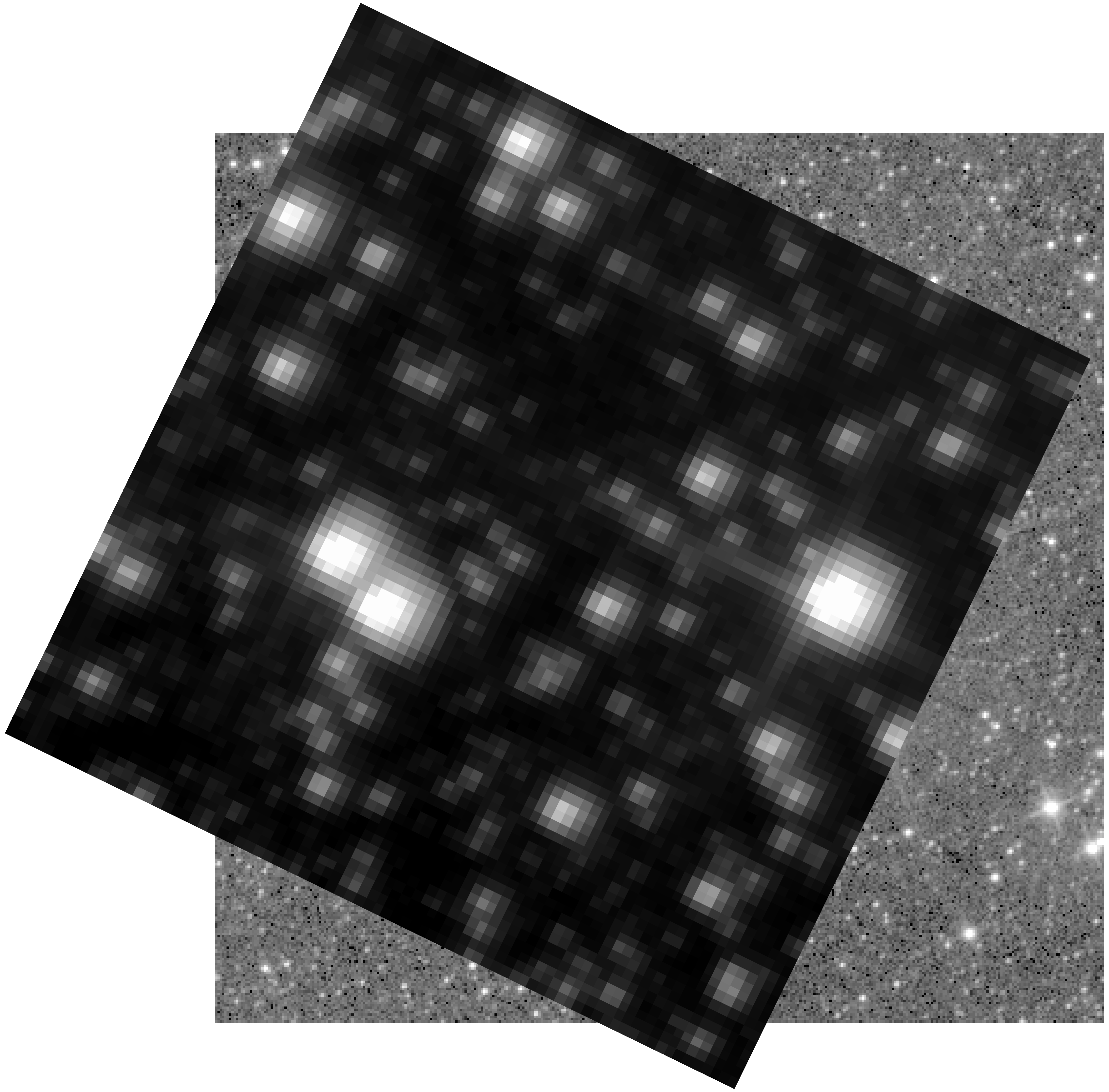

WISE - Wright et al. (2010)
TESS - Ricker et al. (2015)

TESS T

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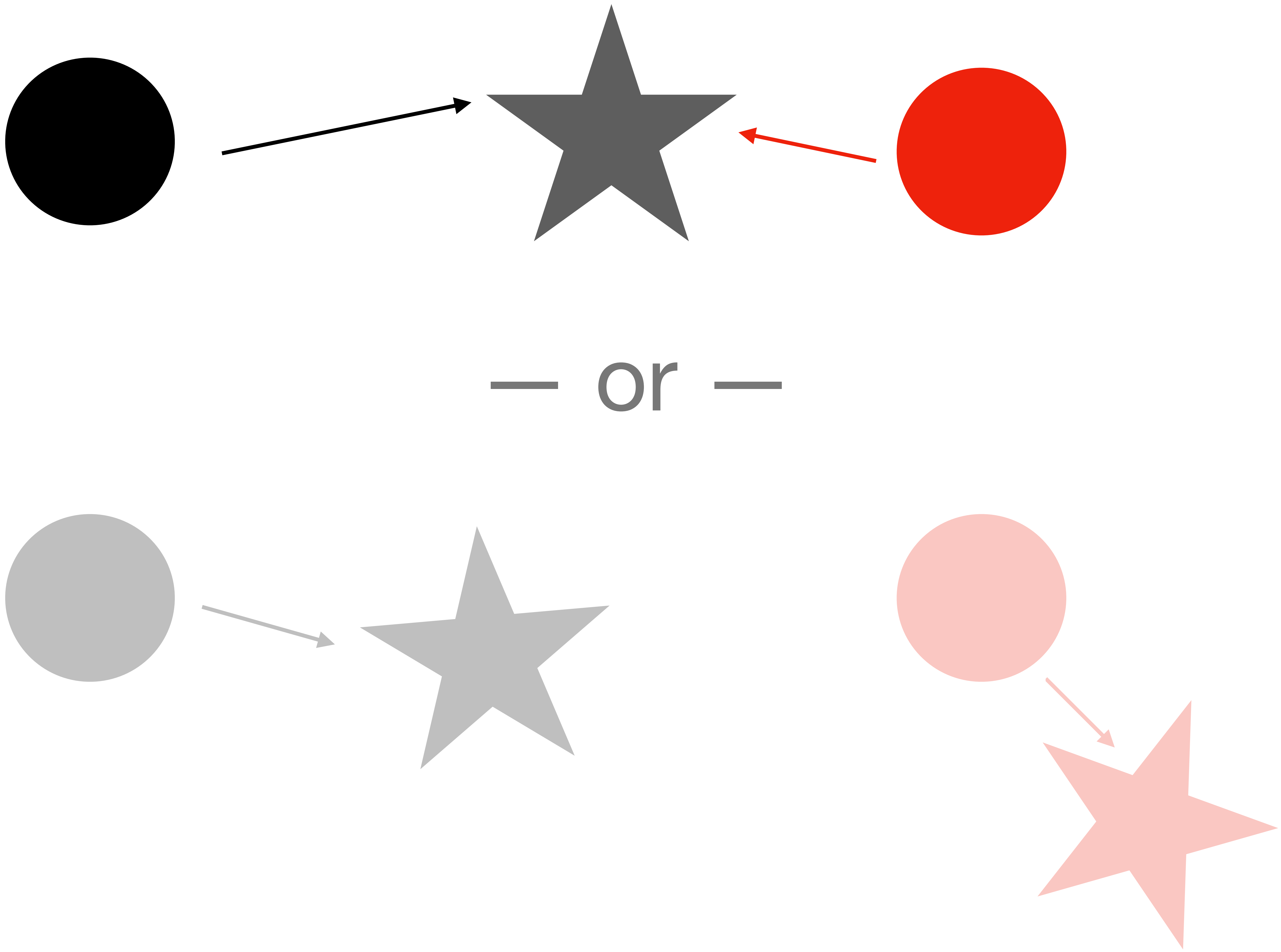
Counterpart Assignment



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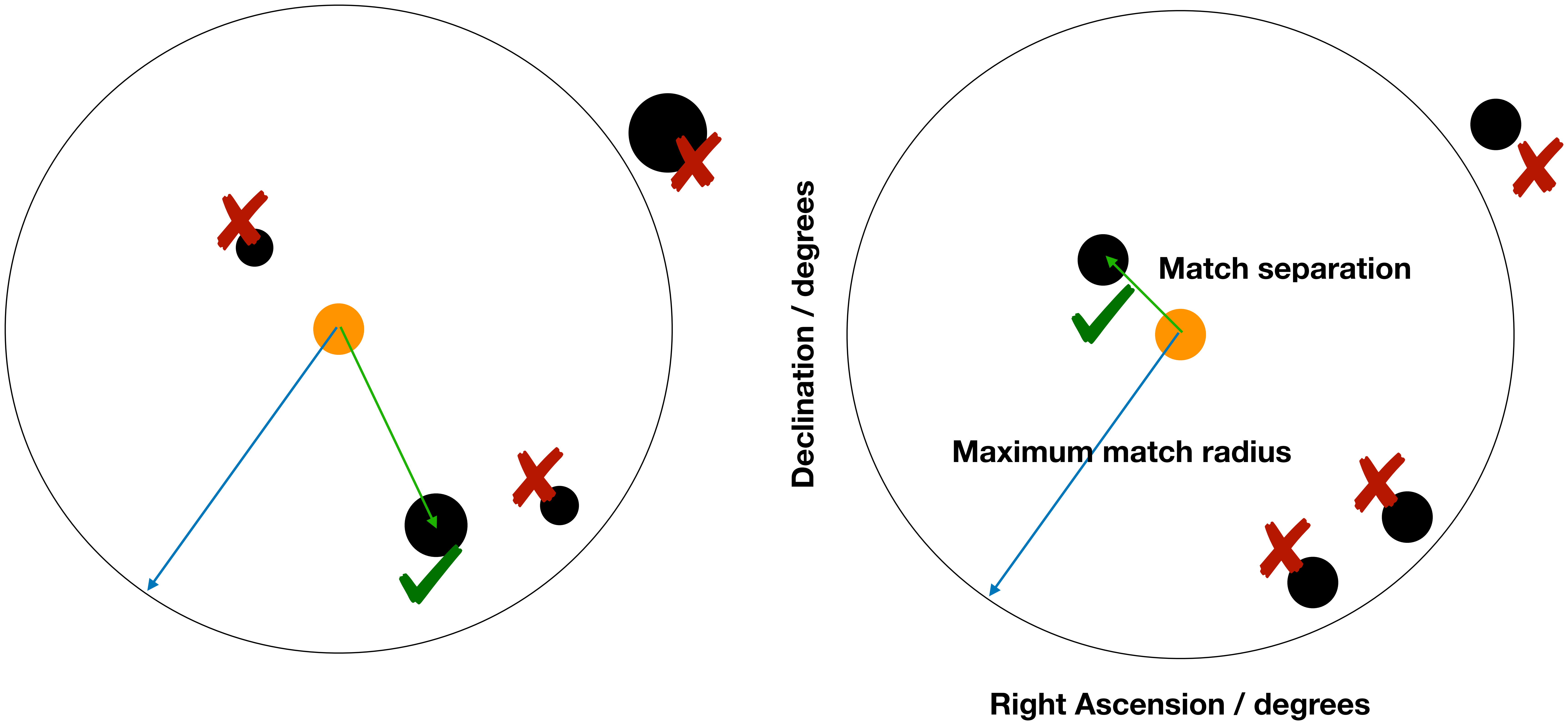
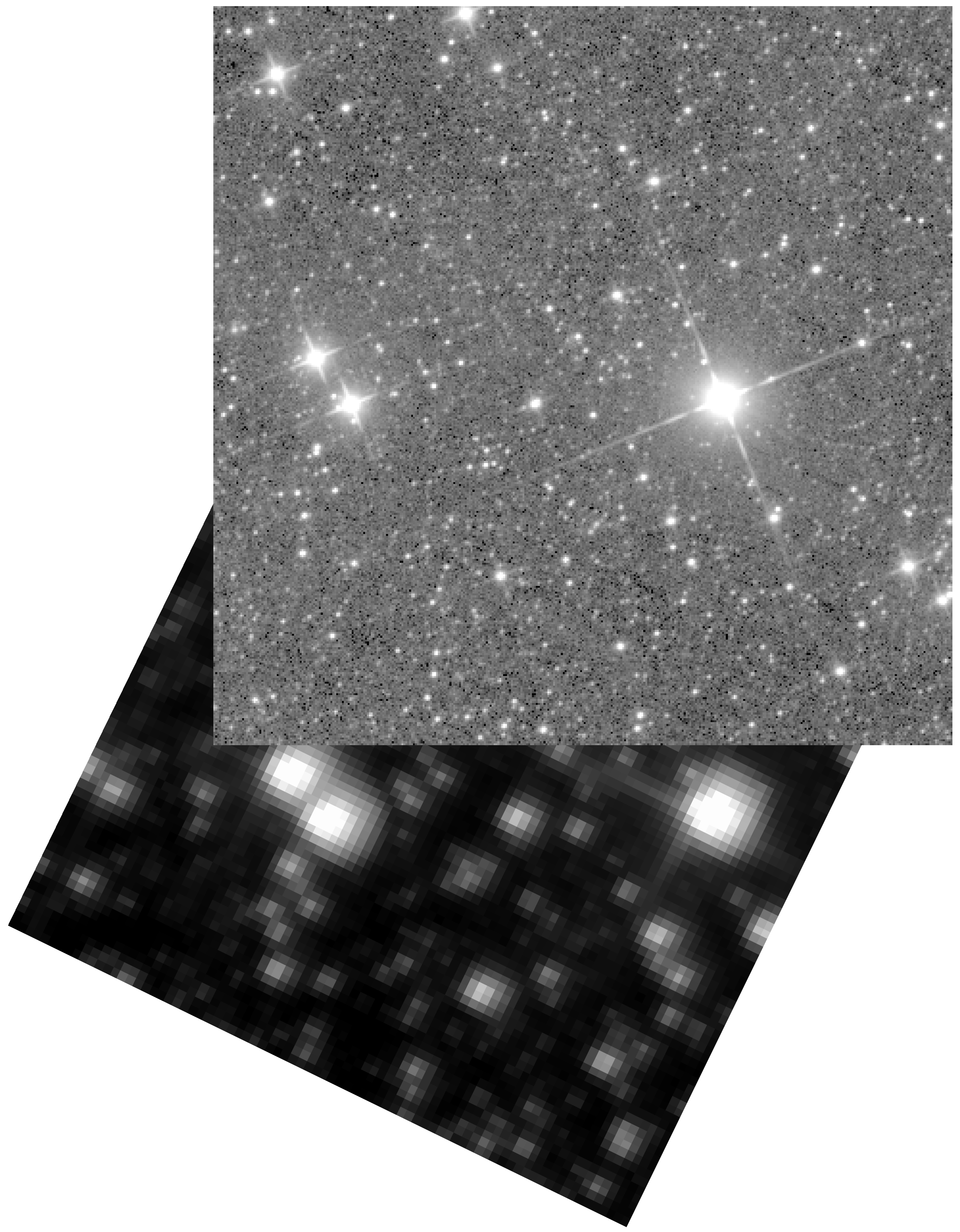
“Traditional” Cross-Matching





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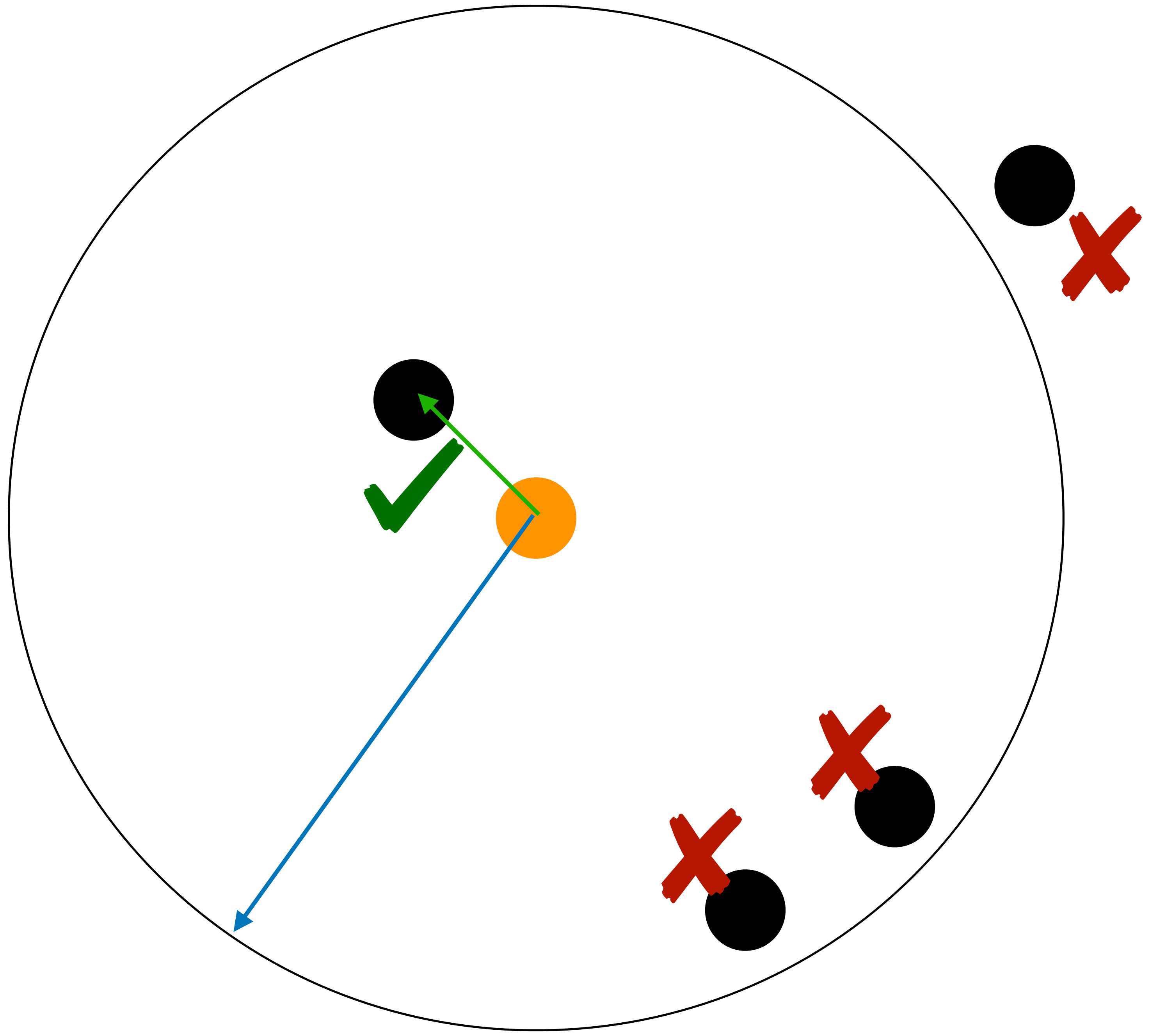
Probabilistic Cross-Matching



$$dp_{\mathrm{id}} = Qr\exp\left(\frac{-r^2}{2}\right)dr \quad dp_{\mathrm{uo}} = 2\lambda r\,dr$$

Wolstencroft et al. (1986)

$$LR(r) = \frac{dp_{\mathrm{id}}}{dp_{\mathrm{uo}}} = \frac{Q\exp(-r^2/2)}{2\lambda}$$

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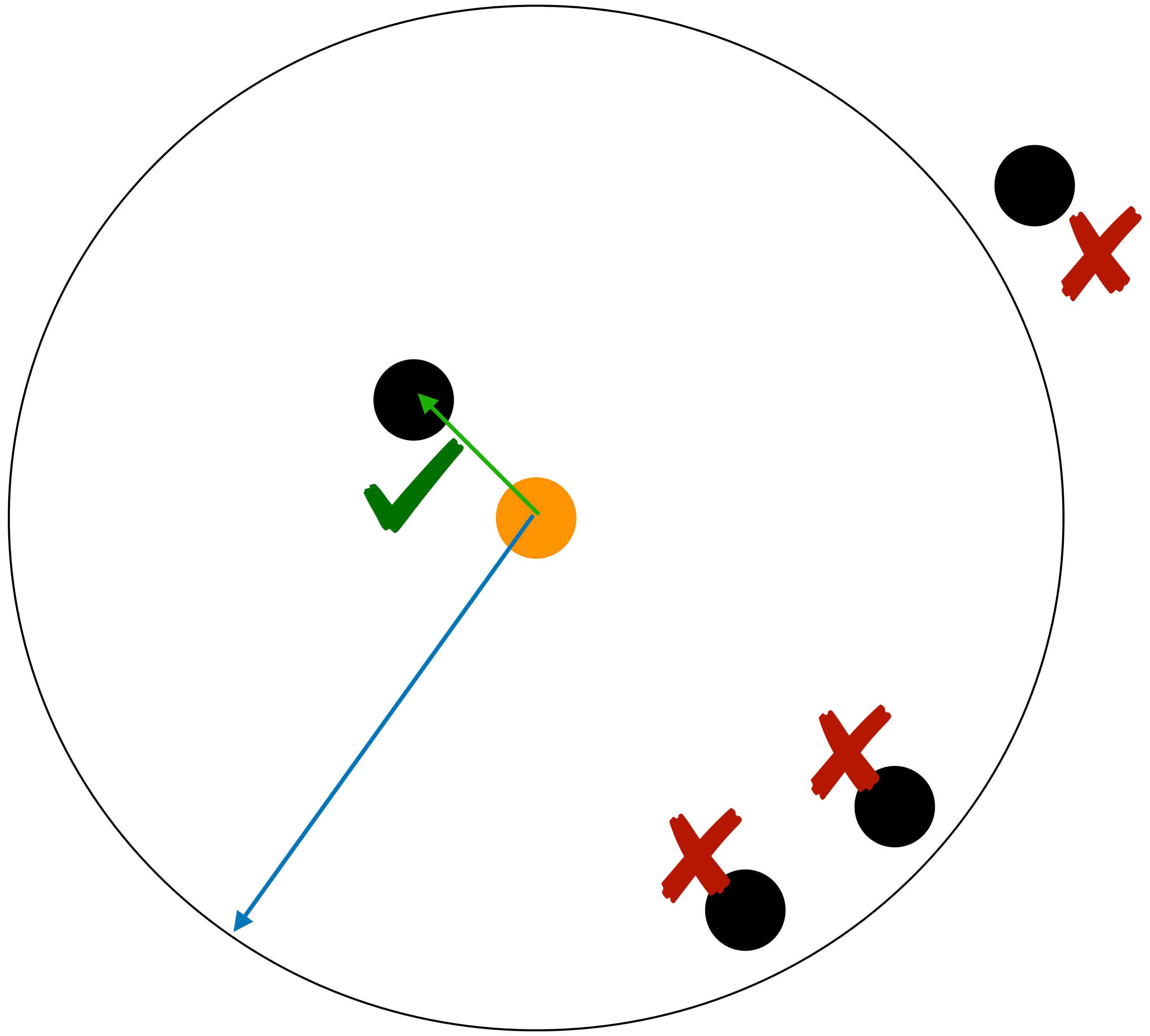

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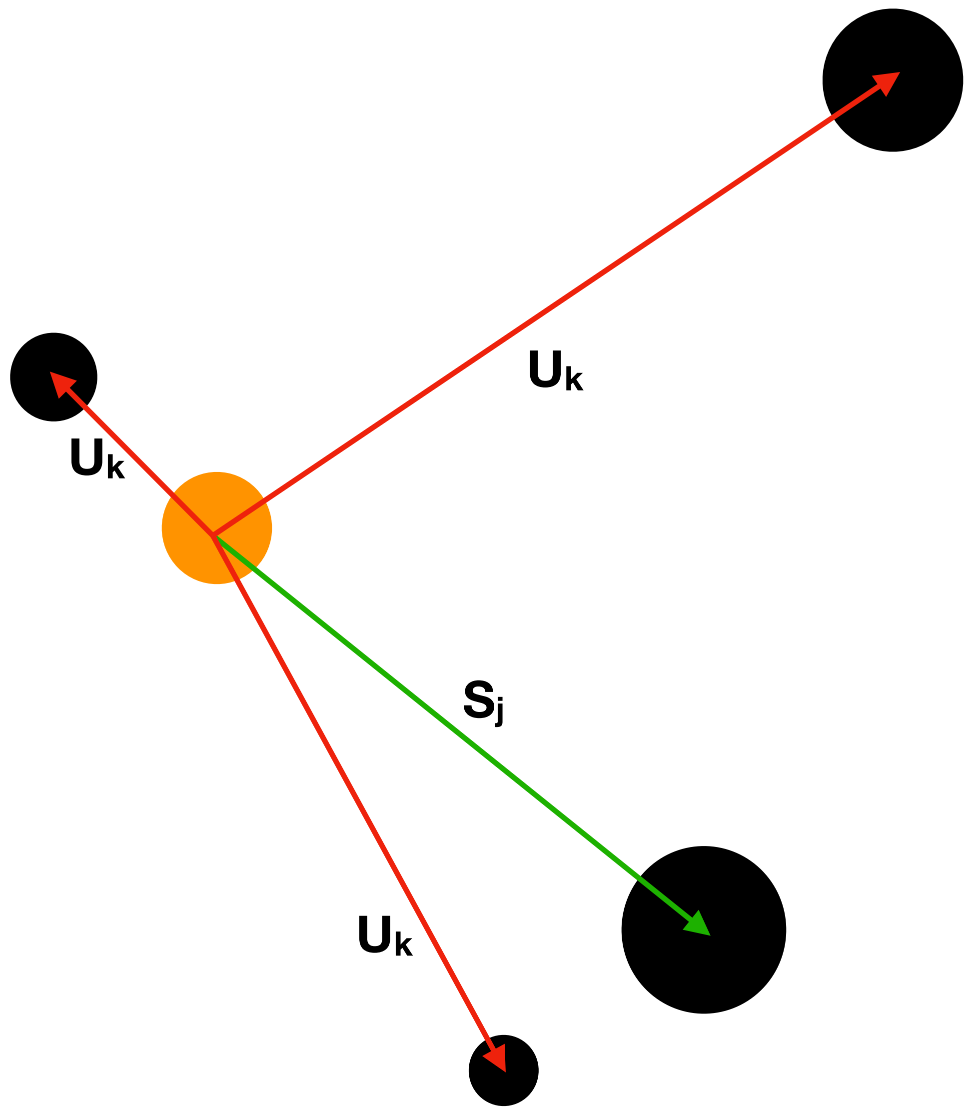
$$L = \frac{q(m,c)\,f(x,y)}{n(m,c)}$$

$$R_j = \frac{\Pr\left[S_j \cap \left(\bigcap_{k\neq j} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]}{\sum_i \Pr\left[S_i \cap \left(\bigcap_{k\neq i} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right] + \Pr\left[(m_S > m_{\lim}) \cap \left(\bigcap_{k} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]} = \frac{L_j}{\sum_i L_i + (1-Q)}$$

Sutherland & Saunders (1992)



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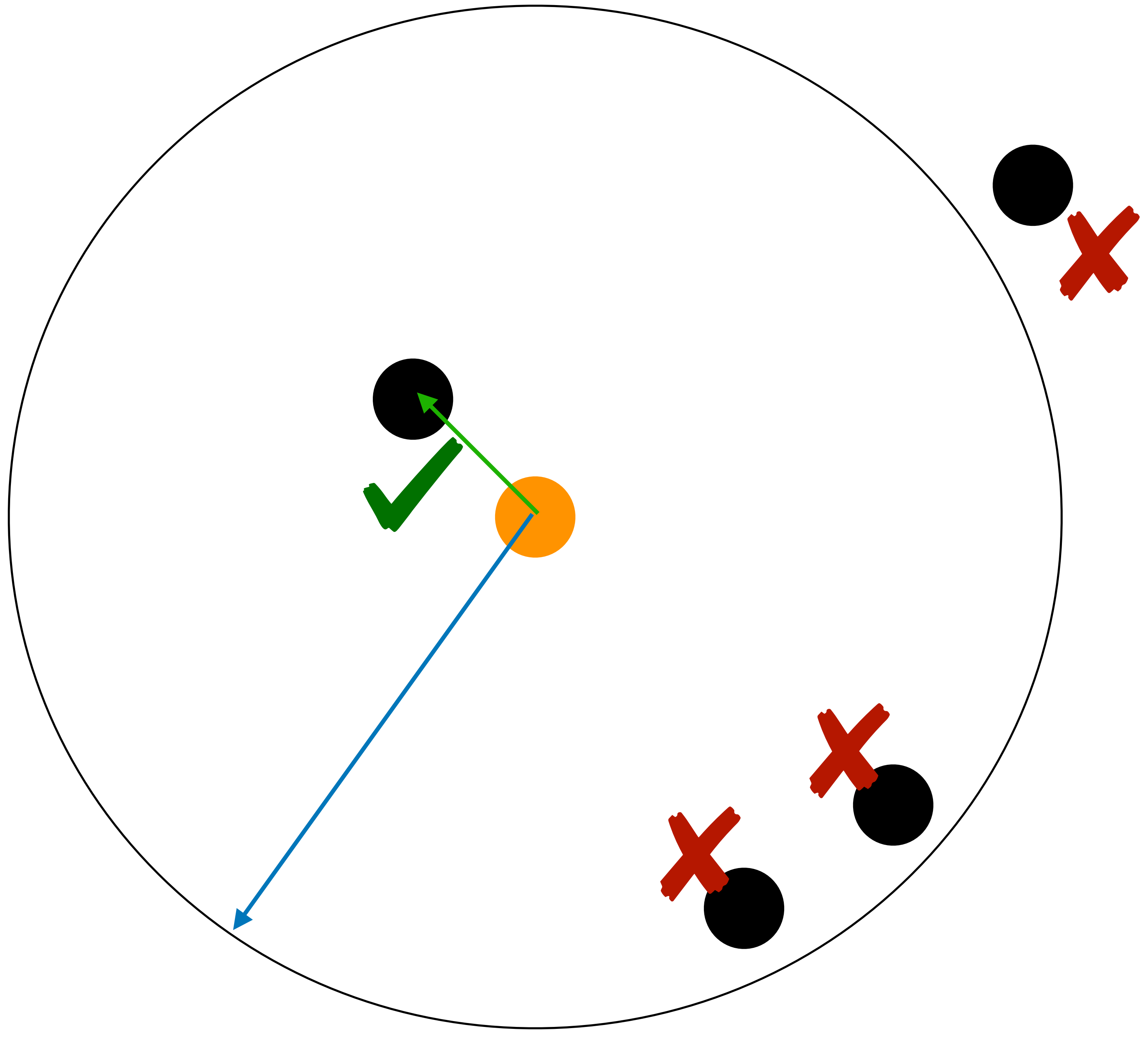

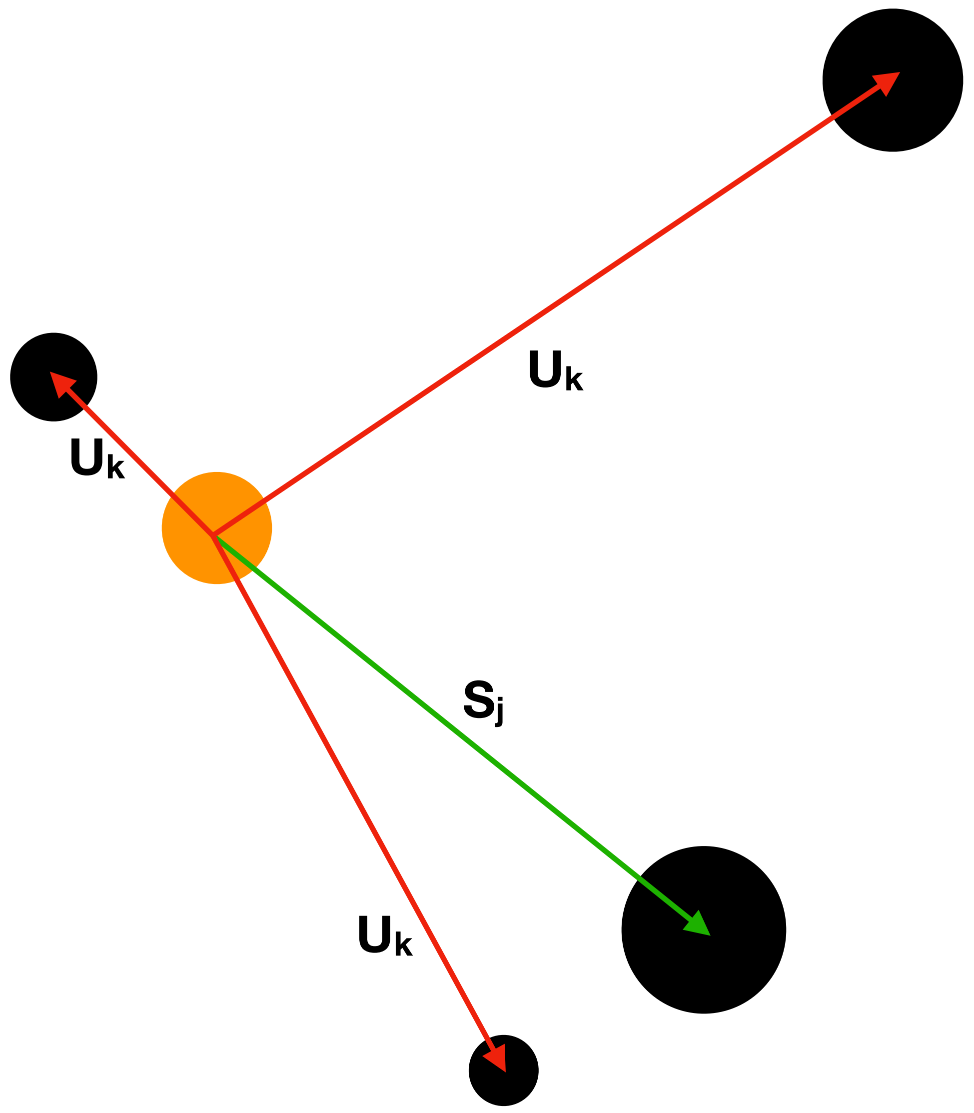

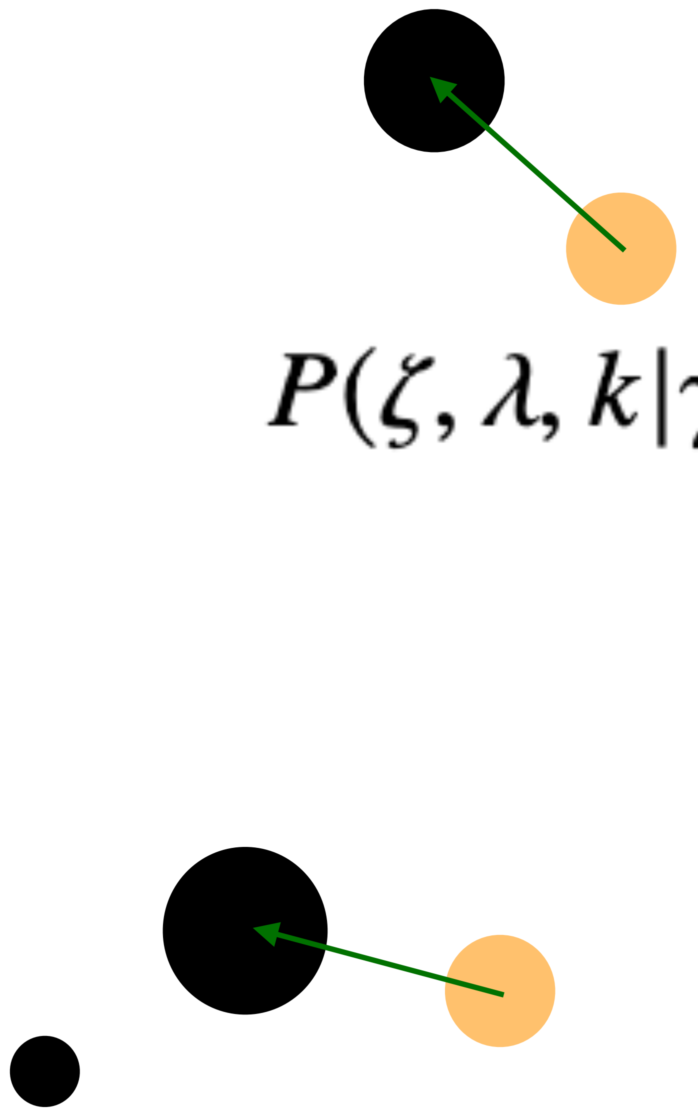

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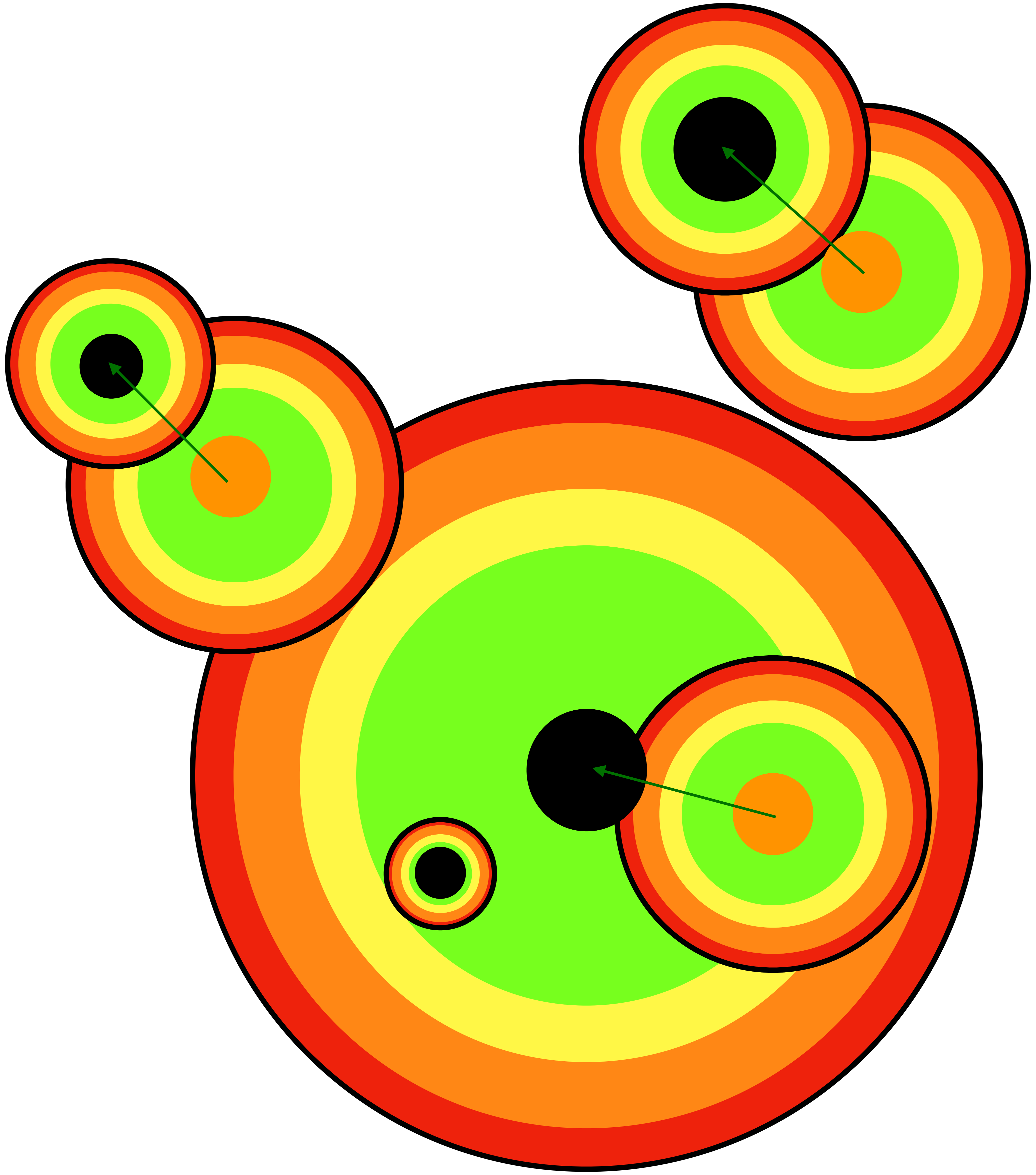

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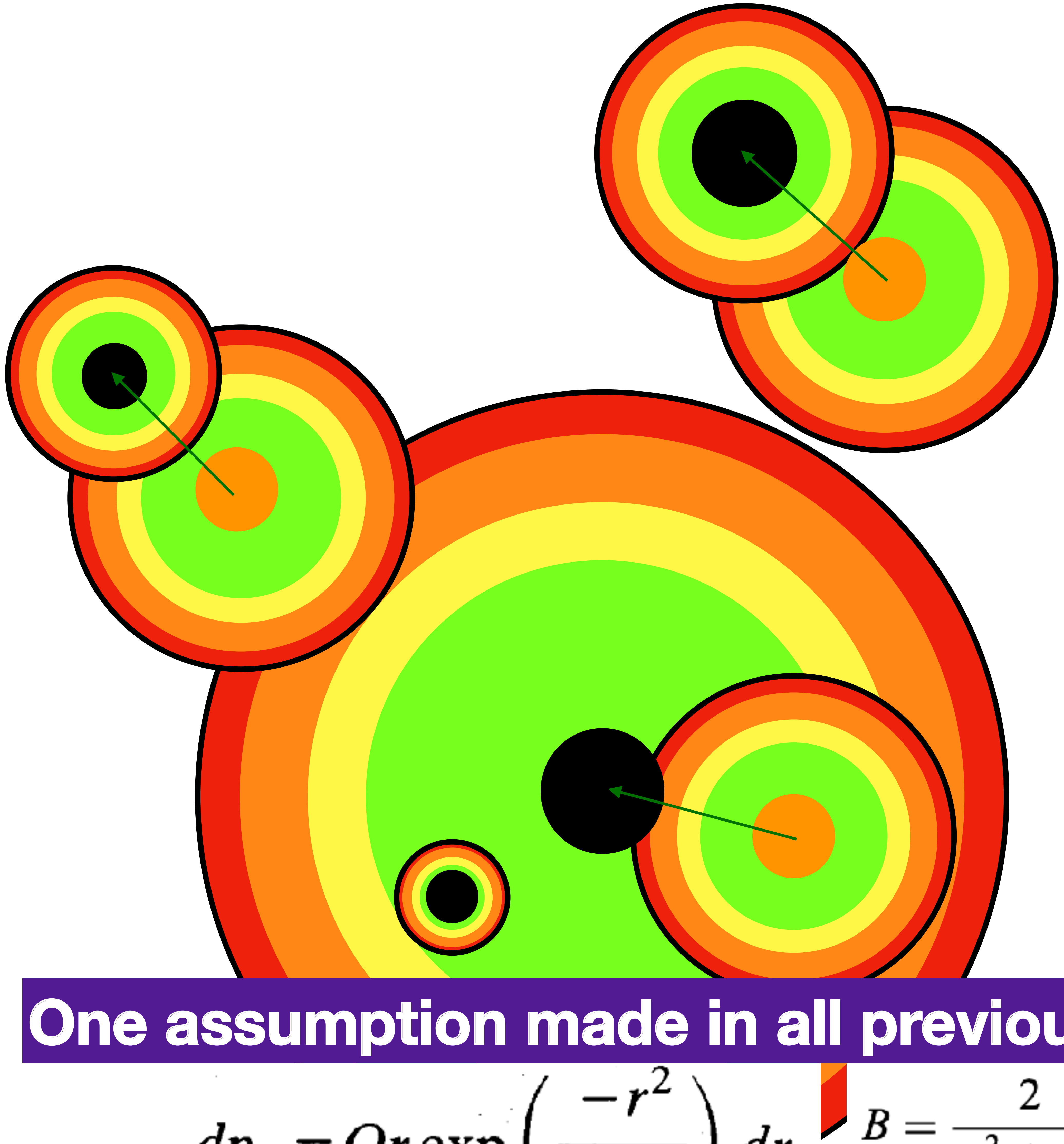

One assumption made in all previous work: positional errors of sources are Gaussian!

$$dp_{\mathrm{id}} = Qr \exp\left(\frac{-r^2}{2}\right) dr.$$

Wolstencroft et al. (1986)

$$B = \frac{2}{\sigma_1^2 + \sigma_2^2} \exp\left[-\frac{\psi^2}{2(\sigma_1^2 + \sigma_2^2)}\right]$$

Budavári & Szalay (2008)

$$e^{-0.5(r^2/\sigma_{39}^2)}$$

Naylor, Broos, & Feigelson (2013)

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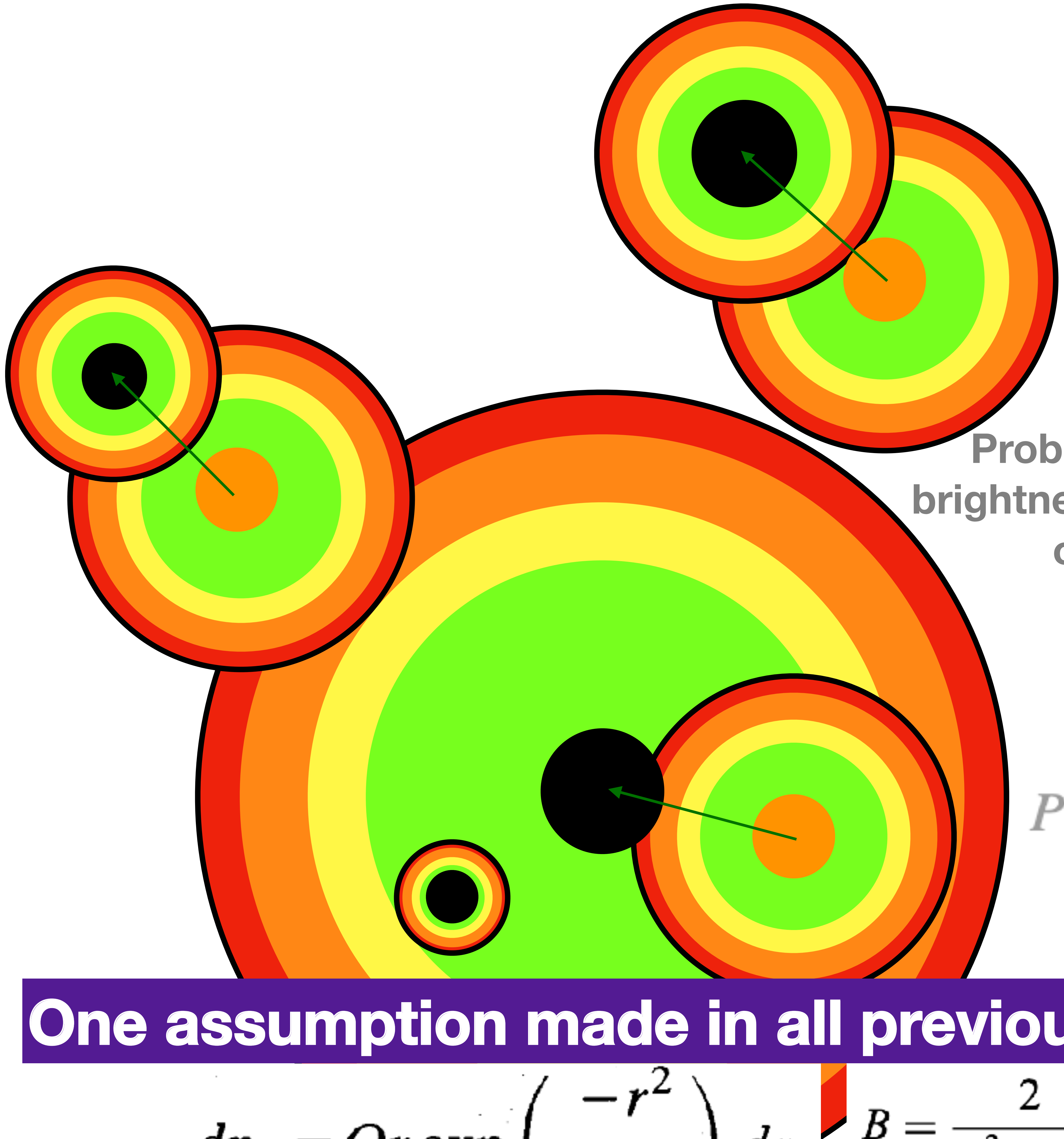

Probability of two sources having their on-sky separation given the hypothesis they are counterparts

Probability of sources having their brightnesses given they are unrelated to one another ("field stars")

Probability of sources having their brightnesses given they are counterparts

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma f_\gamma^\delta \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c \boldsymbol{G}_{\gamma\phi}^{\zeta_i\lambda_i} c_{\gamma\phi}^{\zeta_i\lambda_i}$$

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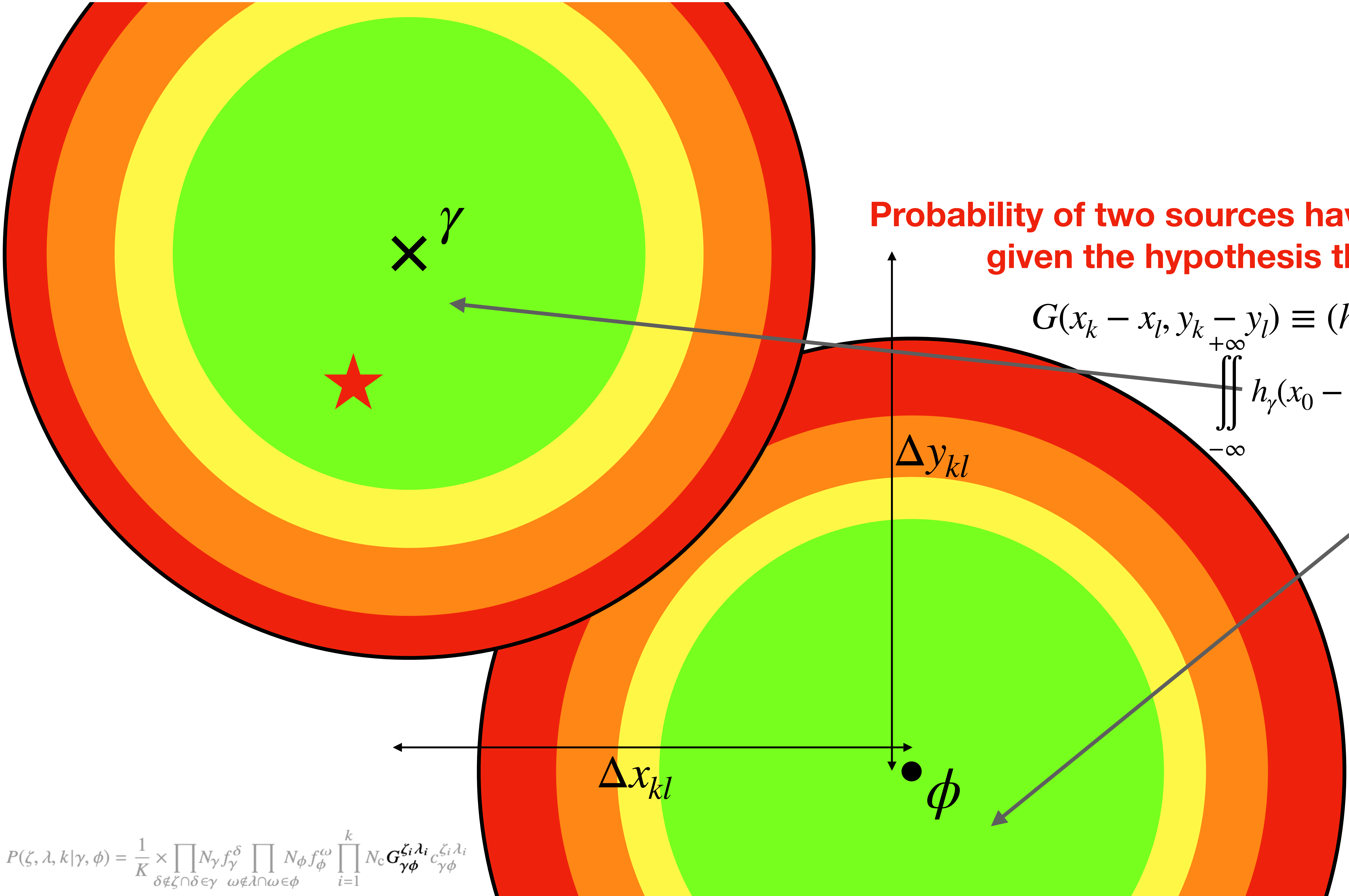
Match Separation Probability



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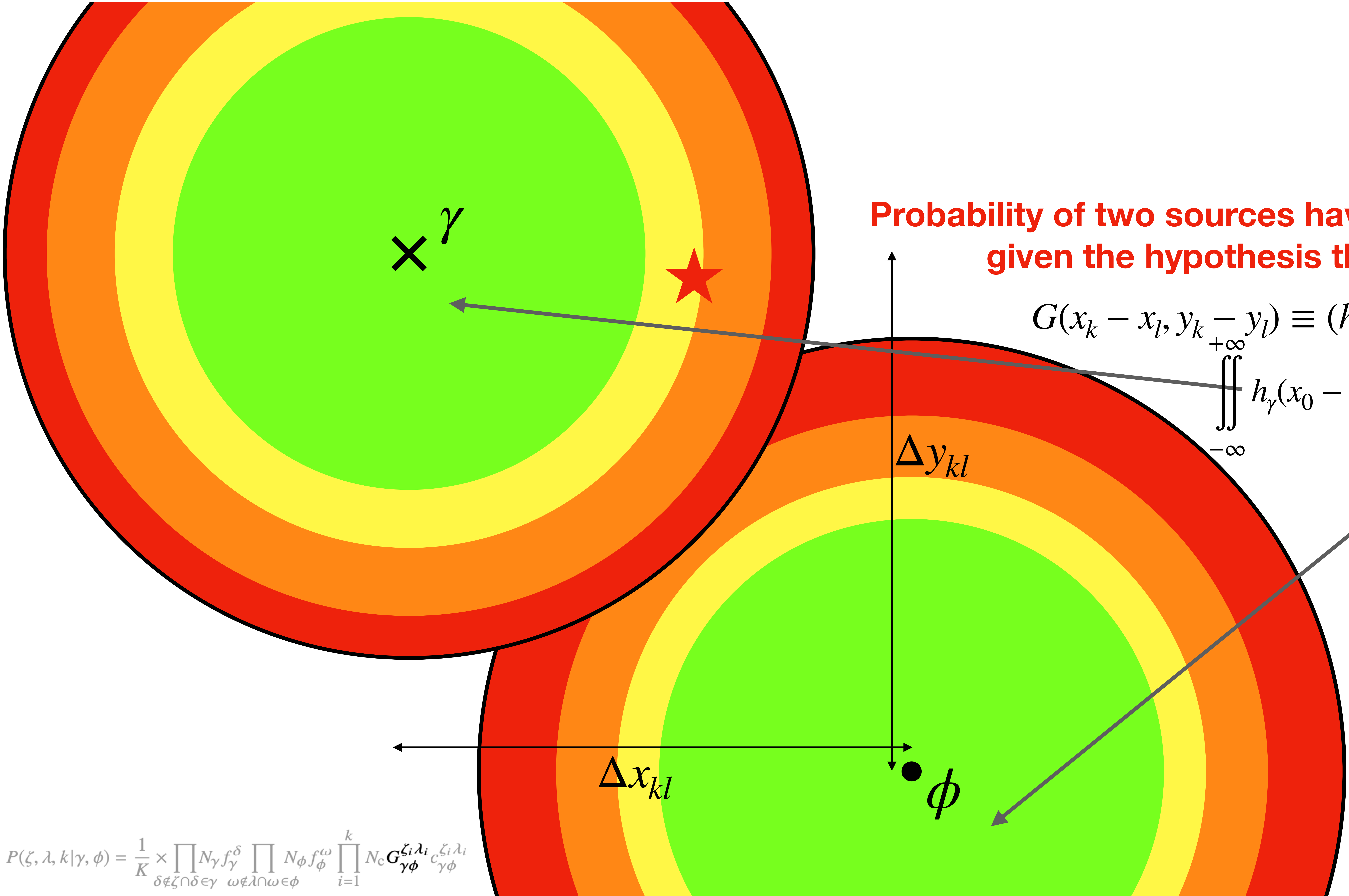

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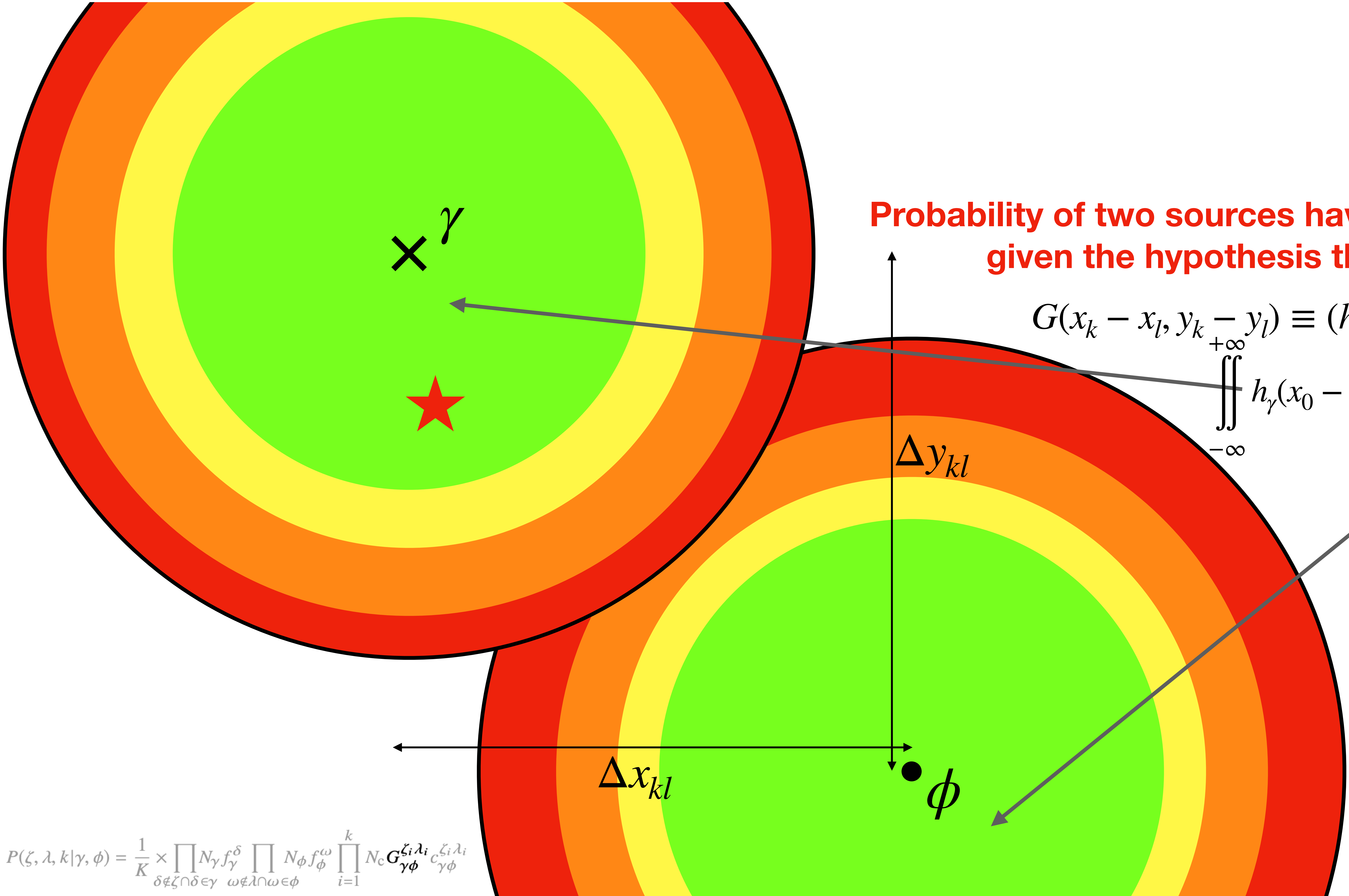

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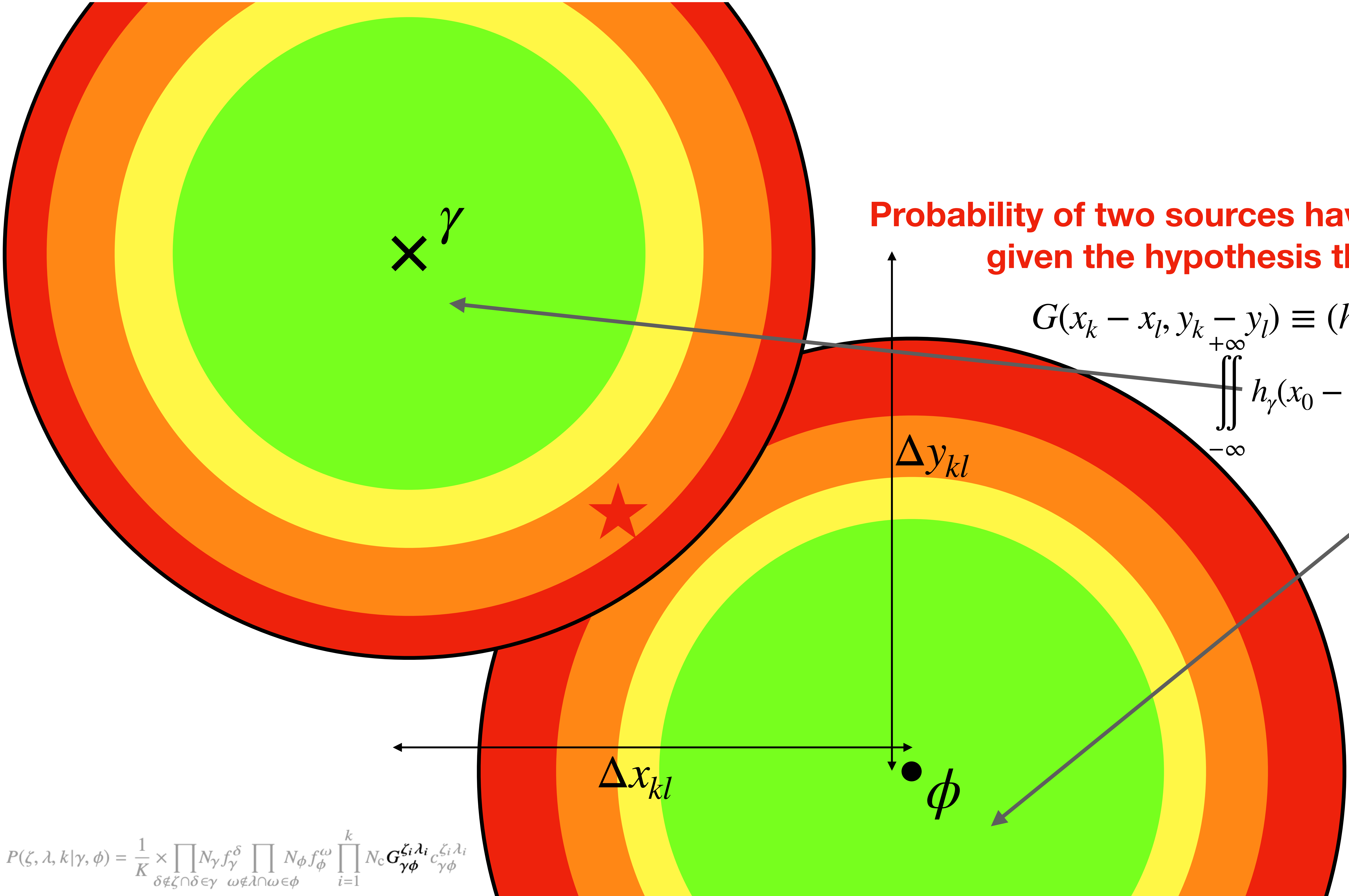

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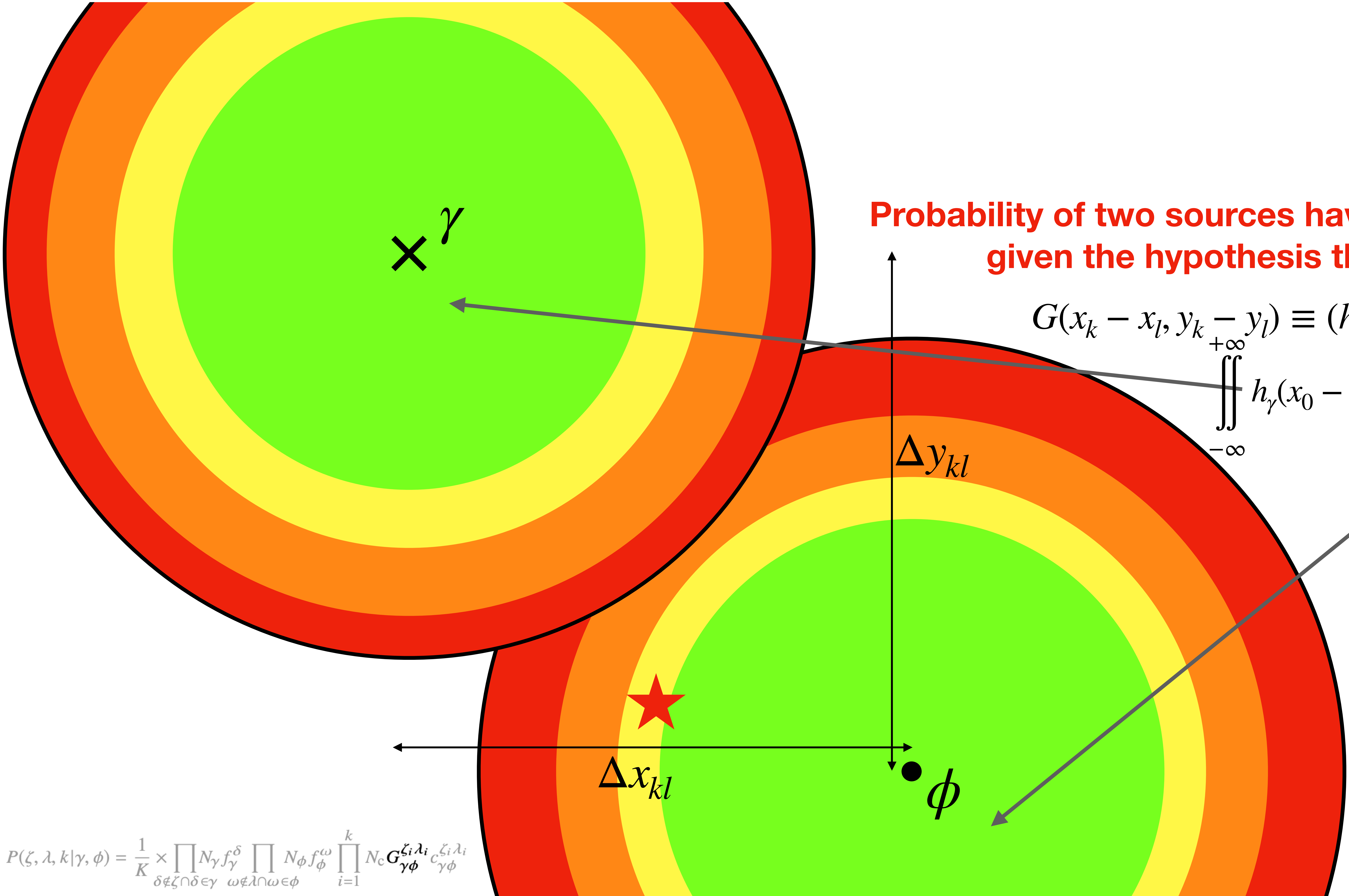

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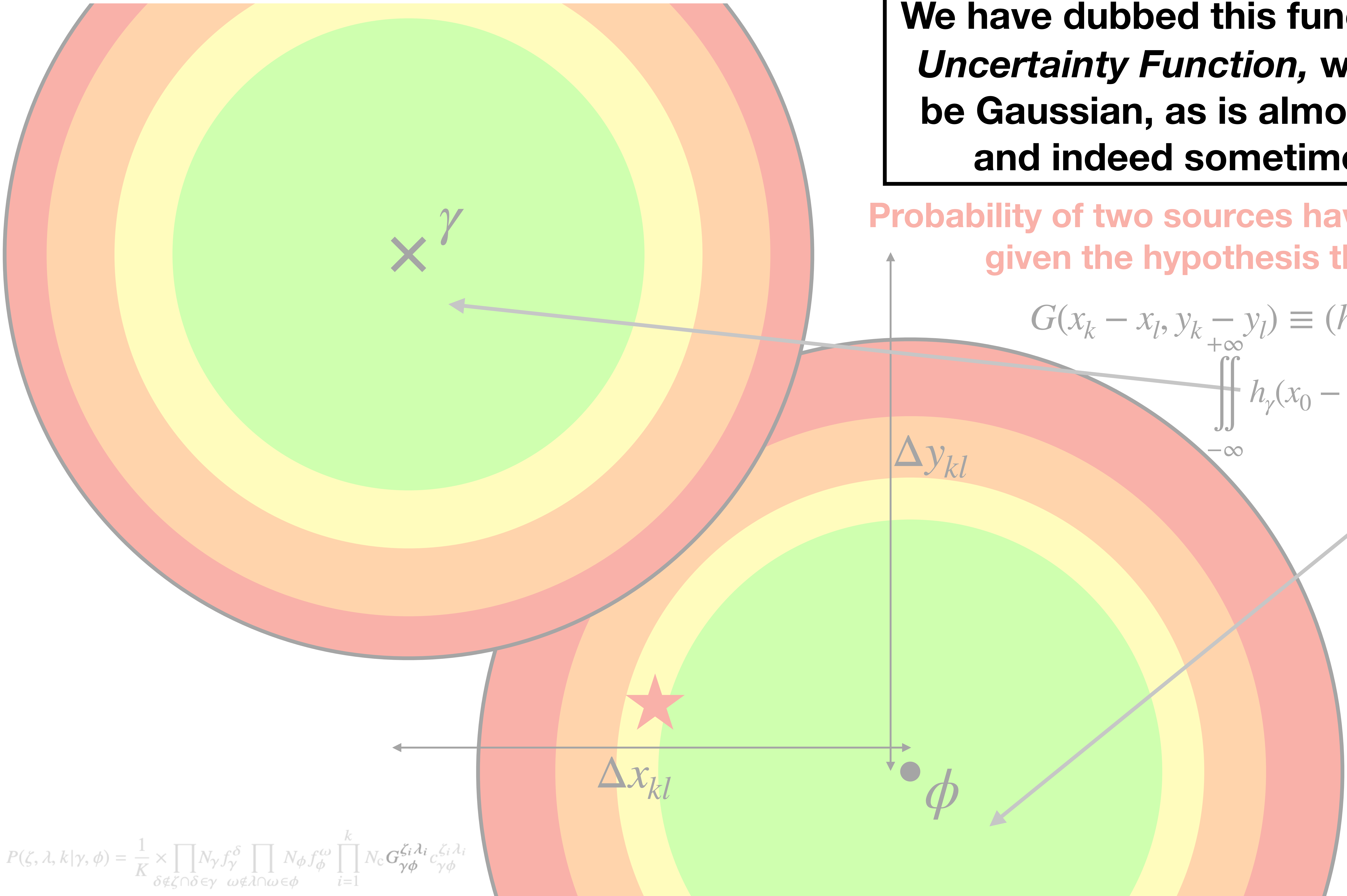
Match Separation Probability

We have dubbed this function h the *Astrometric Uncertainty Function,* which does not need to be Gaussian, as is almost always assumed — and indeed sometimes *needs* not to be!

Probability of two sources having their on-sky separation given the hypothesis they are counterparts



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Additional Components of the AUF

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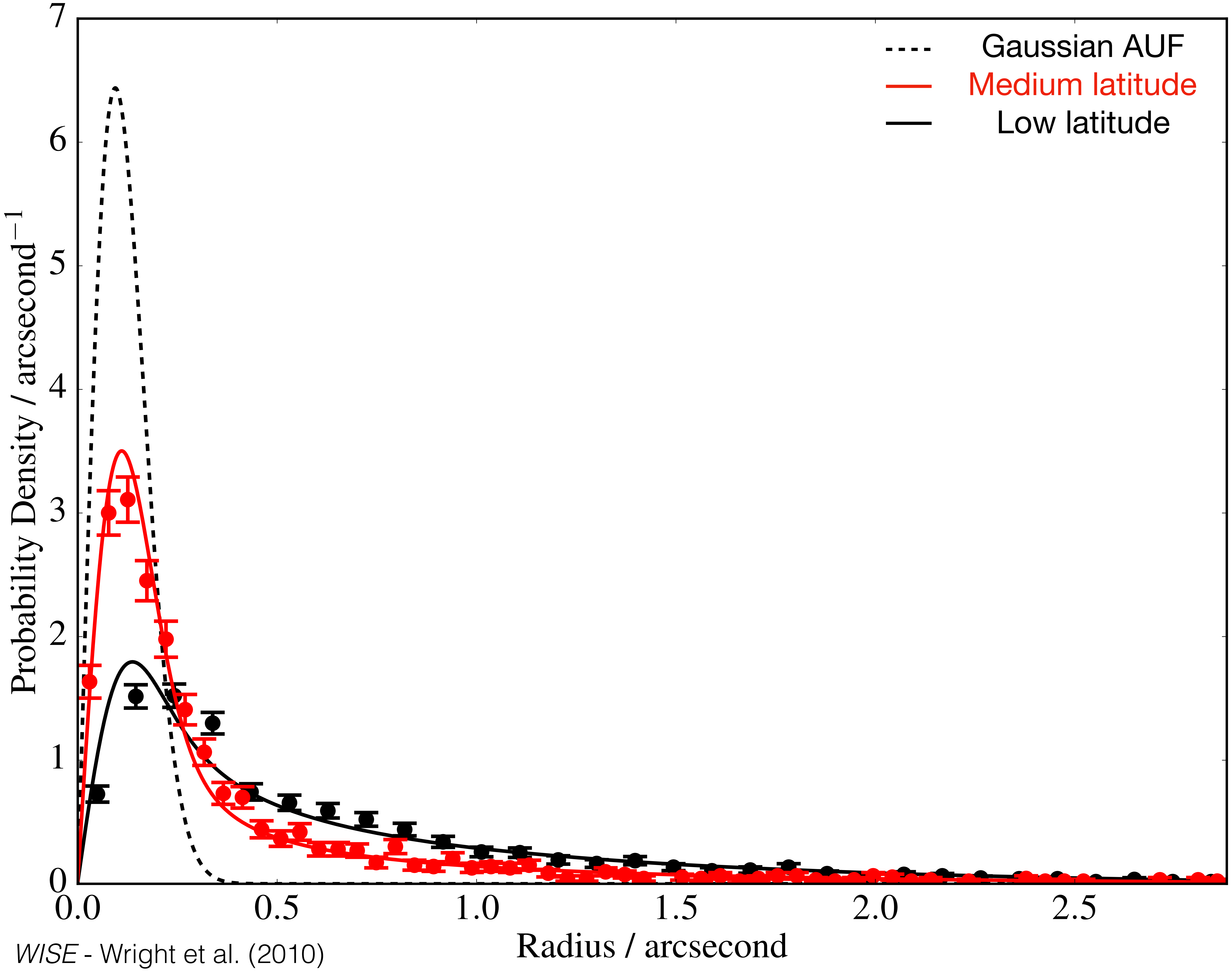

WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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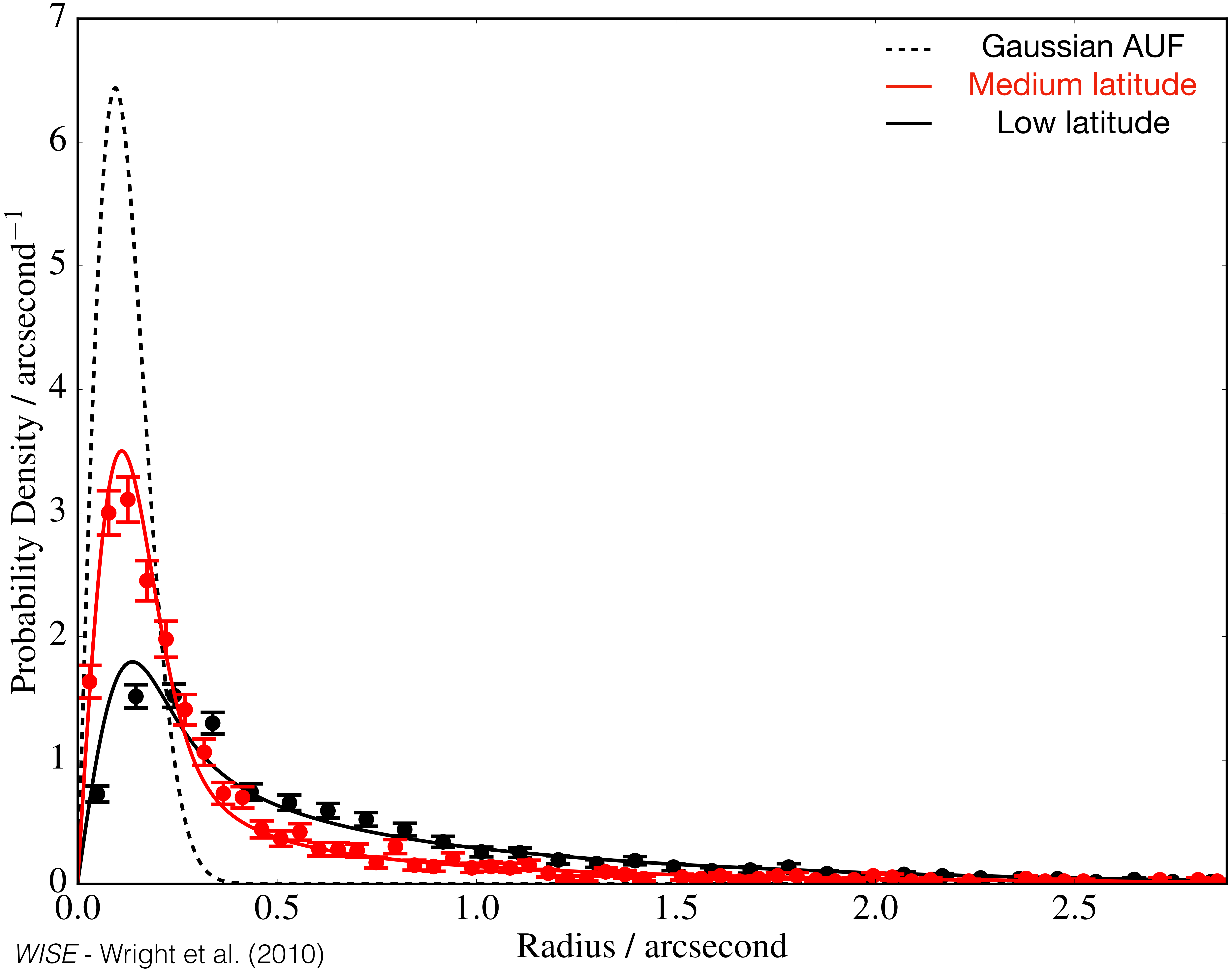

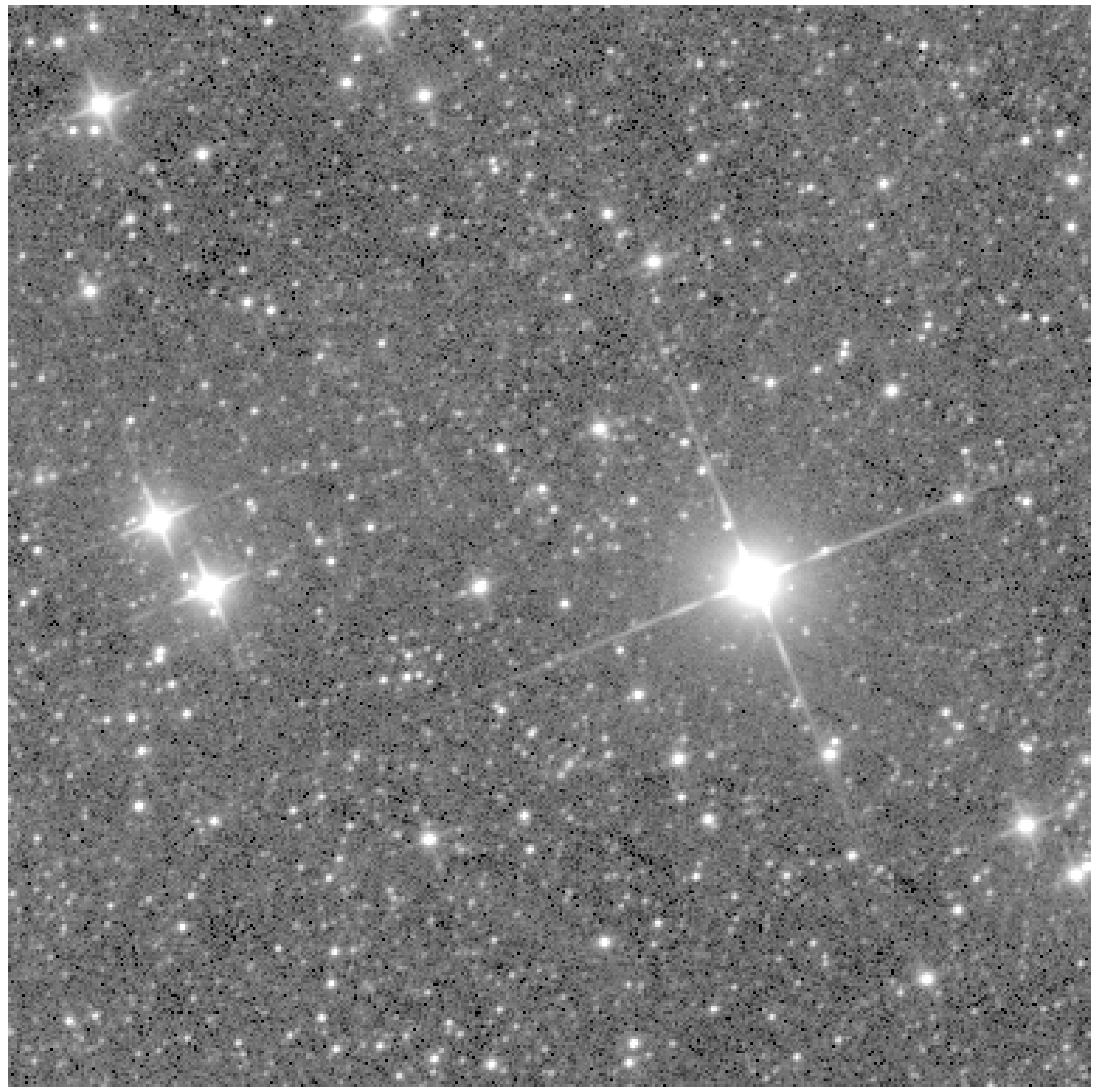

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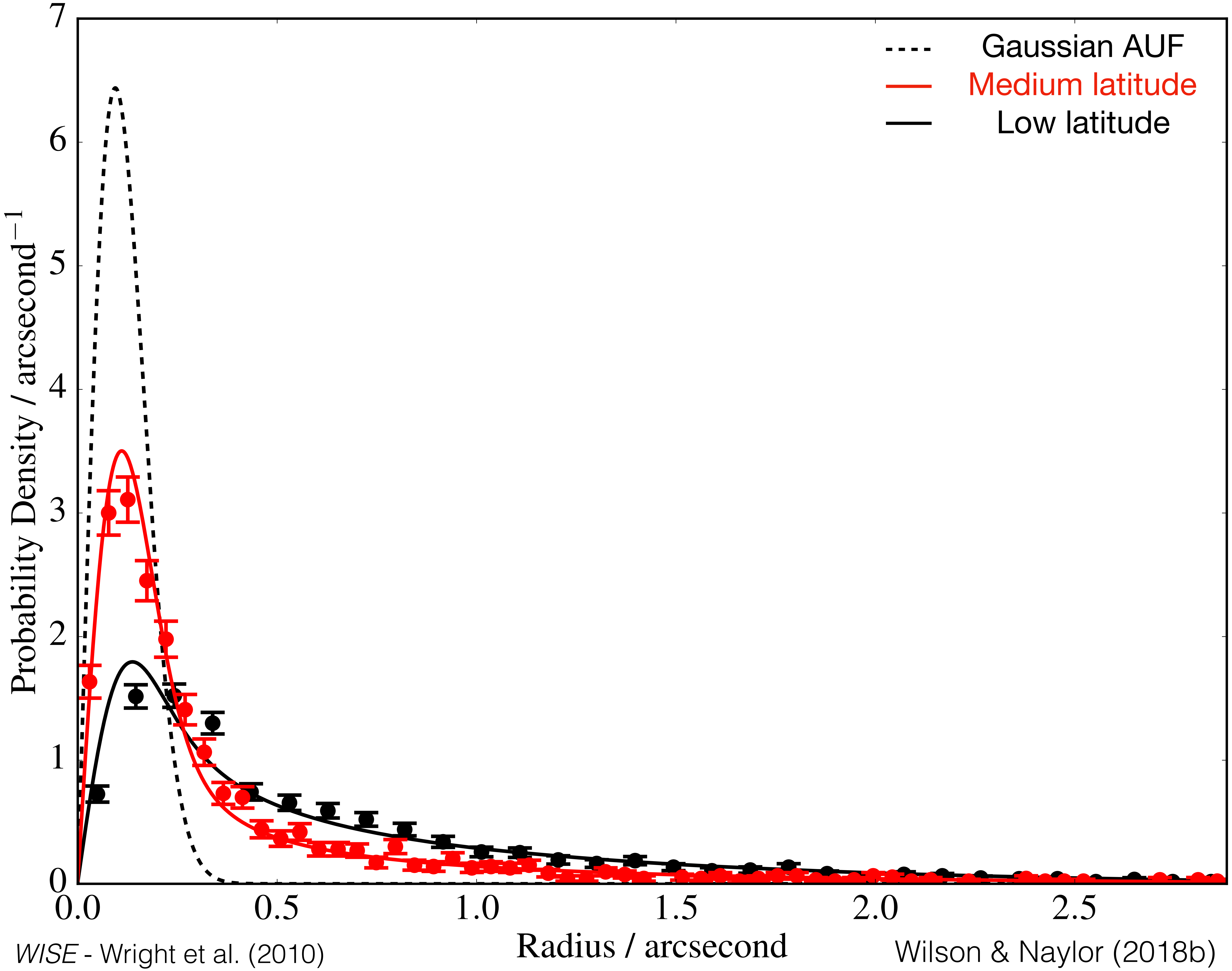

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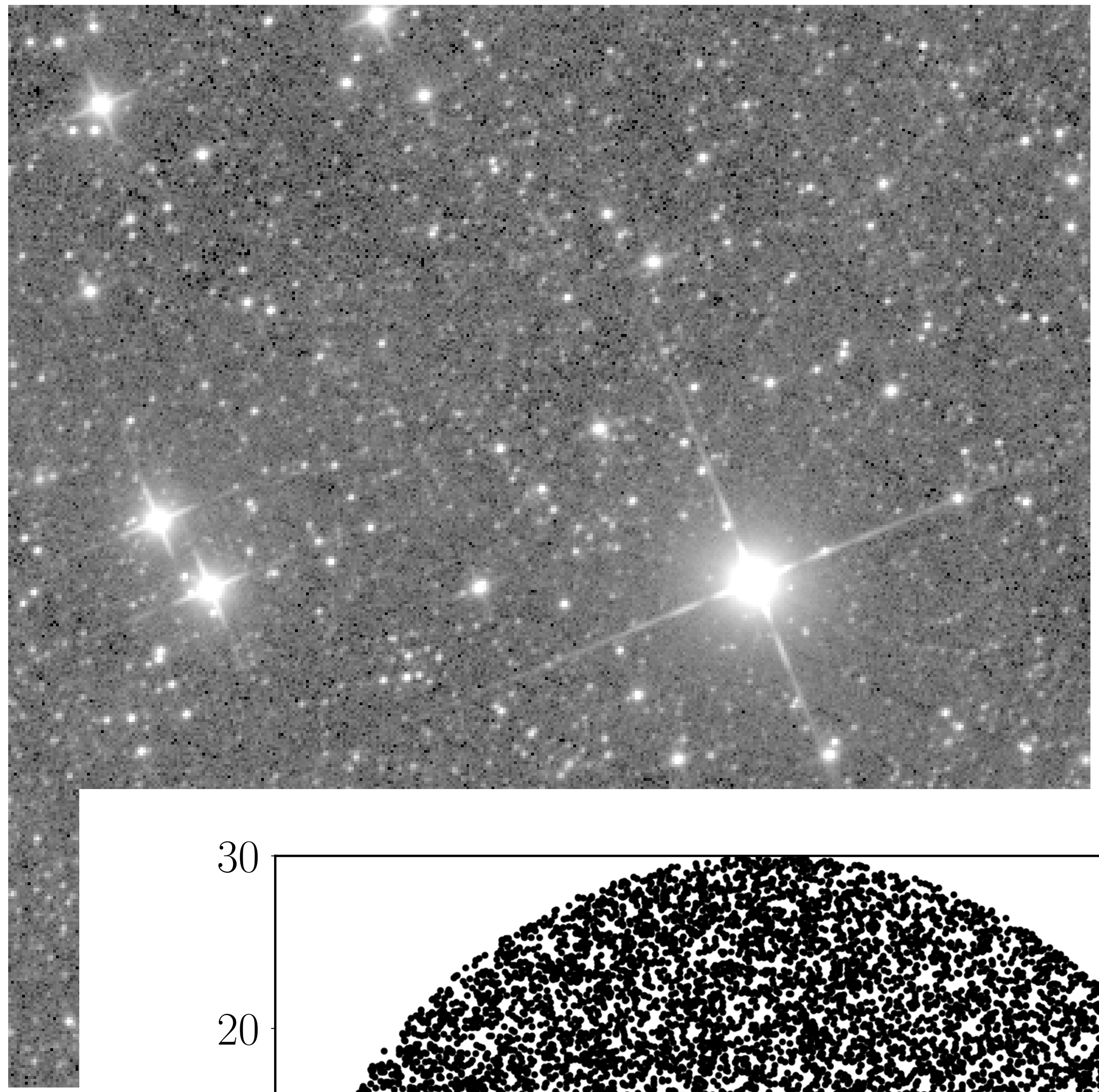
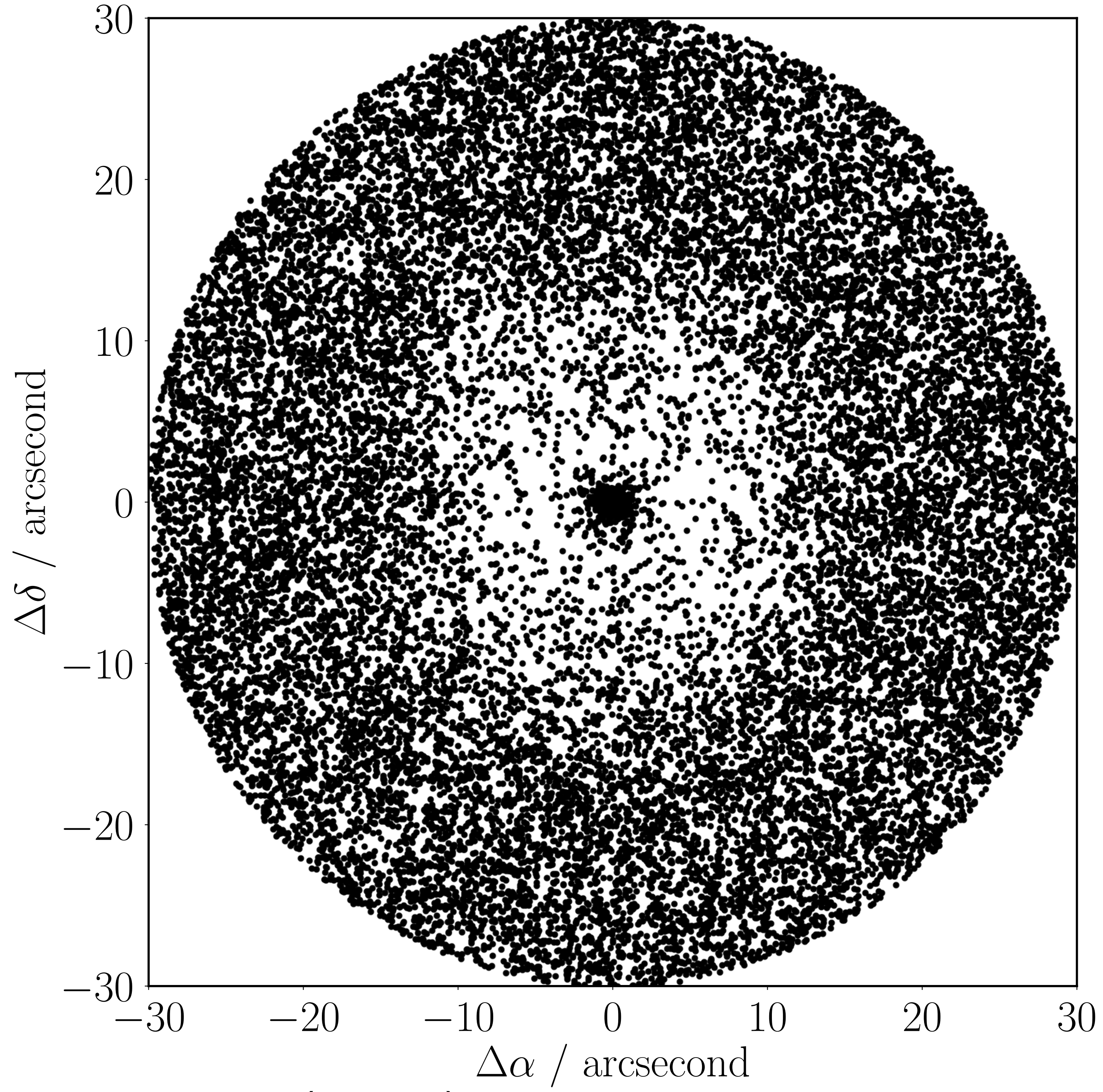
Wilson & Naylor (2018b)





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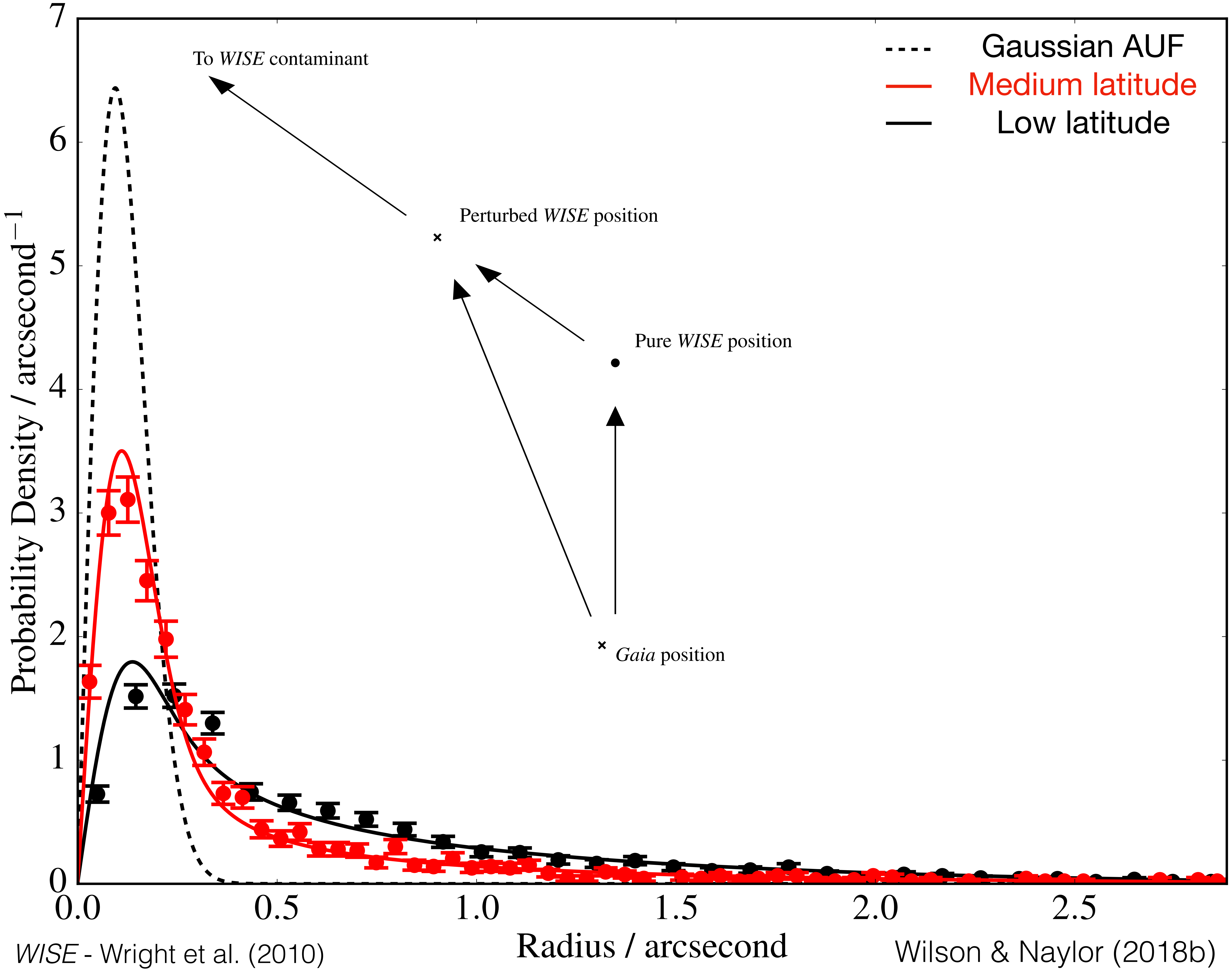

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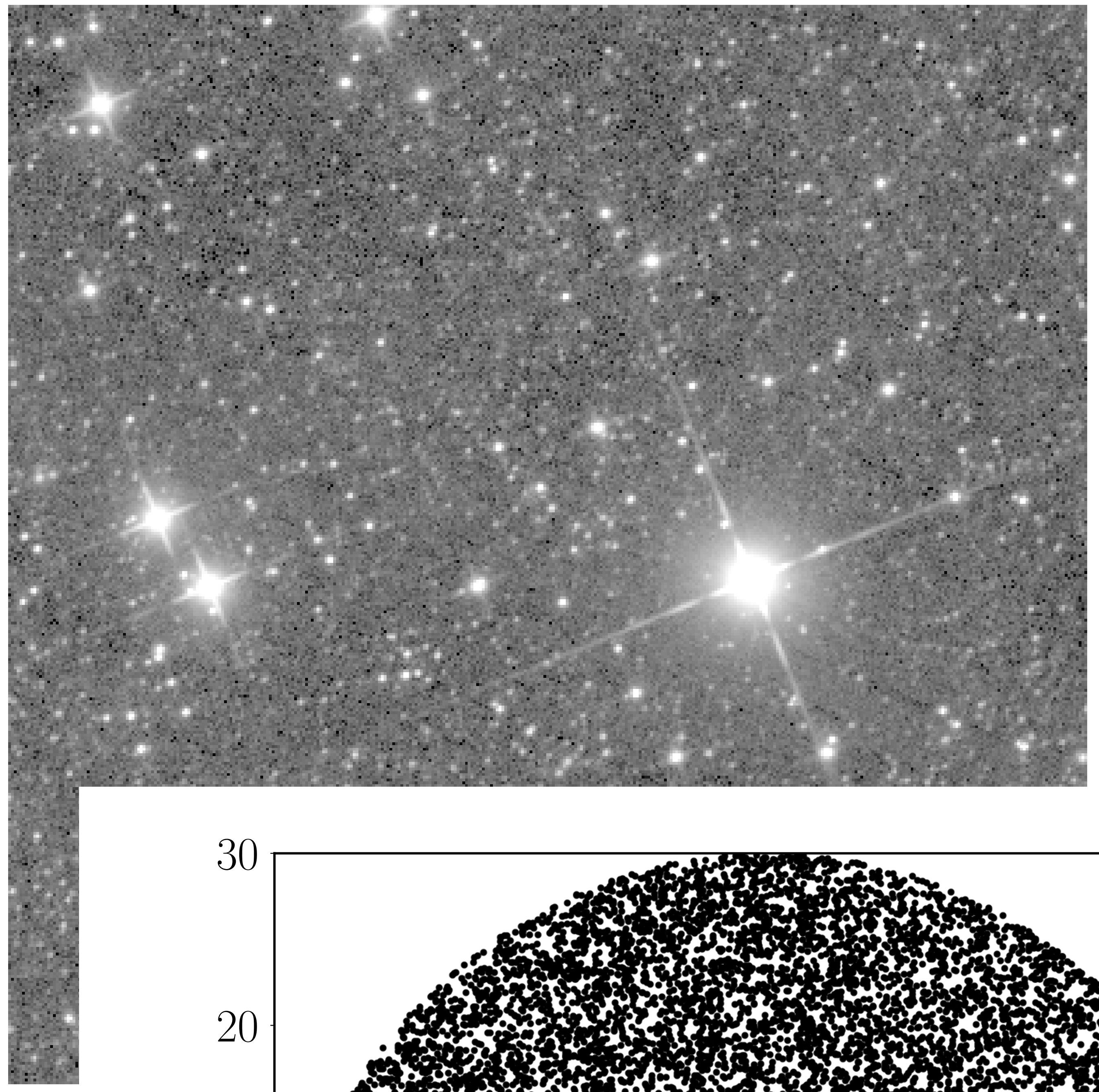

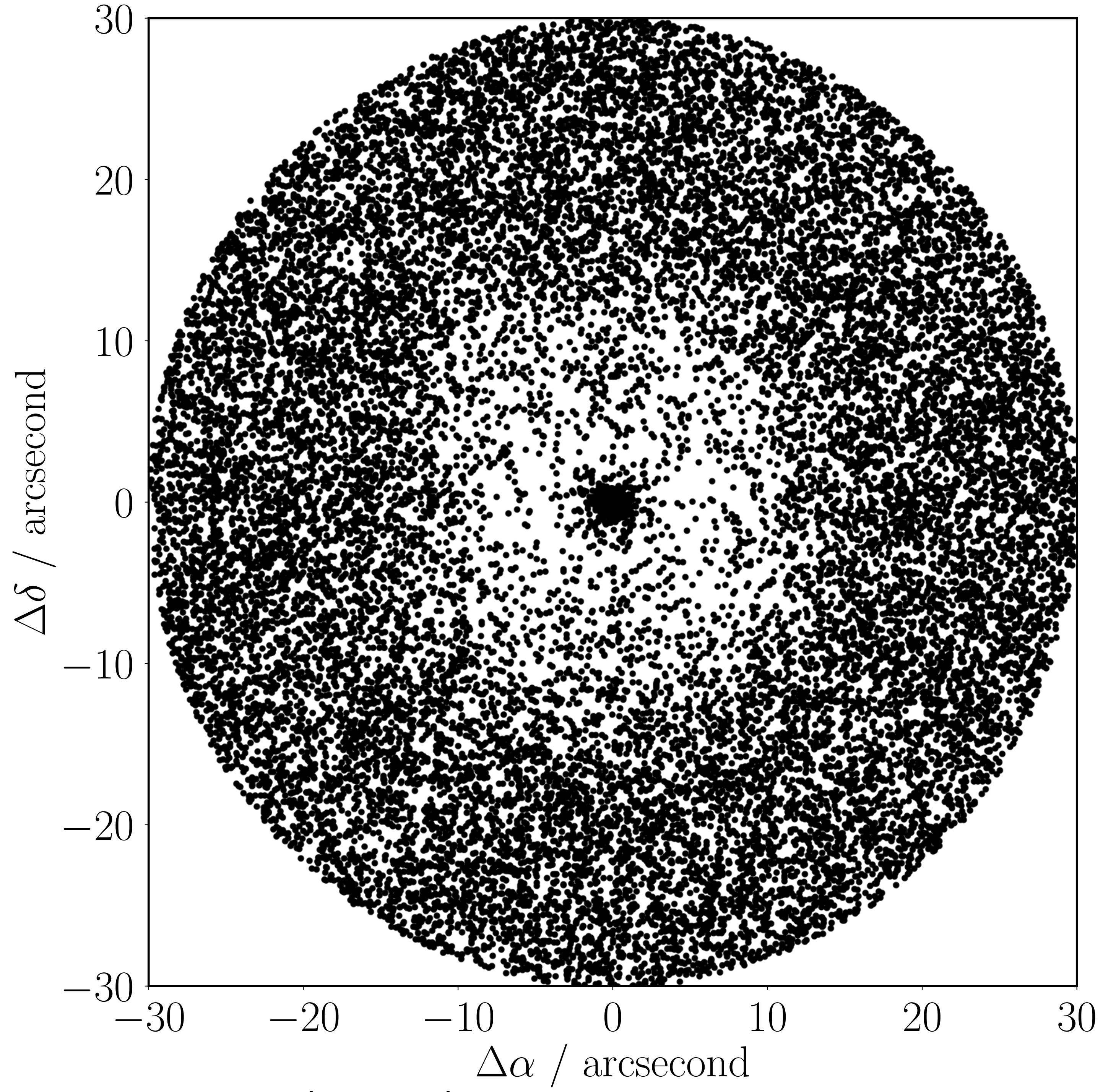

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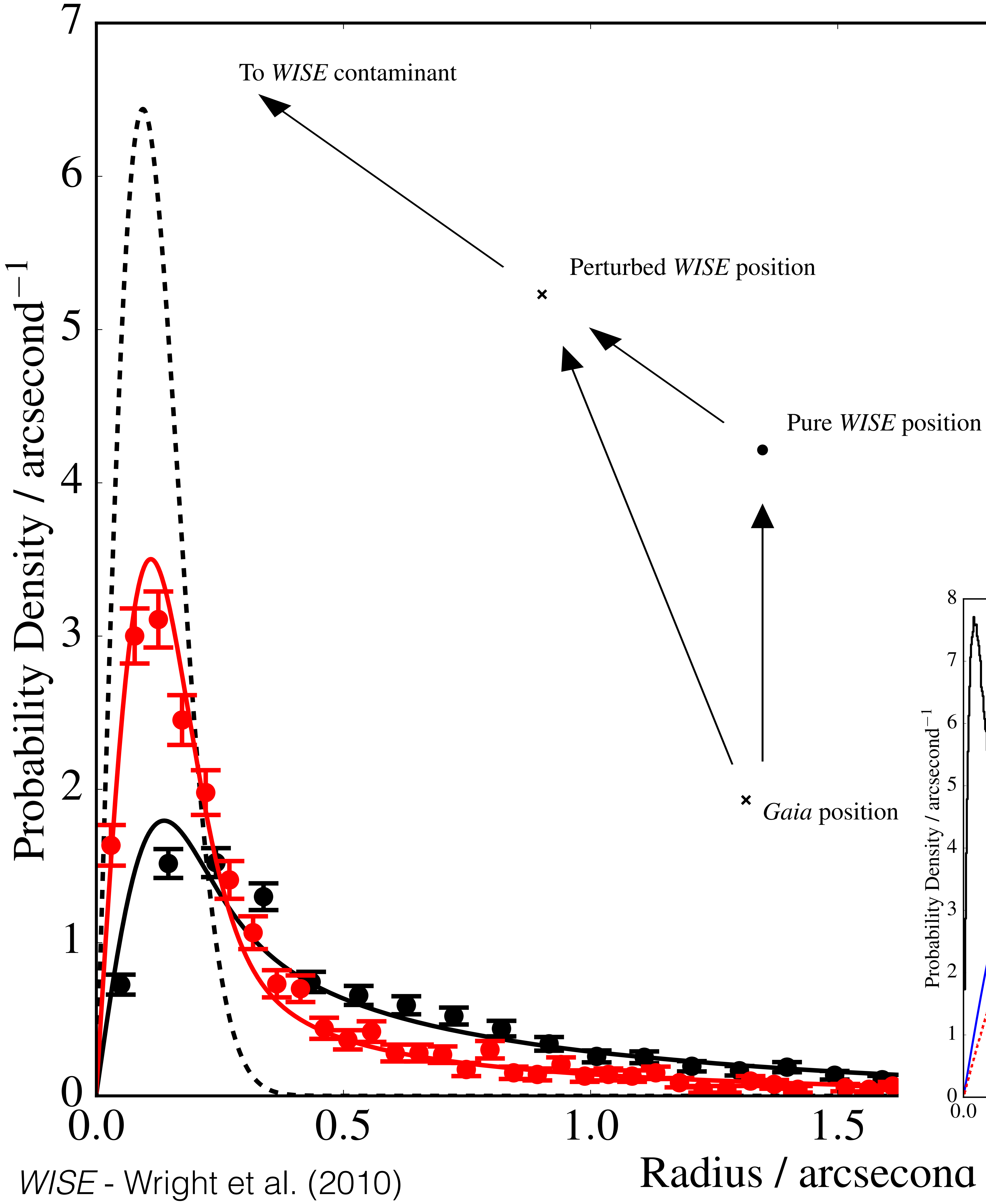
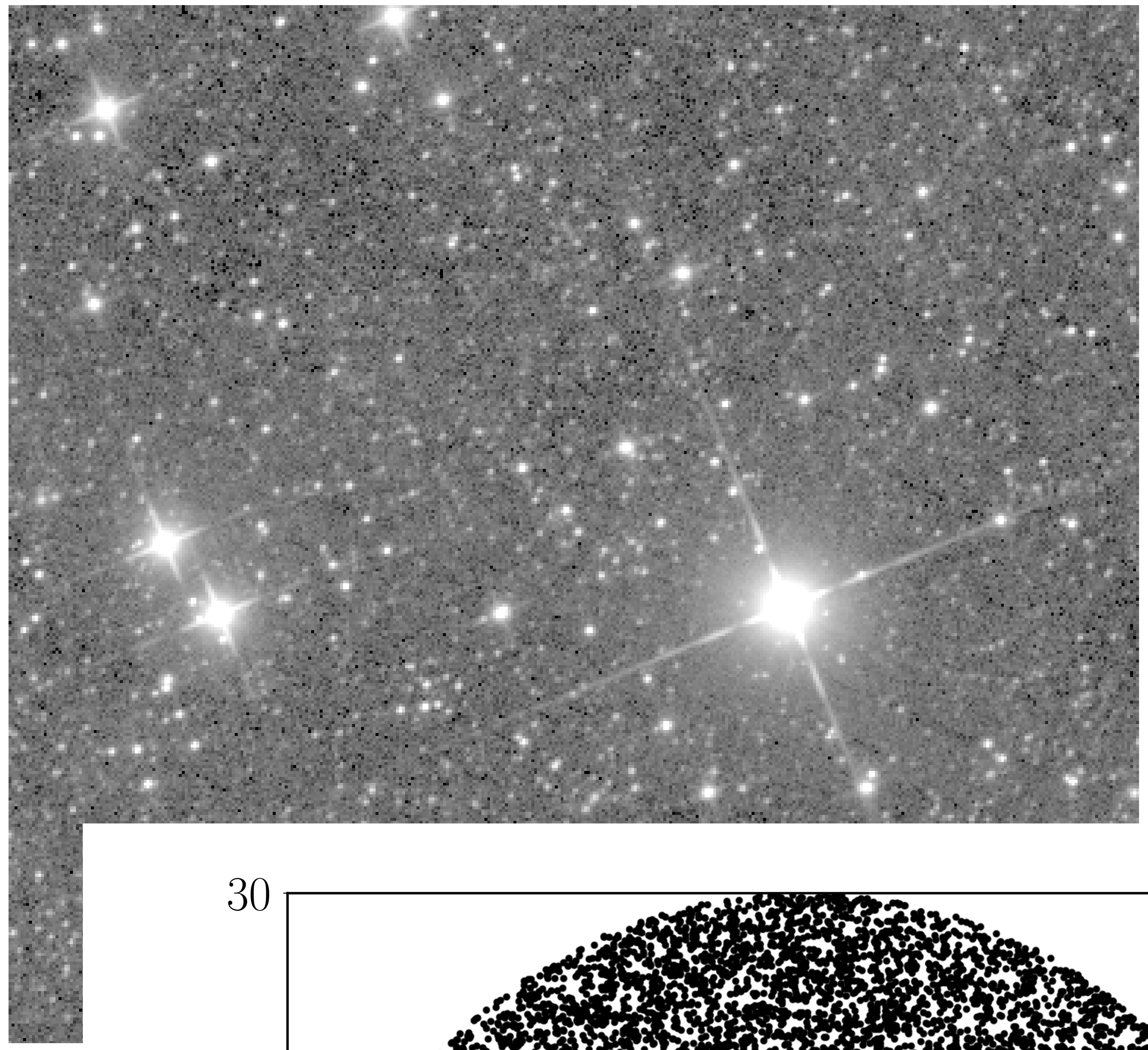
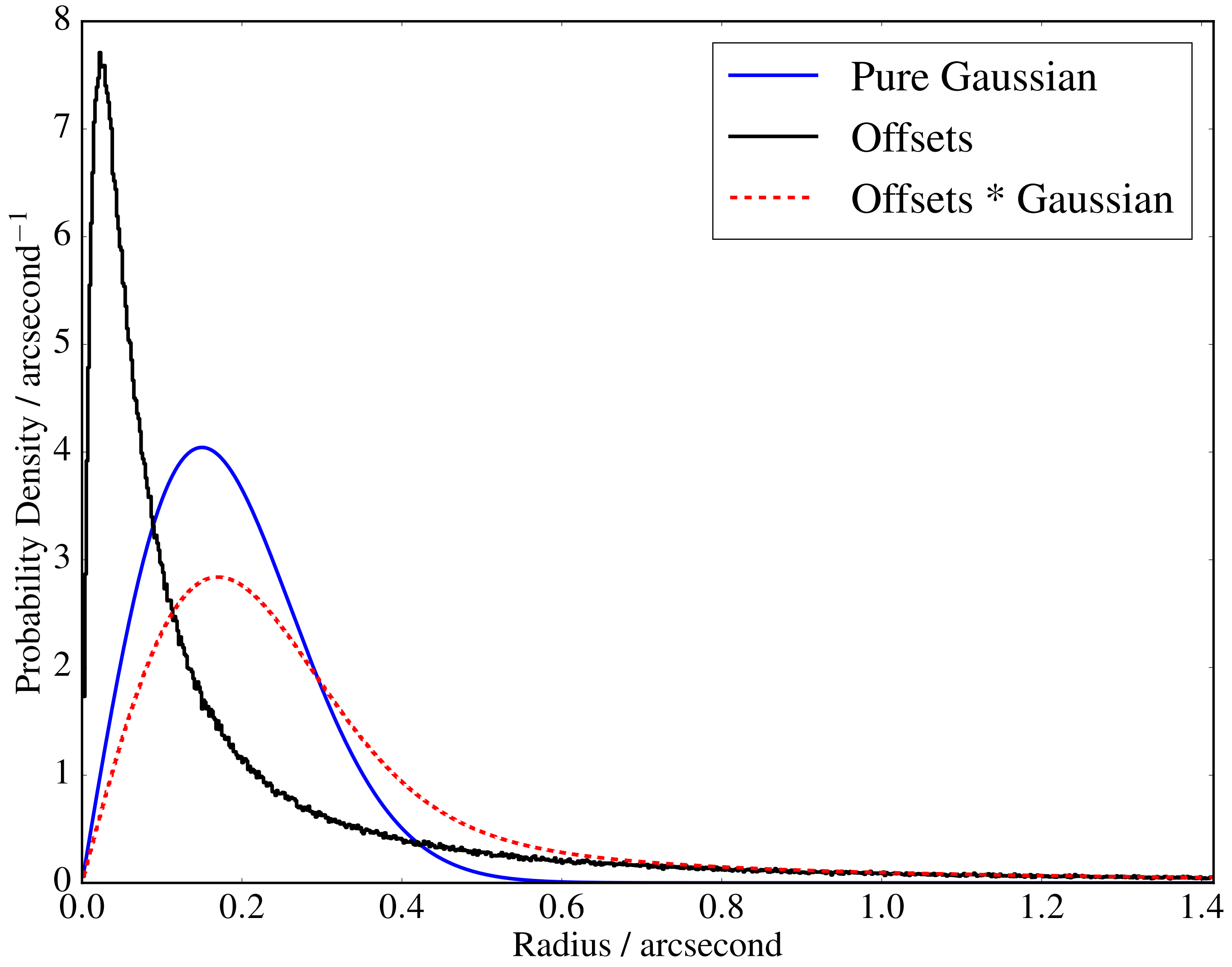
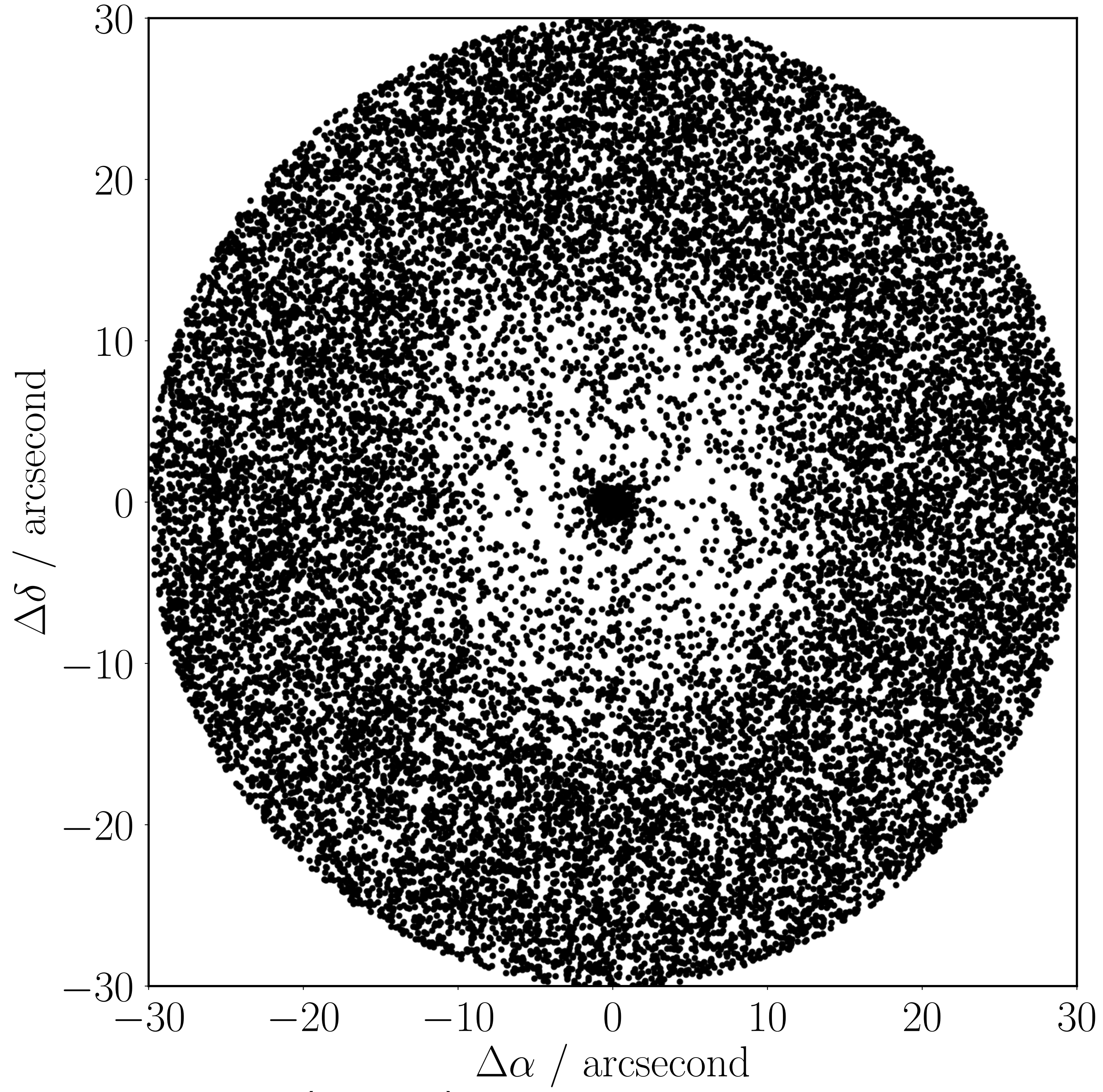






Wilson & Naylor (2018b)



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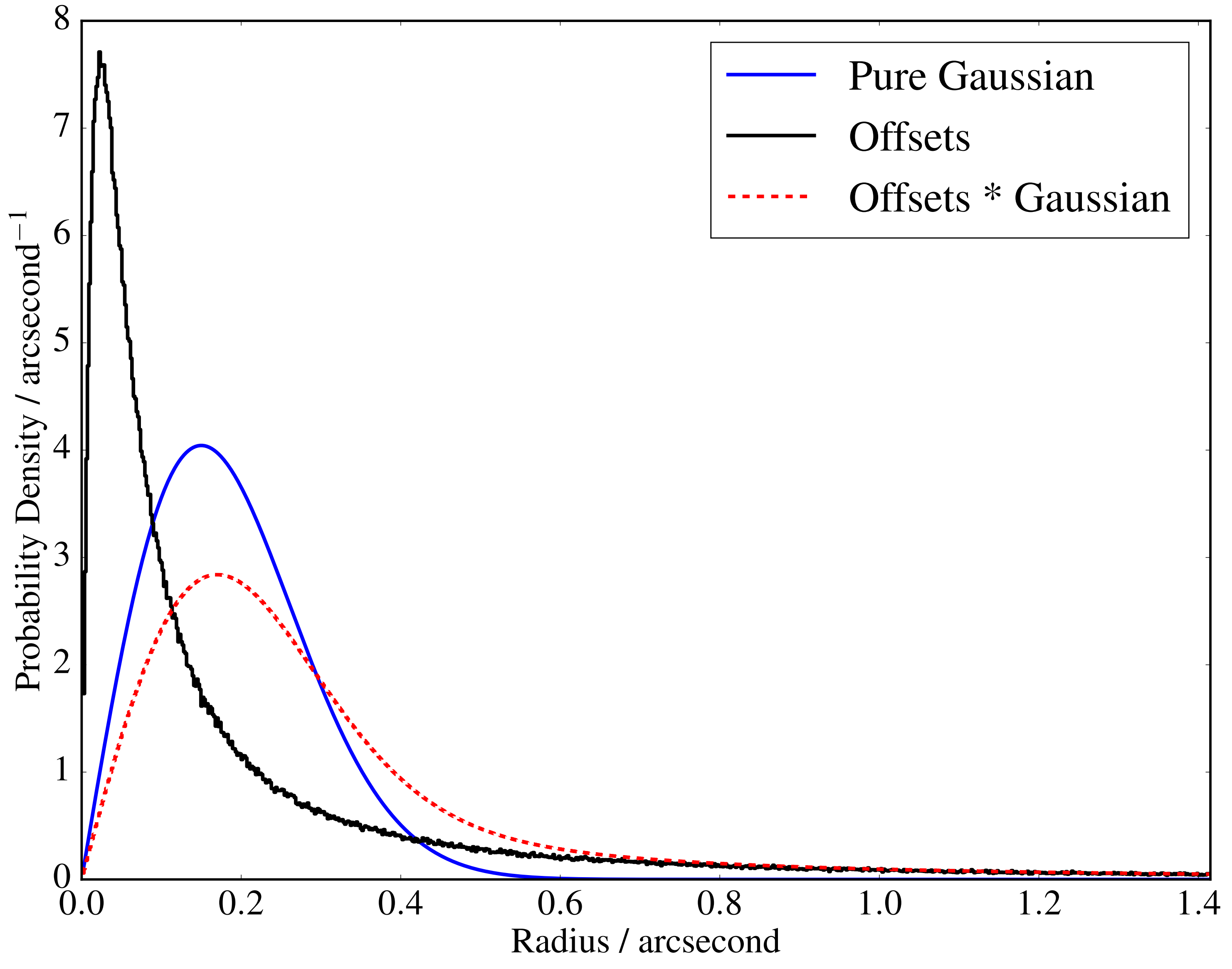

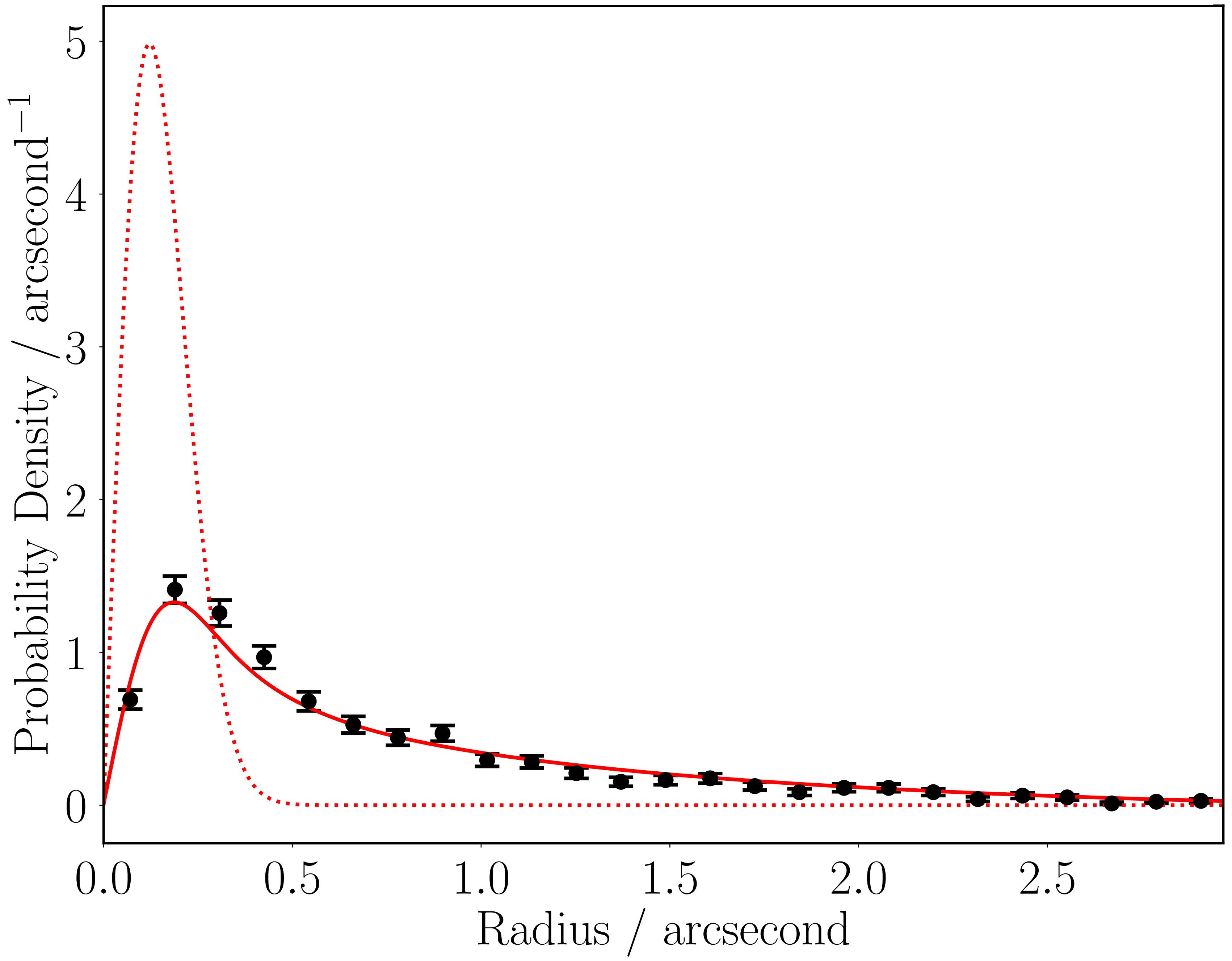

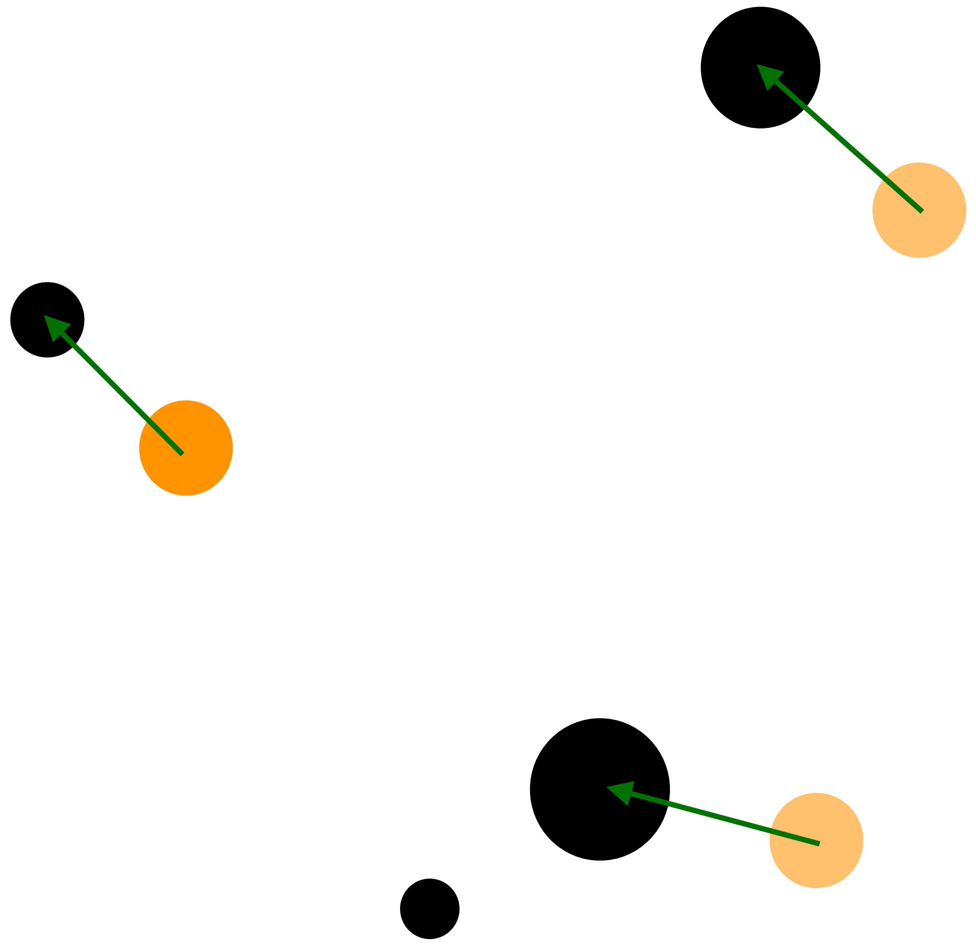

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Probability of sources having their brightnesses given they are unrelated to one another ("field stars")

Probability of sources having their brightnesses given they are counterparts

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma \boldsymbol{f}_{\boldsymbol{\gamma}}^{\boldsymbol{\delta}} \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi \boldsymbol{f}_{\boldsymbol{\phi}}^{\boldsymbol{\omega}} \prod_{i=1}^{k} N_\mathrm{c} G_{\gamma\phi}^{\zeta_i\lambda_i} \boldsymbol{c}_{\boldsymbol{\gamma\phi}}^{\boldsymbol{\zeta_i\lambda_i}}$$

Wilson & Naylor (2018a)

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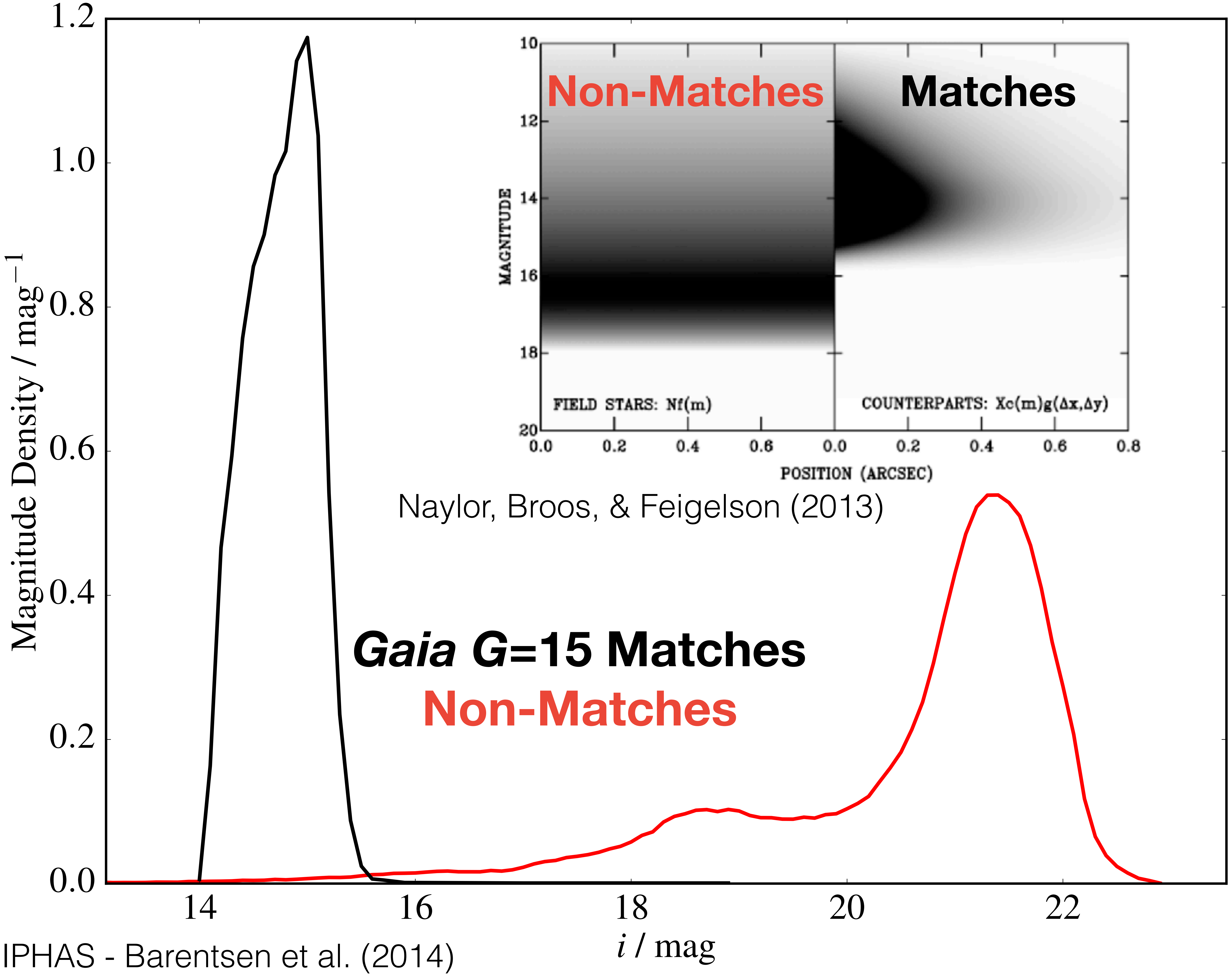
Including Magnitude Information

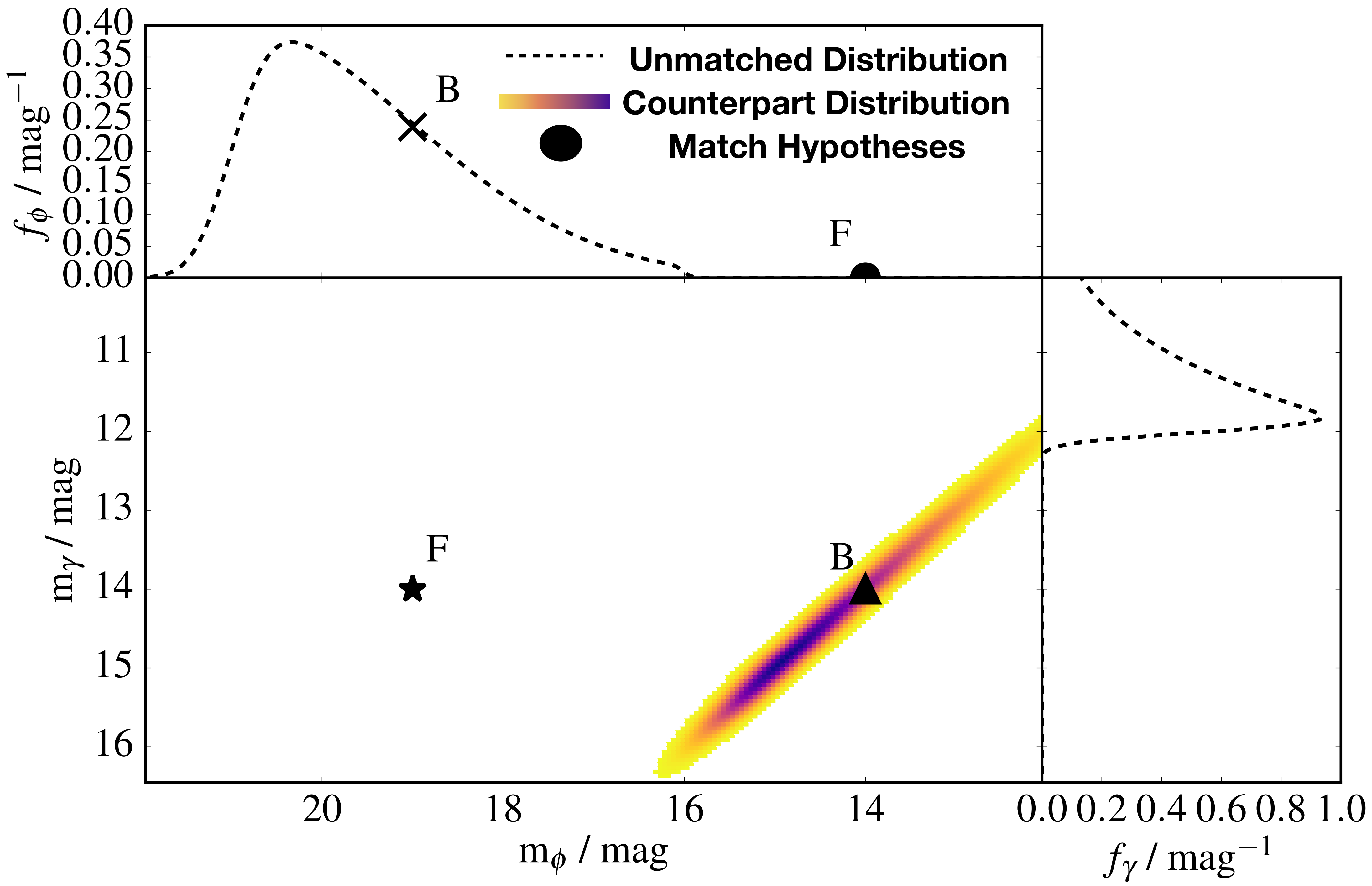
$$P(\zeta,\lambda,k|\gamma,\phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma f_\gamma^\delta \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i\lambda_i} c_{\gamma\phi}^{\zeta_i\lambda_i}$$



Naylor, Broos, & Feigelson (2013)

IPHAS - Barentsen et al. (2014)

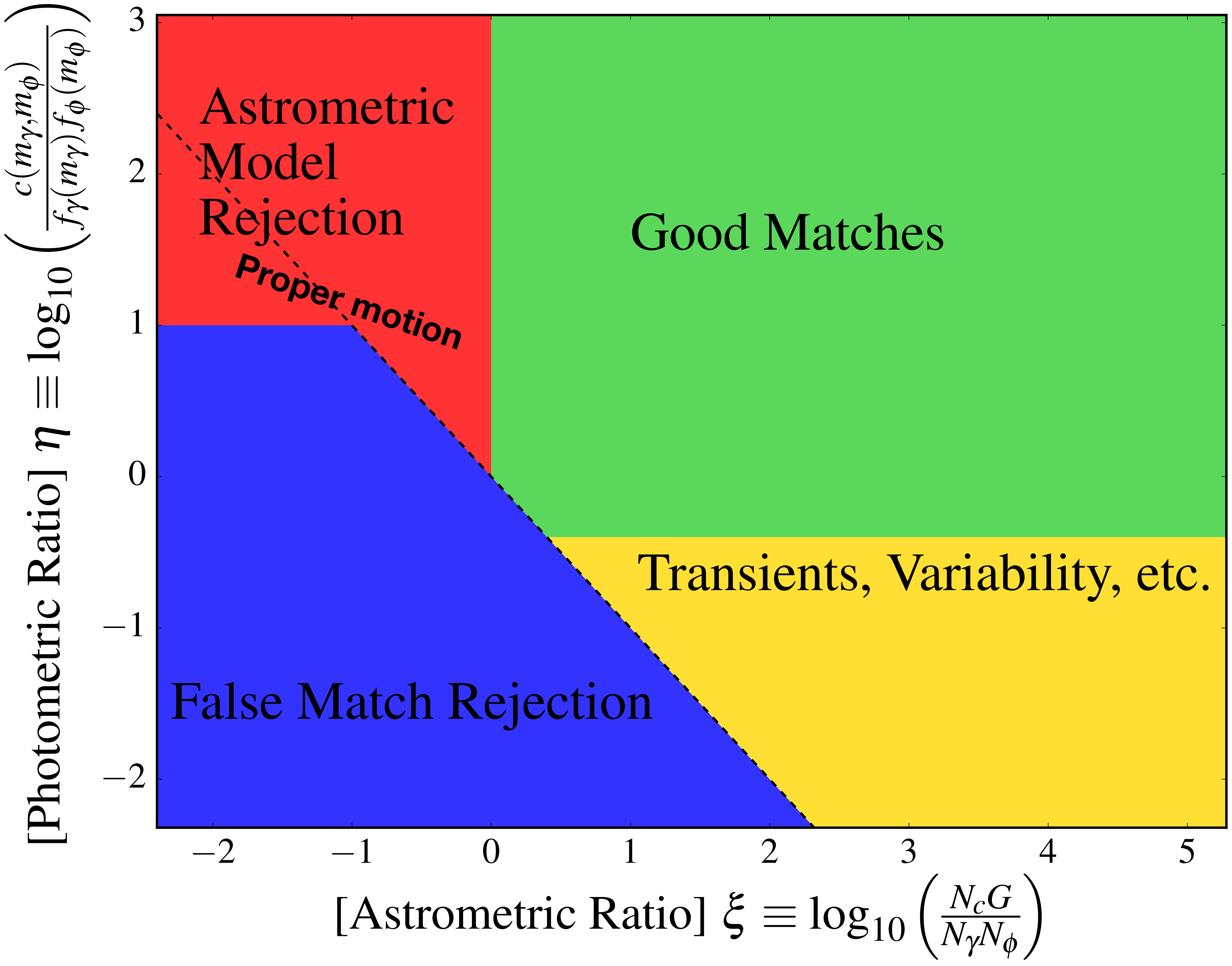
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

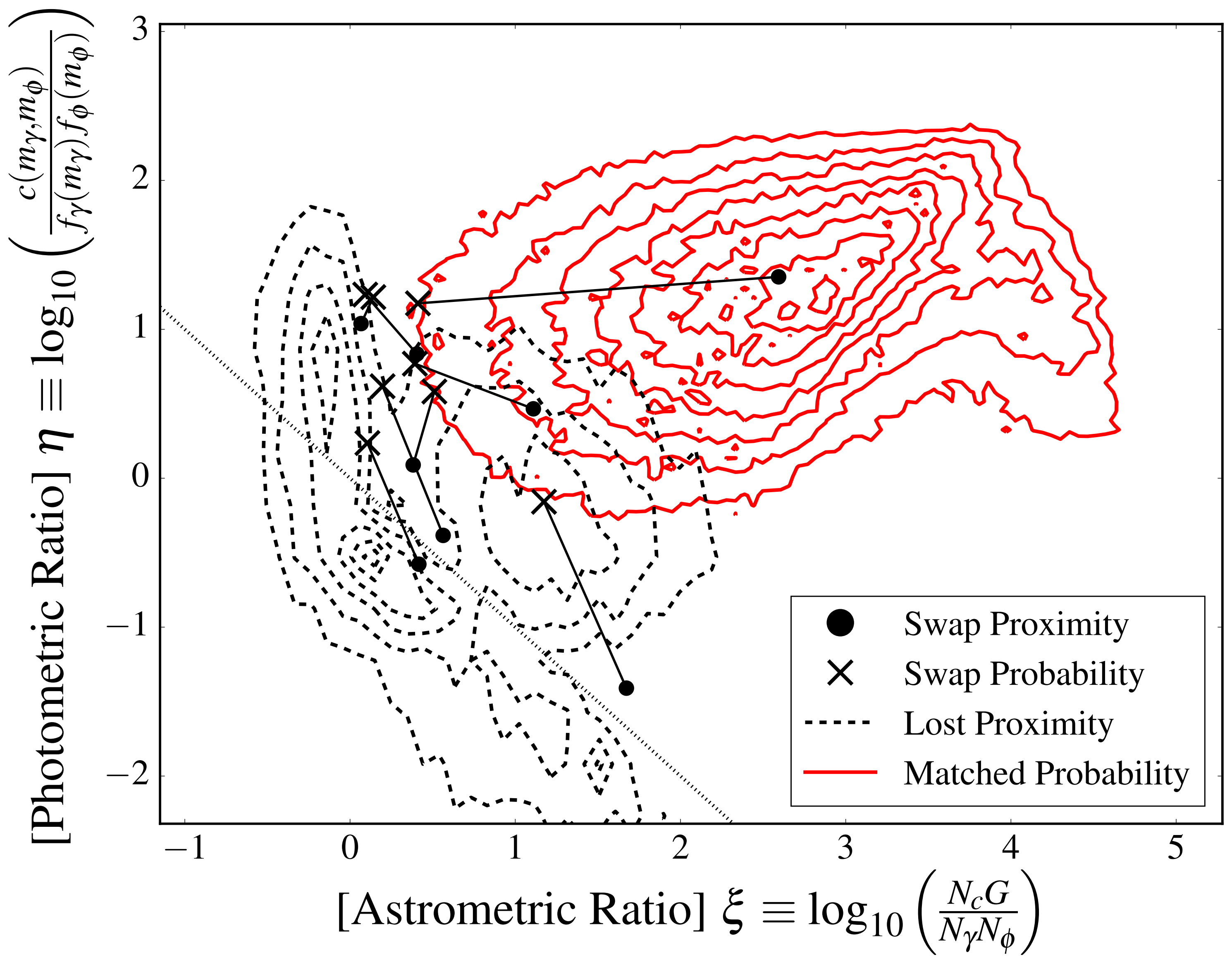


Wilson & Naylor (2018a)

The photometry-based likelihoods (c and f) allow us to reject some matches in crowded fields, but now we need the position-based likelihood G

The Likelihood Ratio Space





Wilson & Naylor (2018b)

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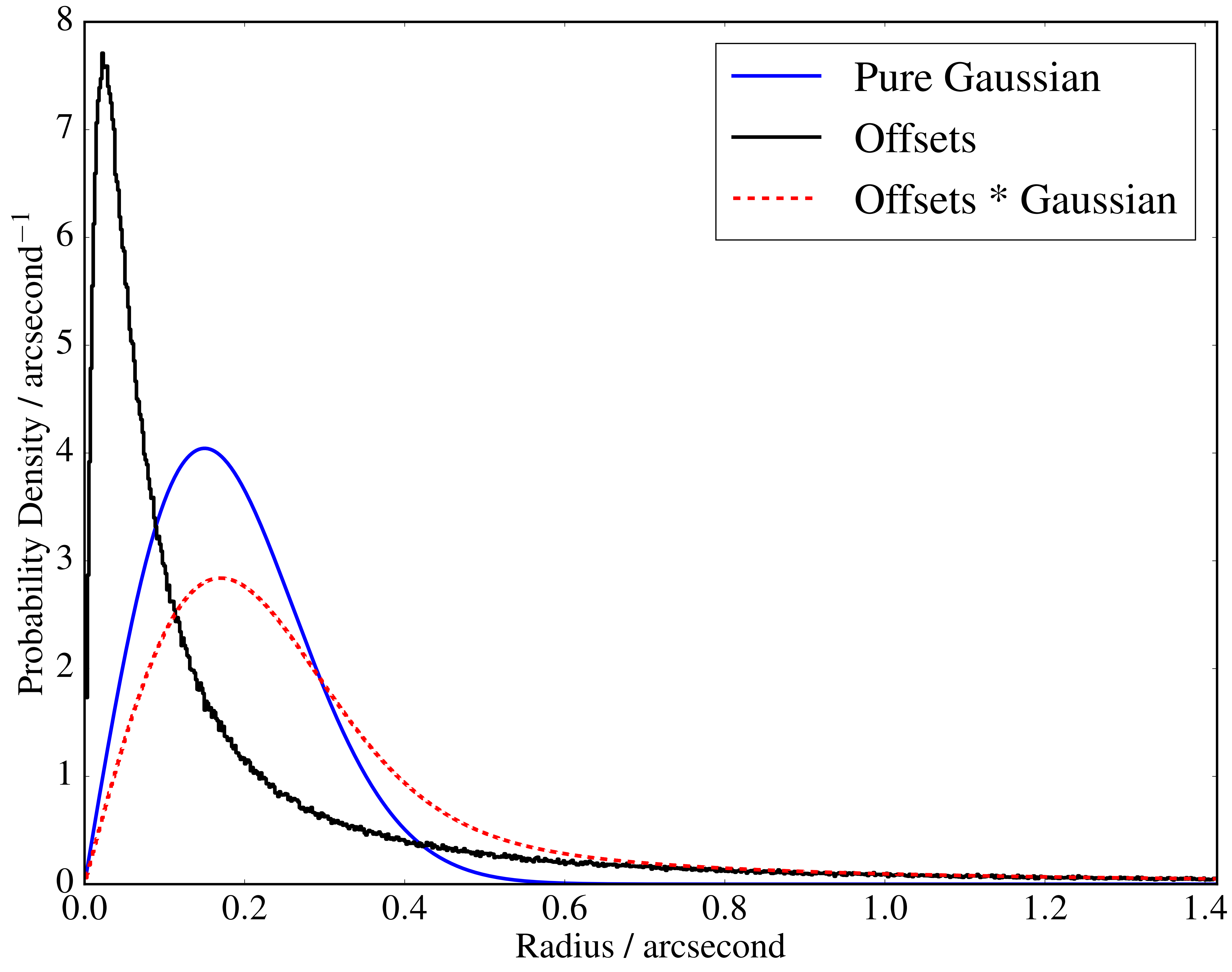
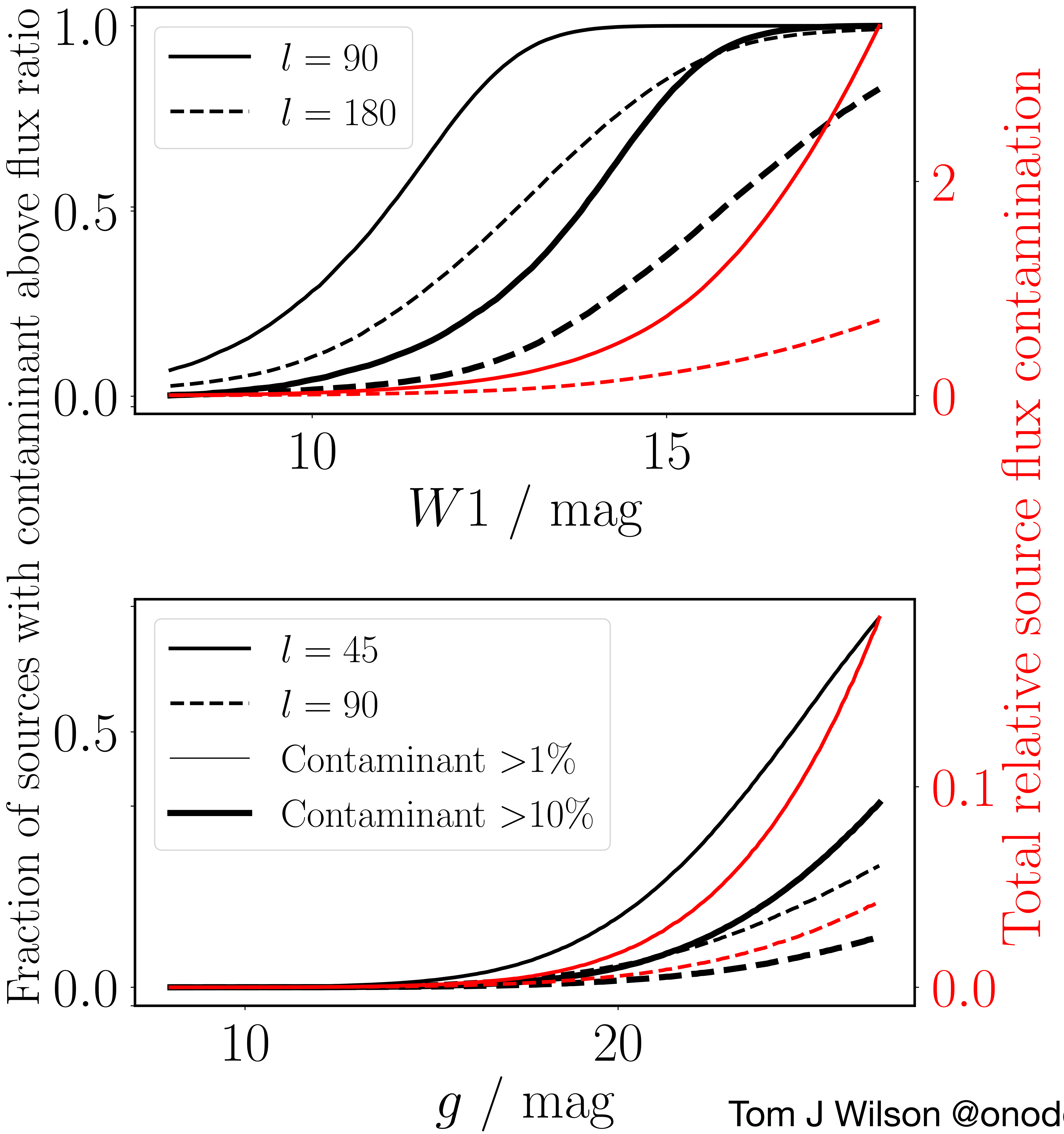
Contamination Rates and Amounts





TRILEGAL - Girardi et al. (2005)
Wilson & Naylor (2018b)

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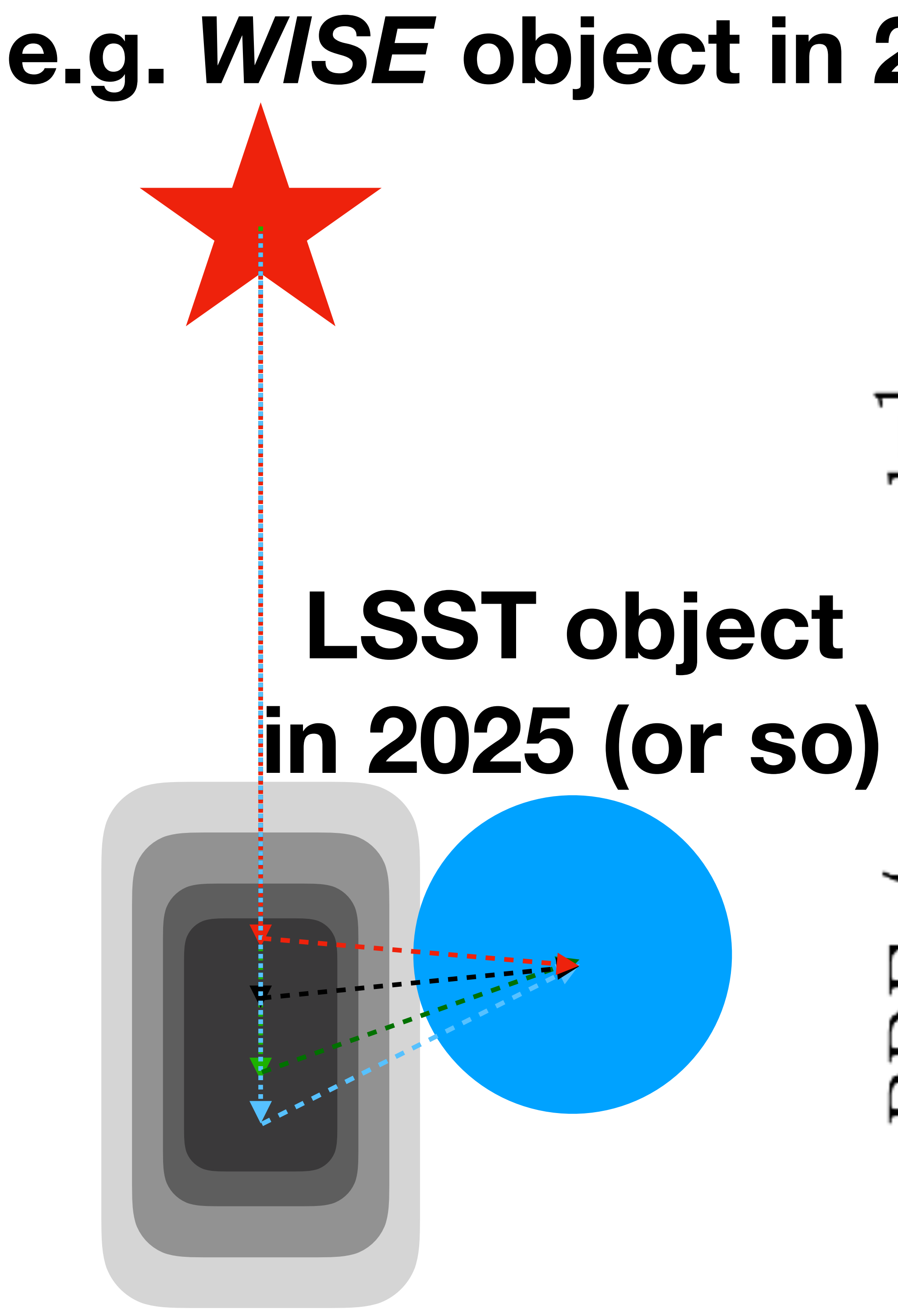
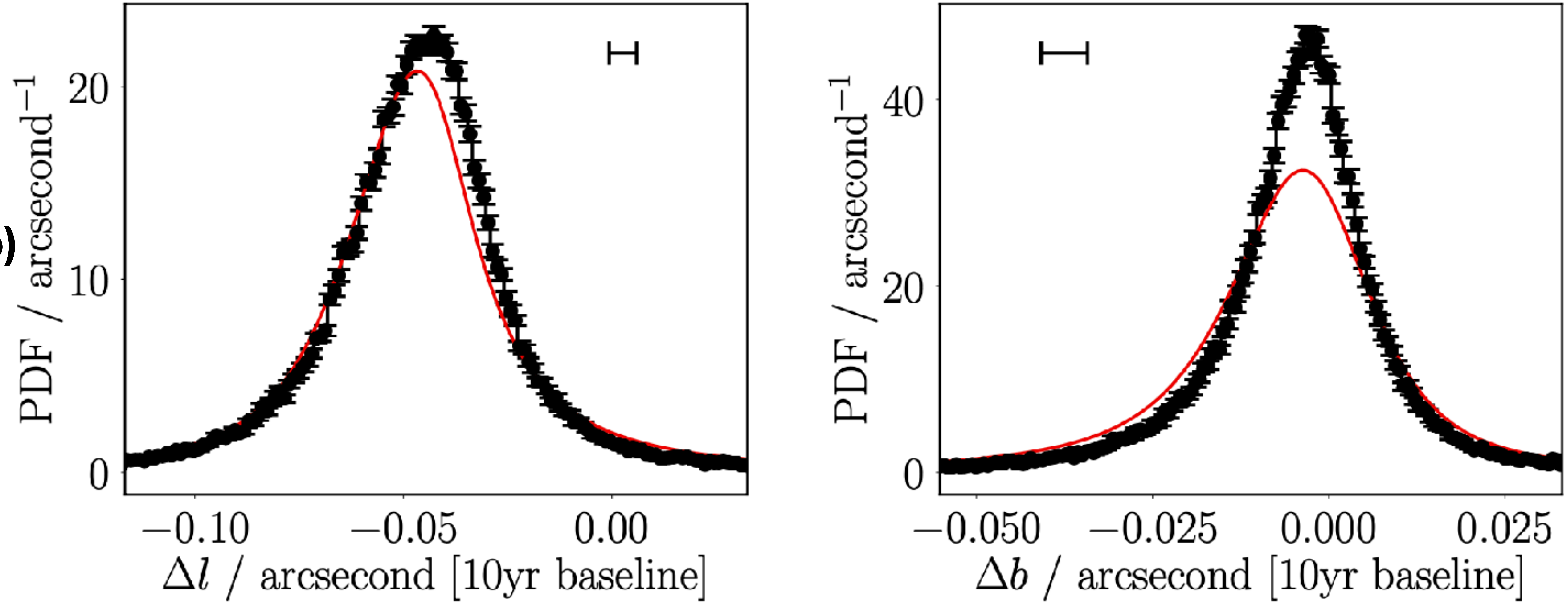
Including Unknown Proper Motions





Because this function works in *separation*, rather than pure *position*, space, we apply the distribution after calculating G.

$$G' = G * h_{\mathrm{pm}}^{'} \qquad G = h_\gamma * h_\phi$$

$$h_\gamma = h_{\gamma,\mathrm{centroiding}} * h_{\gamma,\mathrm{perturbation}} * \ \dots$$

Wilson (2022, RASTI, in review)
Gaia eDR3 - Gaia Collaboration et al. (2021, A&A, 649, A1)

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Open Source Code: macauff

Matching Across Catalogues using the Astrometric Uncertainty Function and Flux

https://github.com/Onoddil/macauff

LSST:UK Consortium

(Points if you know your tartans!)

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Conclusions

- **Blended star contamination causes positional shifts, now modelled robustly for the first time in the AUF**
  - **Crucial for low angular resolution + high SNR data like *WISE* or *TESS* and crowded fields like LSST**
- **Modelling of statistical flux contamination allows for the recovery of “true” fluxes**
- **Can use photometry in catalogues to break false match degeneracies of multiple counterparts**
  - **Symmetric data-driven photometric likelihood now possible**
- **Can include other kinds of offsets, like unknown proper motions, easily within AUF match framework**

- **Upcoming LSST:UK cross-match service macauff — let me know your thoughts/needs/hopes/dreams**

Wilson & Naylor, 2017, MNRAS, 468, 2517
Wilson & Naylor, 2018a, MNRAS, 473, 5570
Wilson & Naylor, 2018b, MNRAS, 481, 2148
Wilson (2022, RNAAS)
Wilson (2022, RASTI, in review)

https://github.com/Onoddil/macauff

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