

Put me here!

Towards Solving the Catalogue Cross-Match Problem

Tom J Wilson (he/him) and Tim Naylor
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University of Exeter



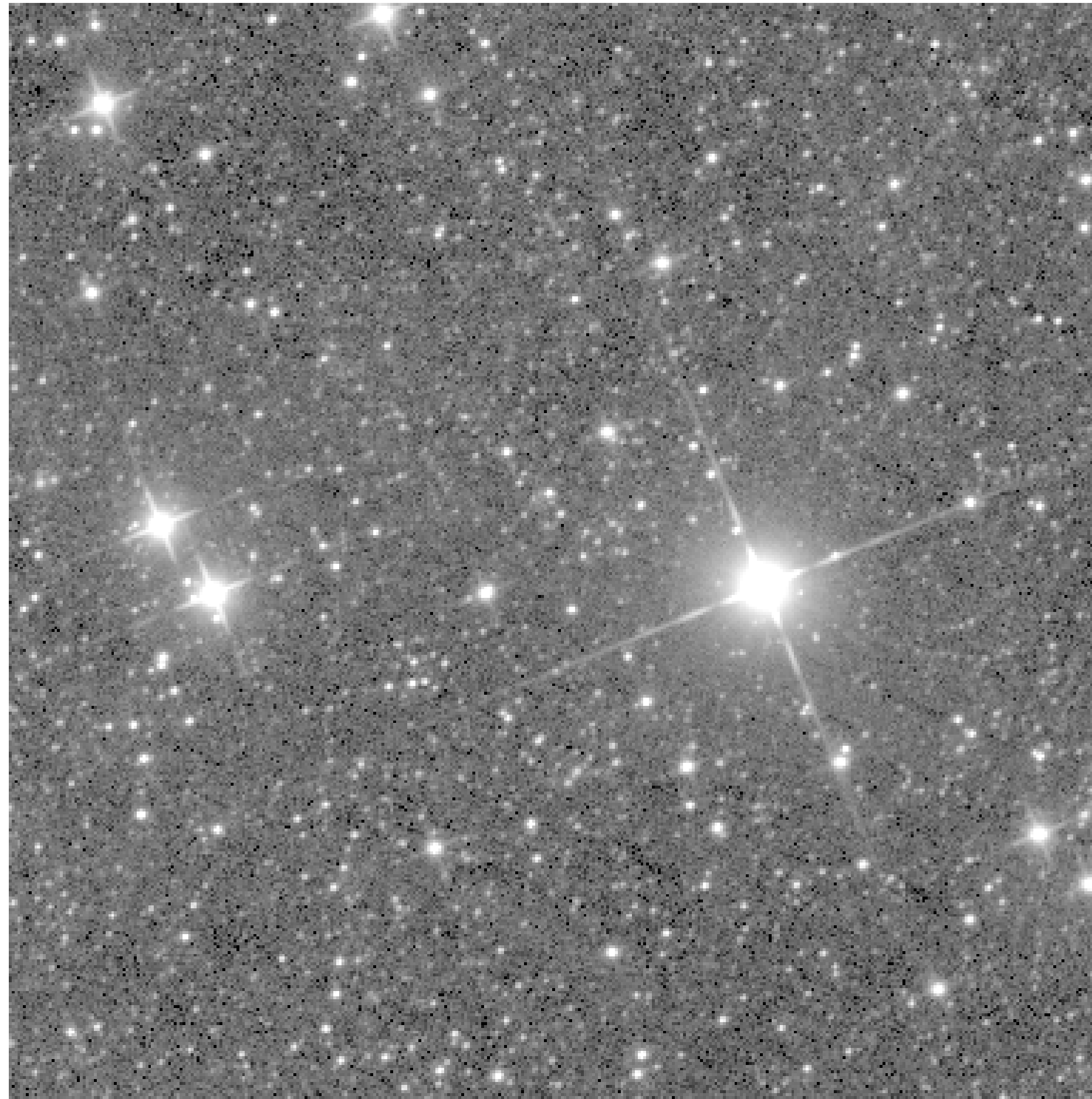
Carnegie EPL, 27/May/22

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What is the Cross-Match Problem?

- **We extend the probability-based catalogue cross-match schemes of others in a few key areas:**
 - * **include additional descriptions of detected source deviation from “true” location**
 - ★ **crowded fields (*WISE*, *LSST*) significantly affected by unresolved hidden contaminants**
 - ★ **but we also model unknown proper motions**
 - * **false matches can be solved by the use of the photometry of the objects**
- **This new framework then allows for useful, secondary information that has wide scientific application**
 - * **simulations of unresolved objects allow us to estimate how much additional flux is hidden within a detection, crucial for confidence in things like IR excesses or extinction**
 - * **finding “odd” objects based on whether they match or not when including photometric information; useful for rare objects, transients, variability etc.**

Photometric Observations

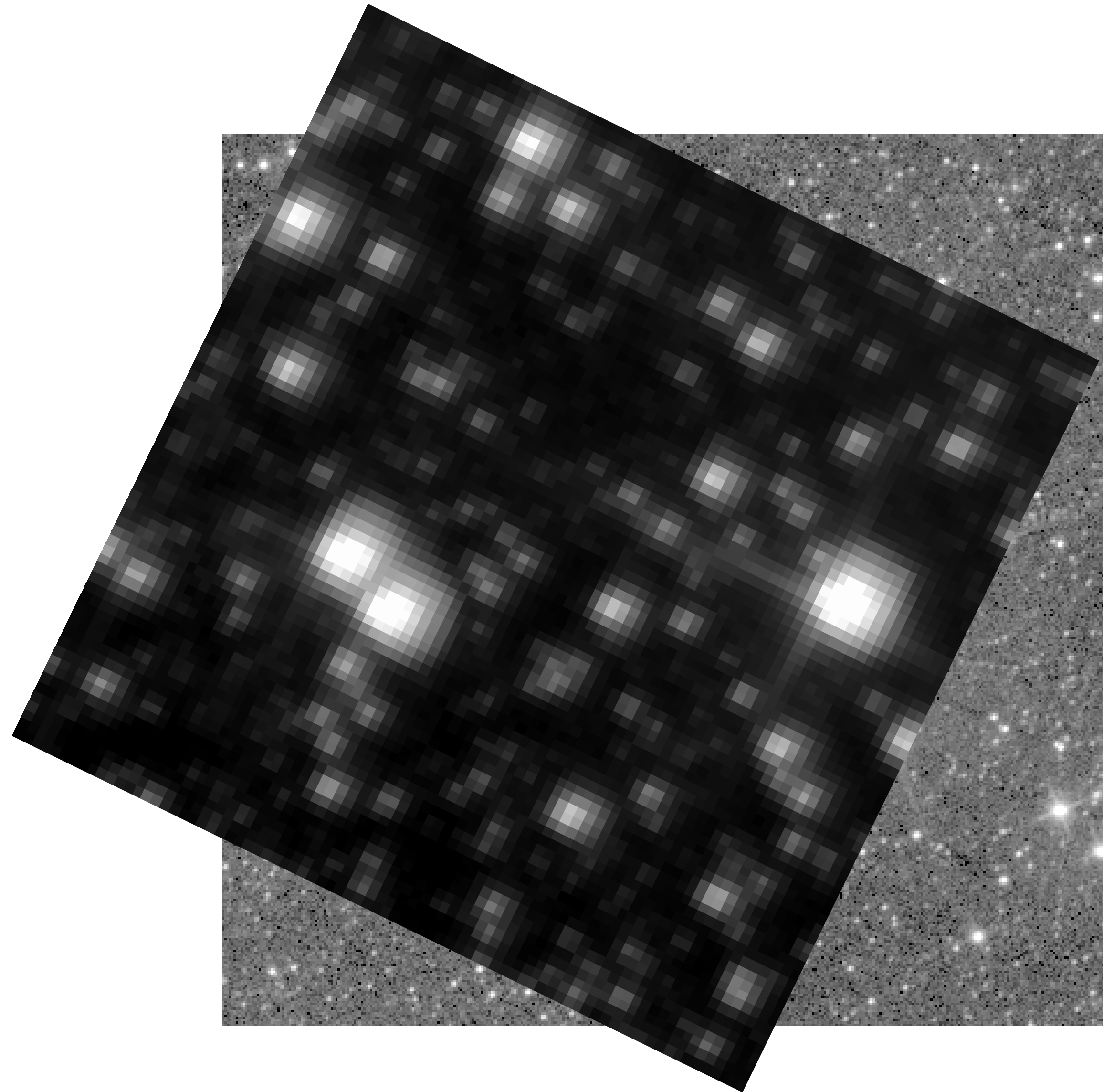


WISE - Wright et al. (2010)

WISE W1

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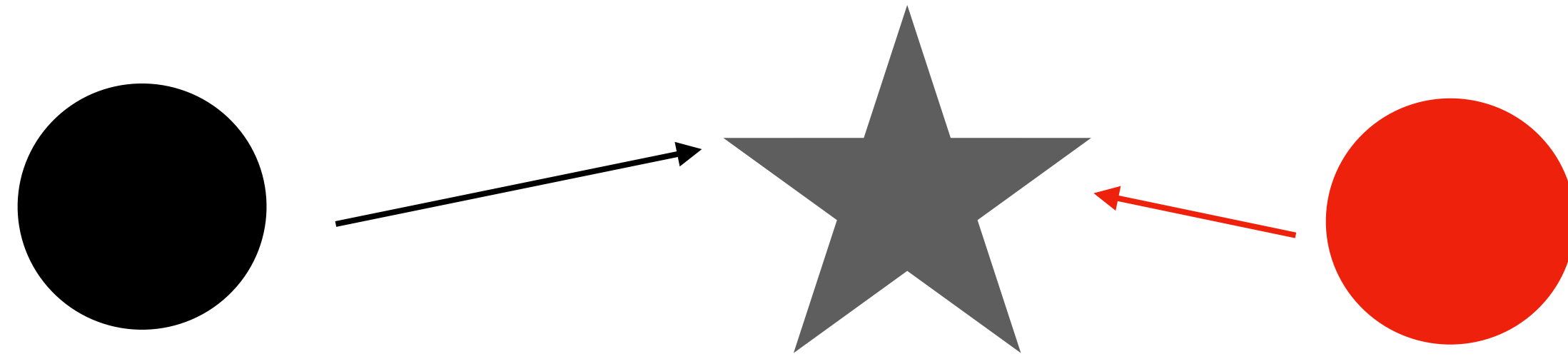
Photometric Observations



WISE - Wright et al. (2010)
TESS - Ricker et al. (2015)

TESS T
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Counterpart Assignment



— or —



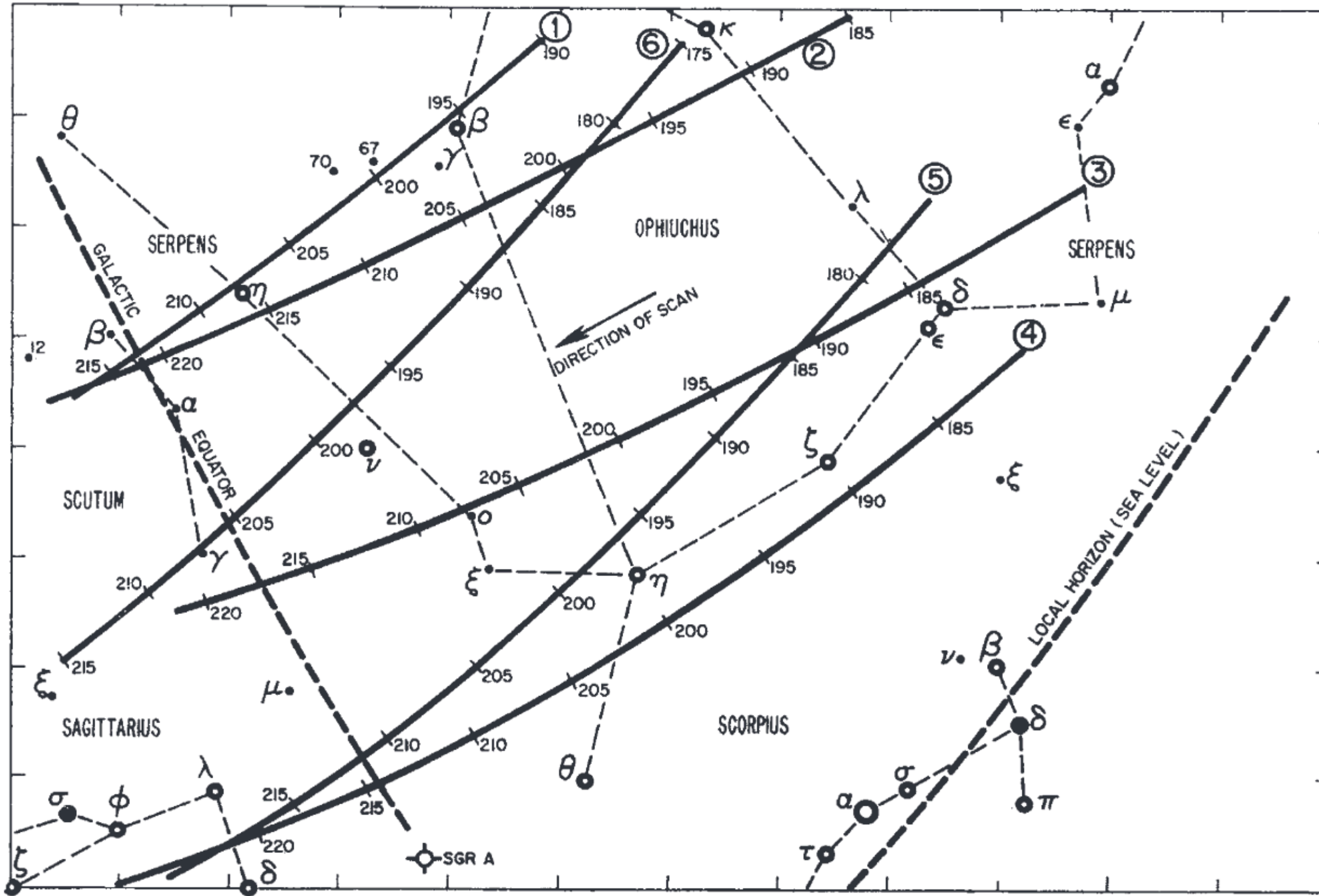
Matching Constellations



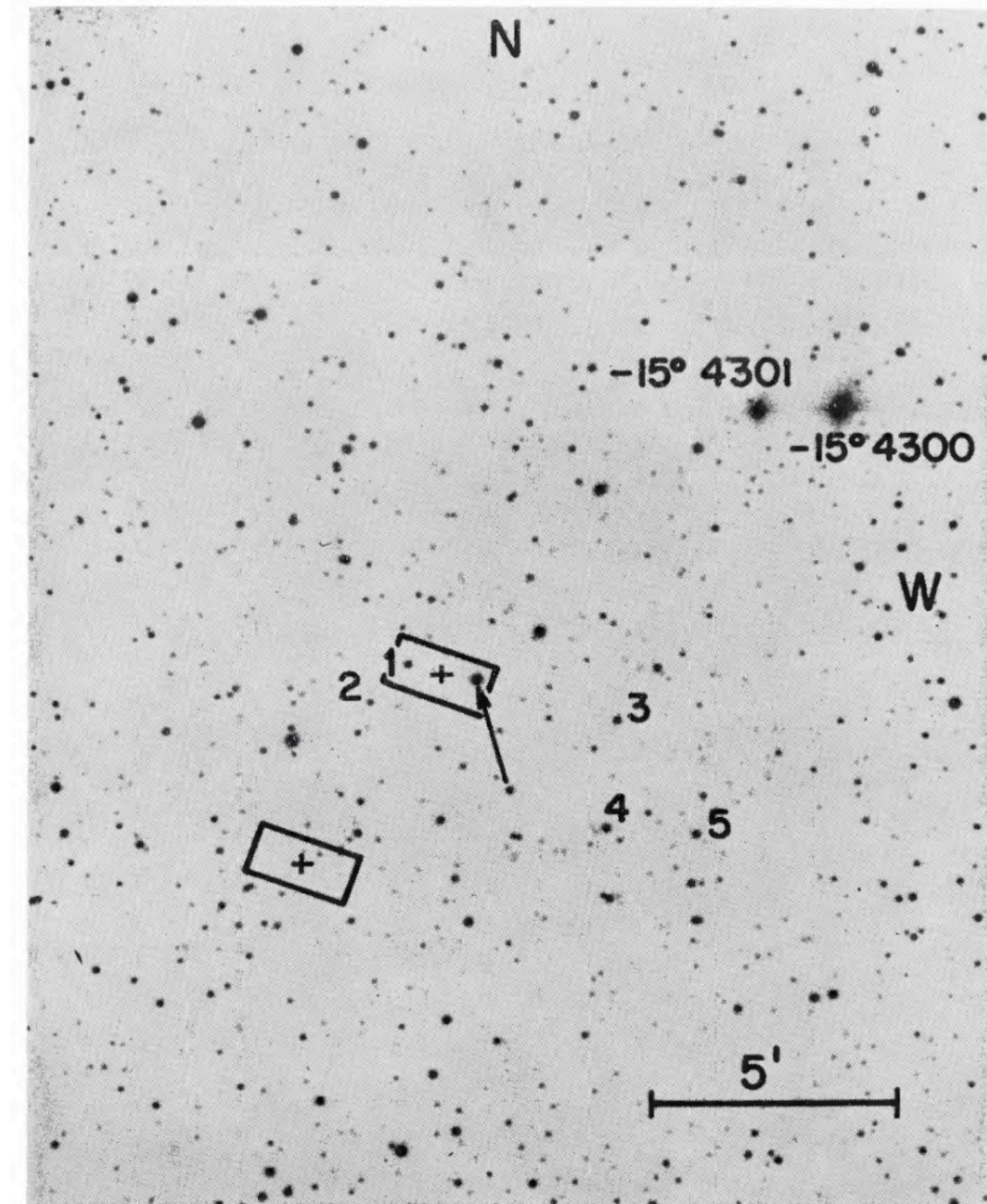
Technology Abounds

- **Ancient lists of stars (Ptolemy, 150; Brahe, 1598)**
- **Galileo invents the telescope (1610)**
- **Greenwich Observatory catalogues (e.g. Bradley, 1798)**
- **Astrophotography invented (Bond & Whipple, 1850)**
- **Harvard Observatory surveys (8th magnitude, 1882-1886)**
- **Astrographic Chart (11th magnitude; 1887-1962)**
- **Carte Du Ciel (14th magnitude; 1880s-never finished)**
- **Invention of the CCD (Boyle & Smith, 1970)**
- **InfraRed detector invented (Forrest et al. 1985)**
- **4- and 5-m class telescopes (1970s-1980s; e.g. LAT, MMT, UKIRT, CFHT, WHT)**
- **Space Telescopes (1980s-2010s; e.g. IRAS, ISO, AKARI, *WISE*, *Spitzer*)**
- **All-sky ground-based surveys (e.g. 2MASS, 1997-2001; SDSS, 2000-; Pan-STARRS, 2010-).**

X-ray Detections: Hunting for Sco X-1

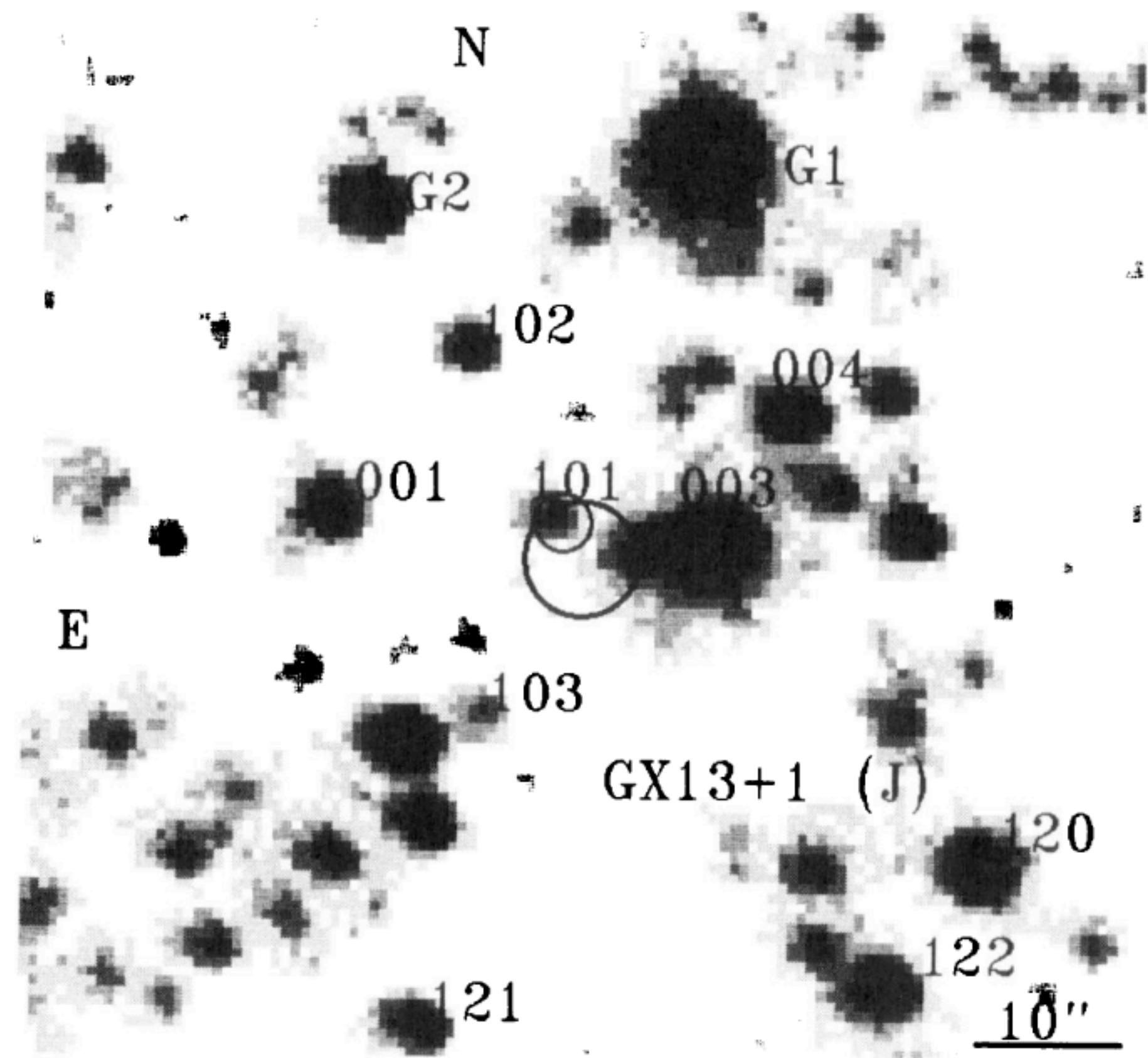


Giacconi, Gursky, & Waters (1964)

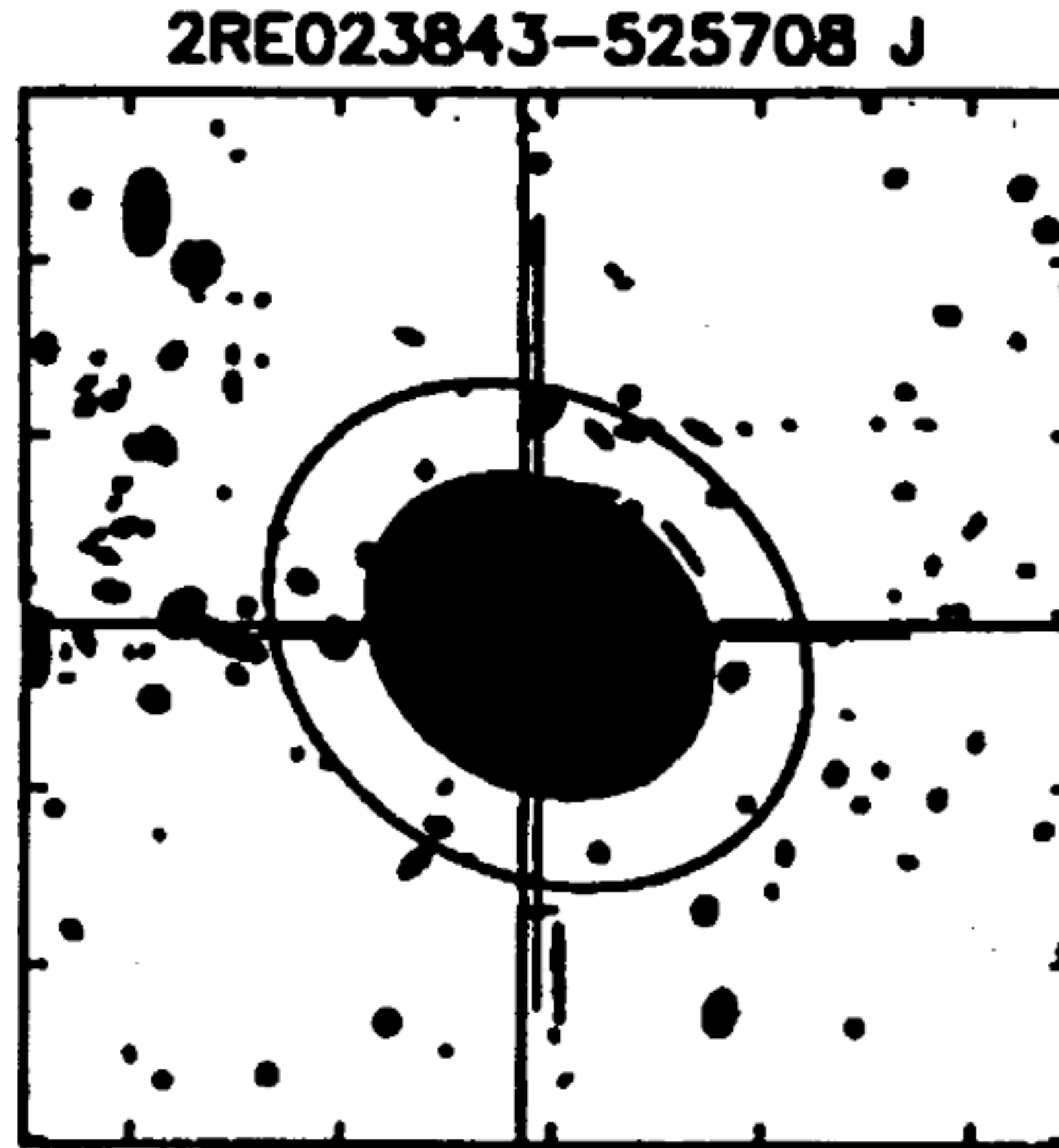


Sandage et al. (1966)

The Brightest Star in the Sky



Naylor, Charles, & Longmore (1991)

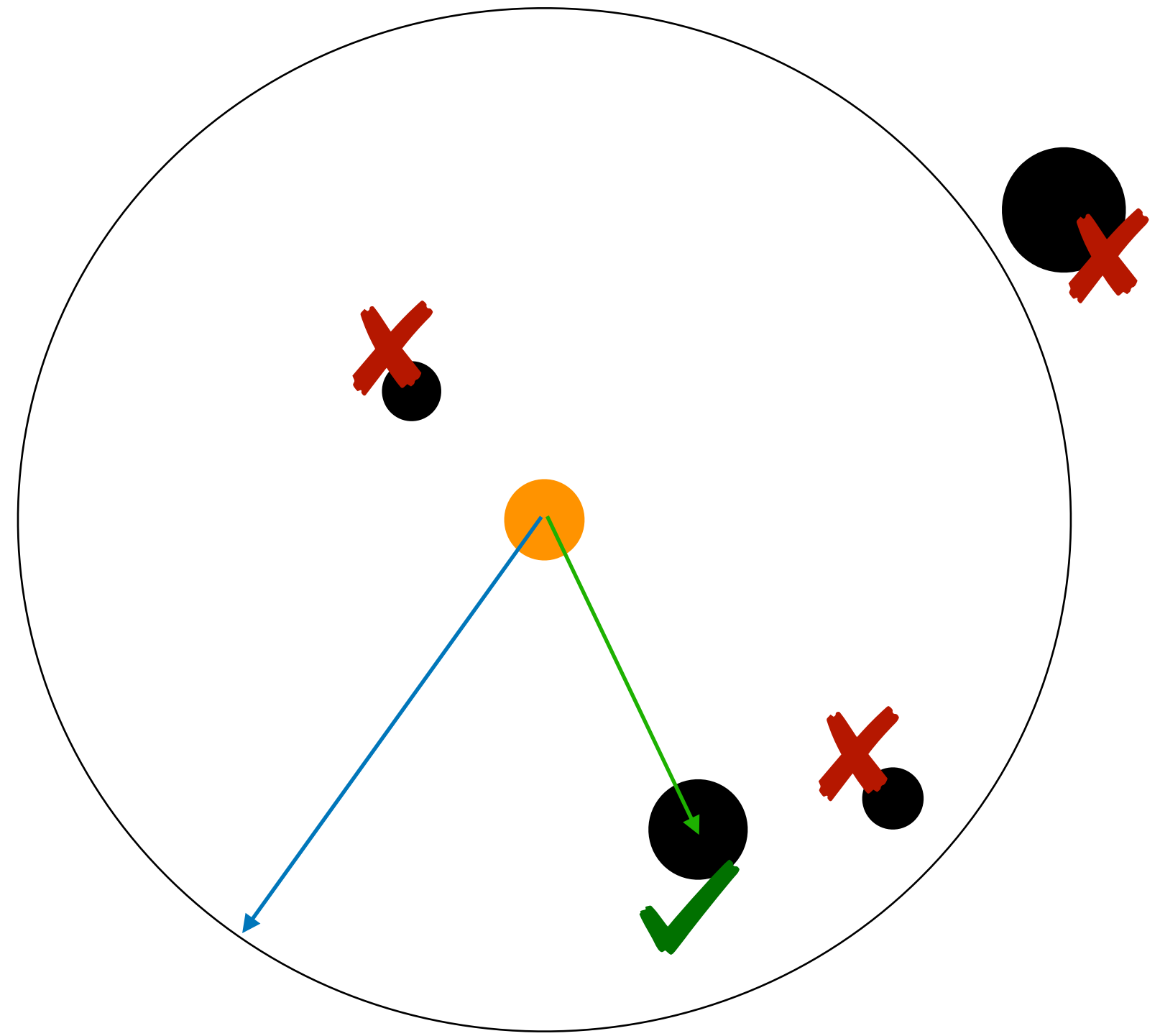


Mason et al. (1995)

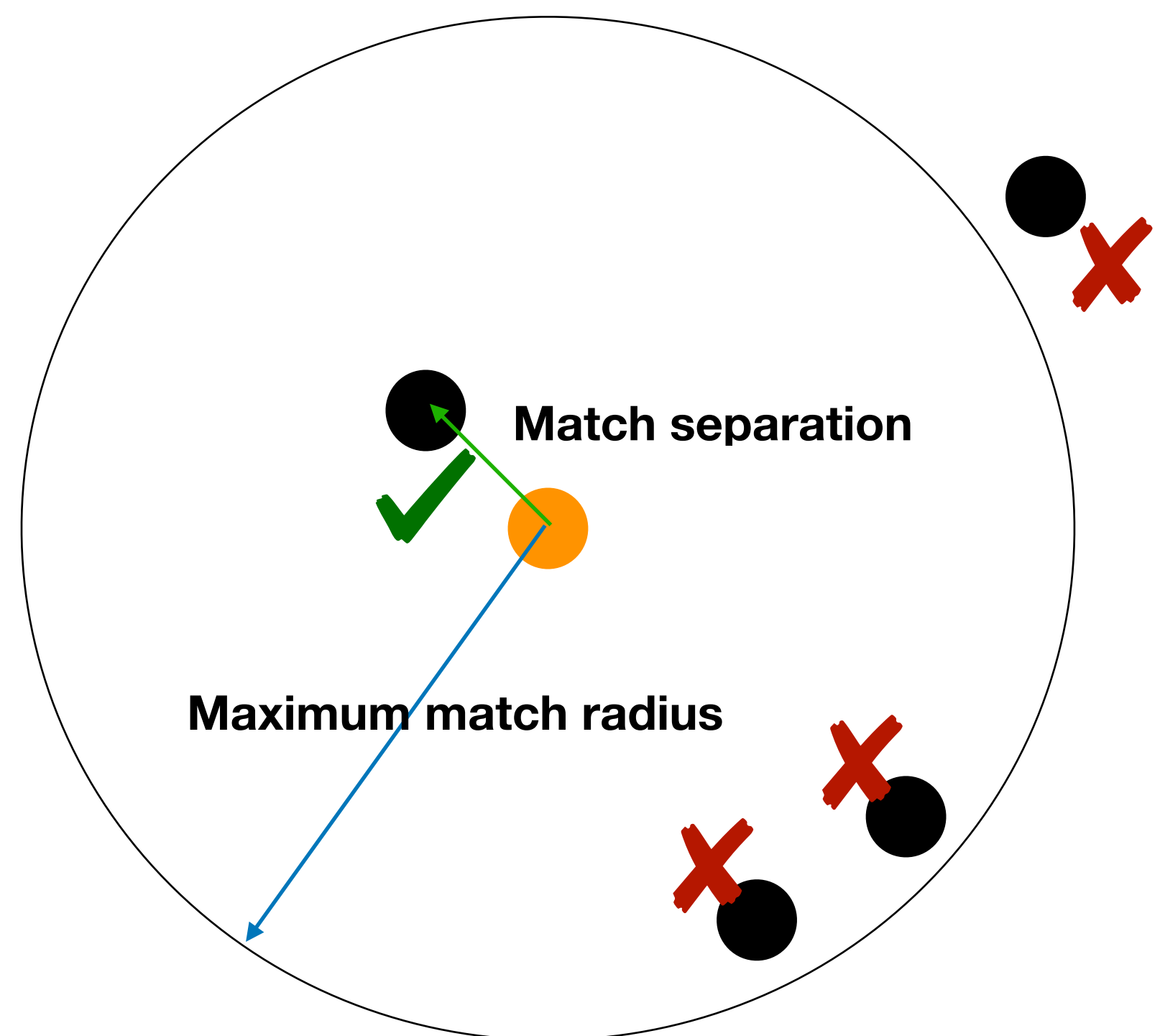
“...X-ray sources are rare events; bright optical sources are also rare events, so the observation of an X-ray source and a bright optical source in the same region of the sky is considered a non-random event”

Fotopoulou et al. (2016)

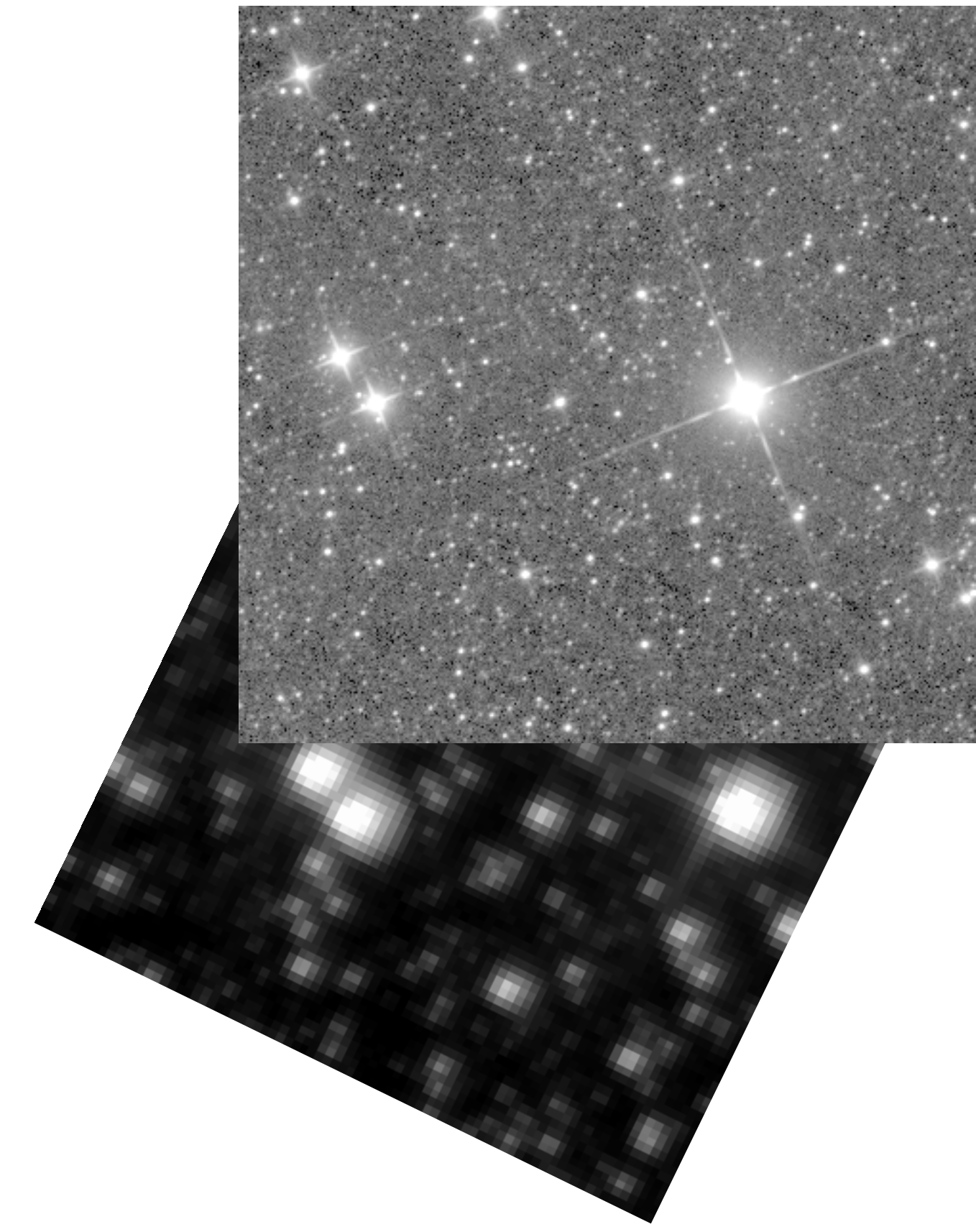
“Traditional” Cross-Matching



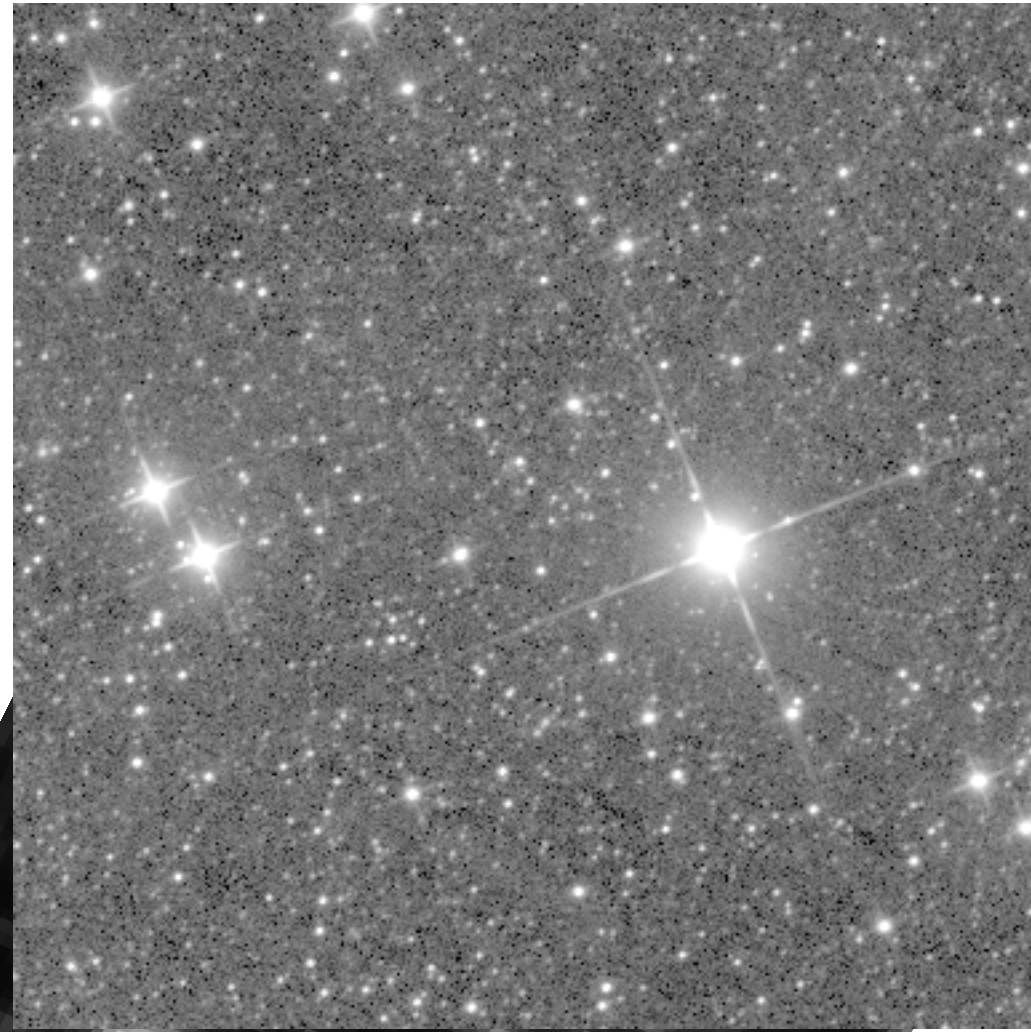
Declination / degrees



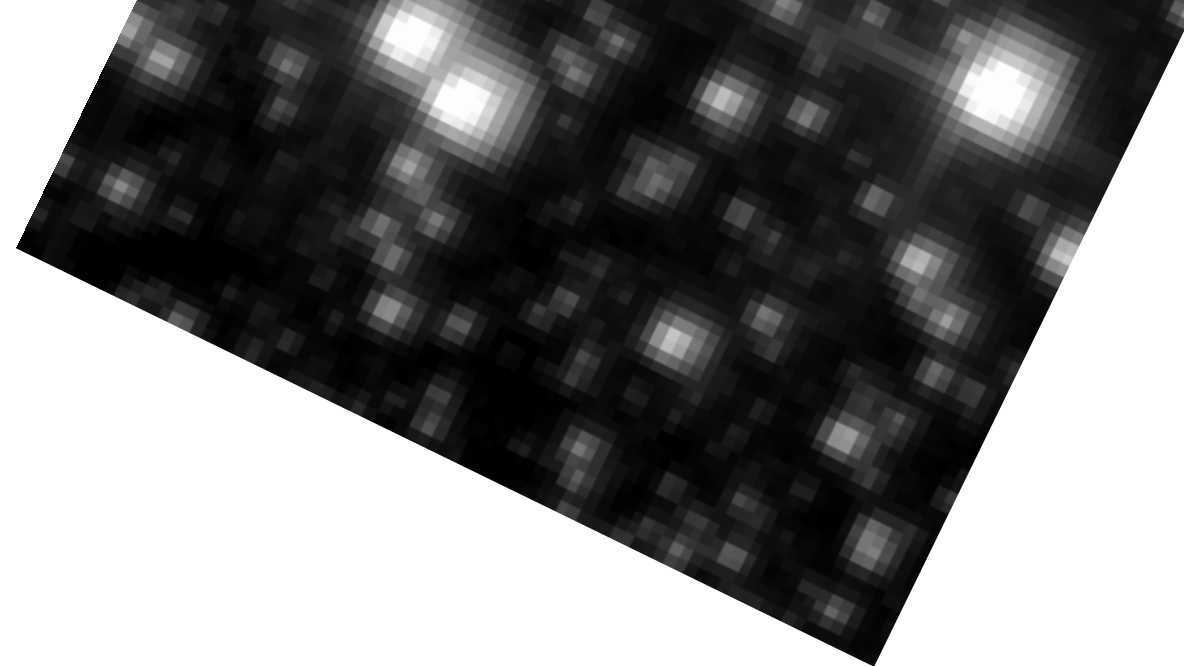
Right Ascension / degrees



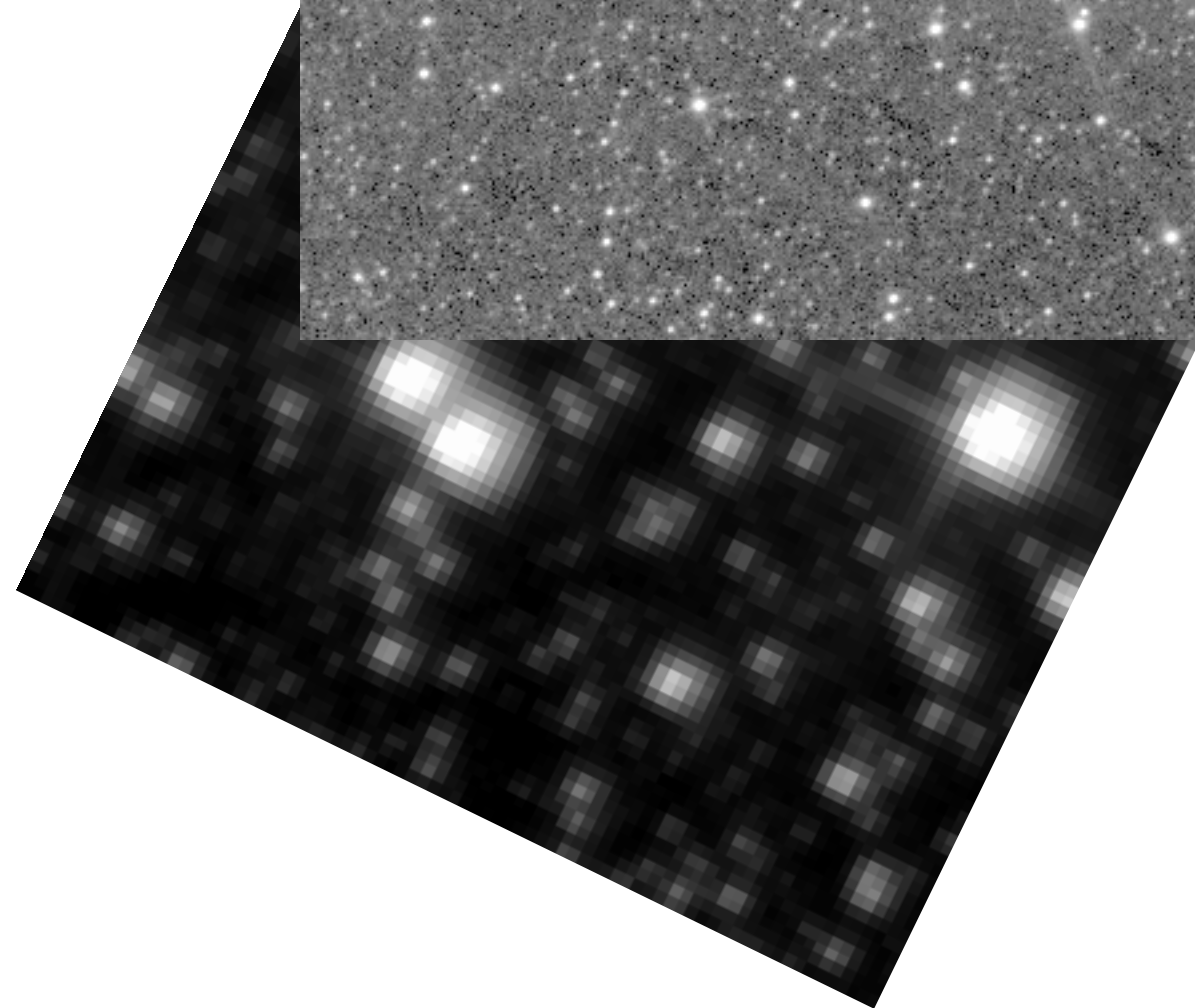
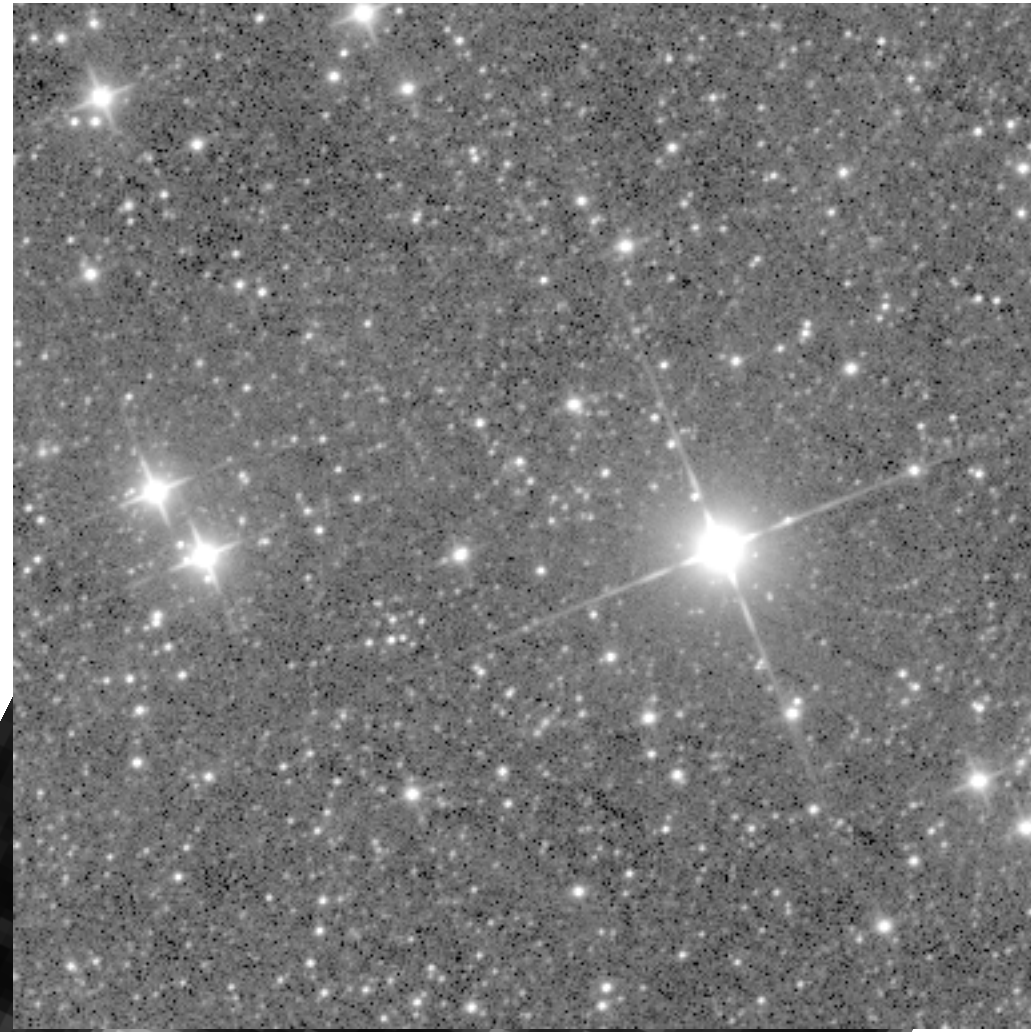
The Astronomy Error Function



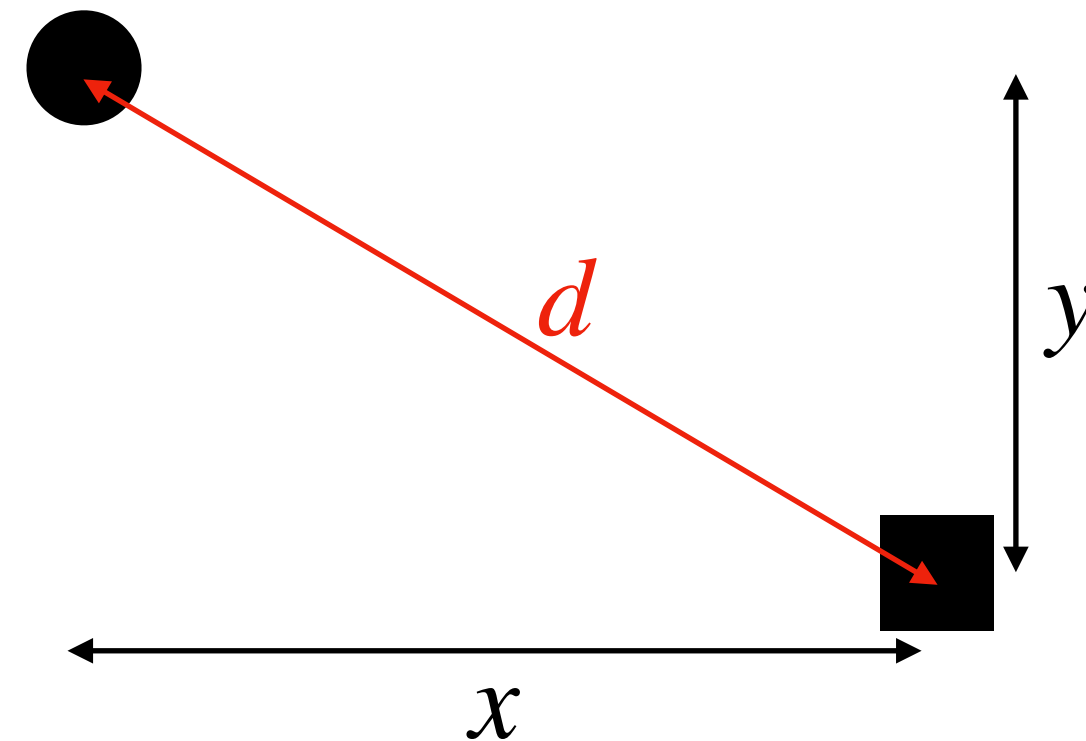
- 1) $p(x \text{ and } y) = p(x)p(y)$
- 2) $p(x)$ decreases as x increases
- 3) $p(x) = p(-x) \Rightarrow p(x^2)$



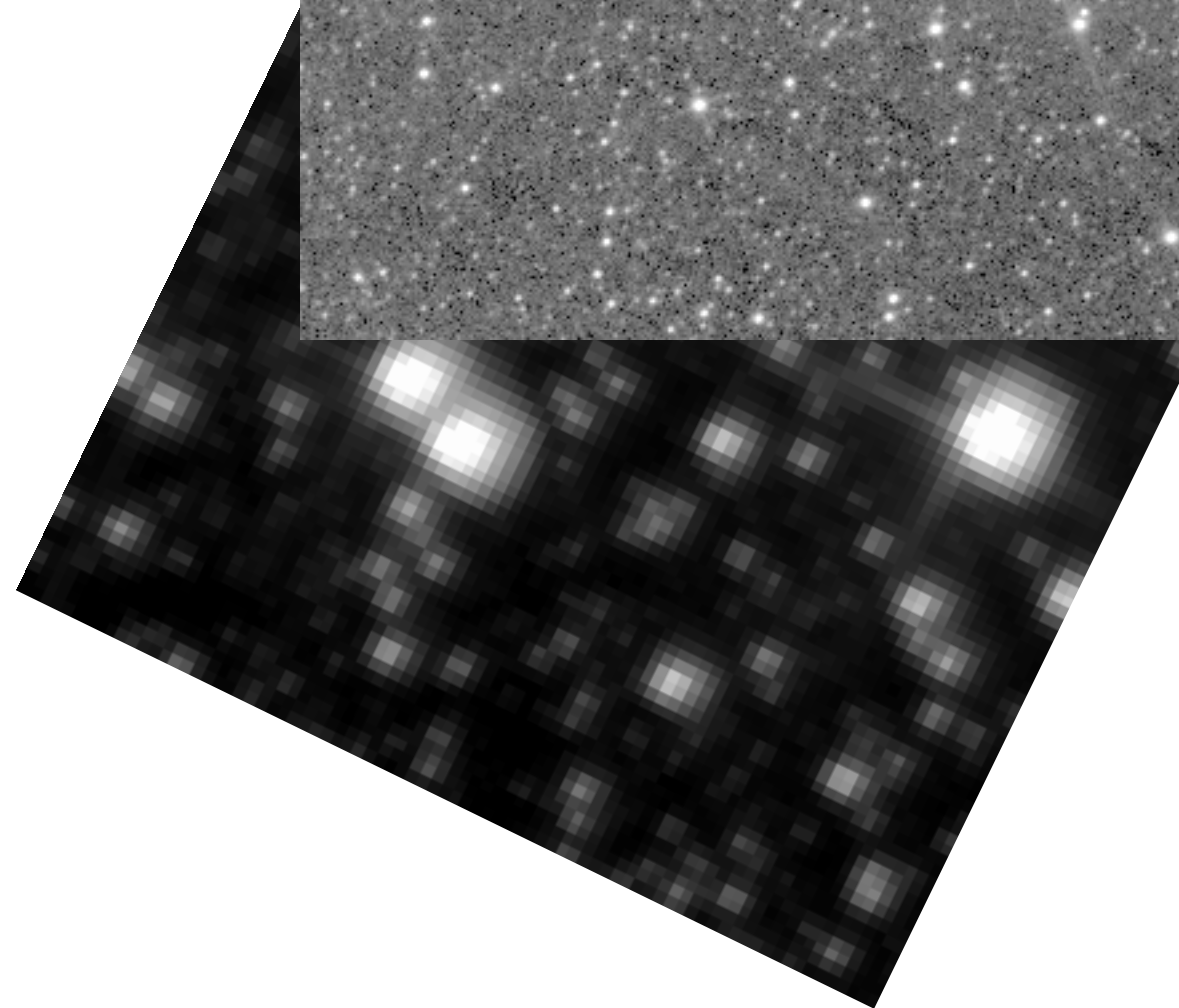
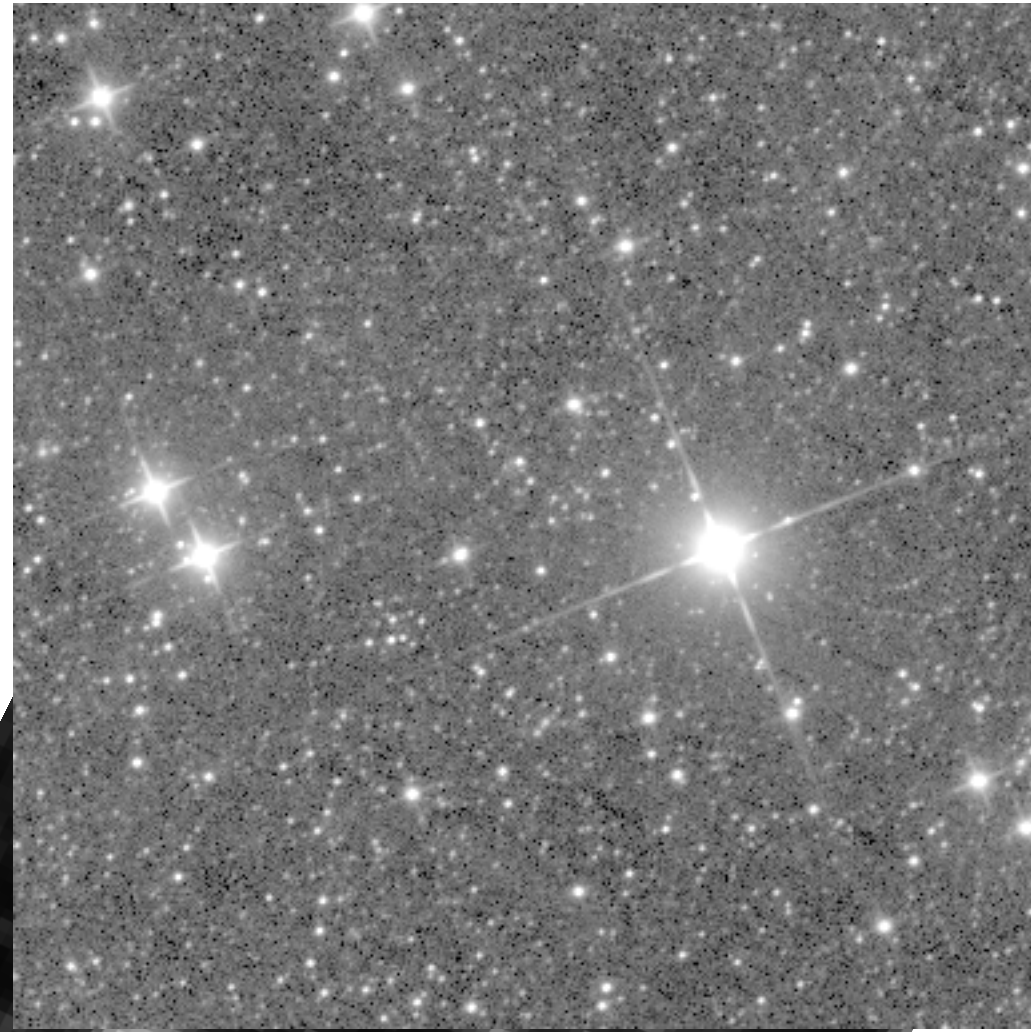
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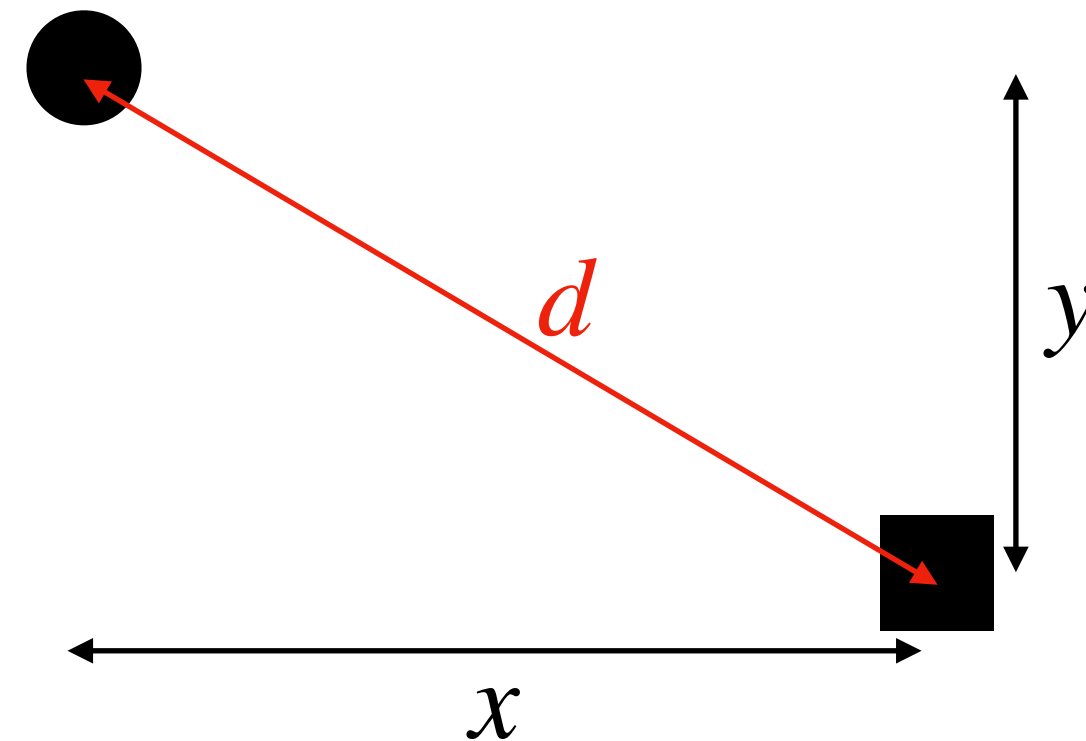
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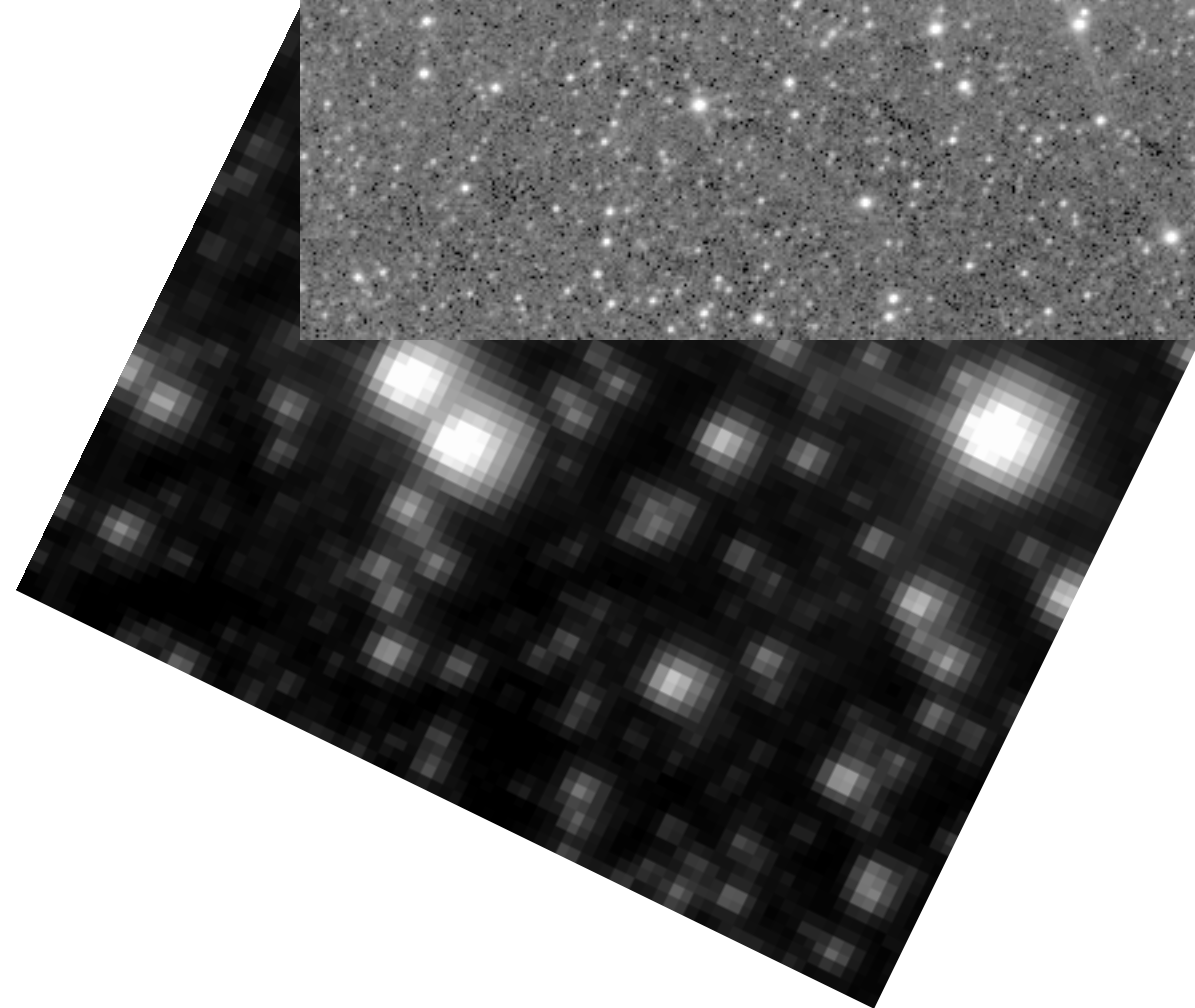
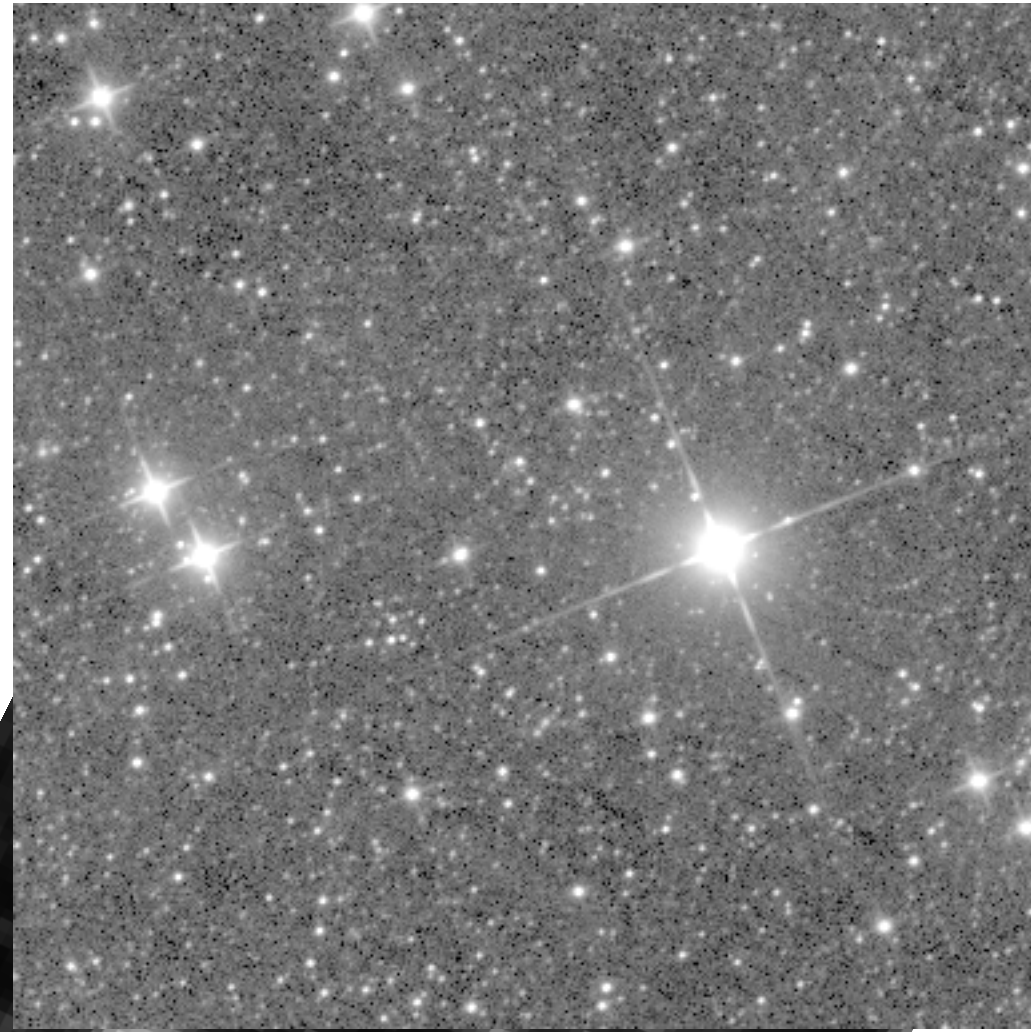


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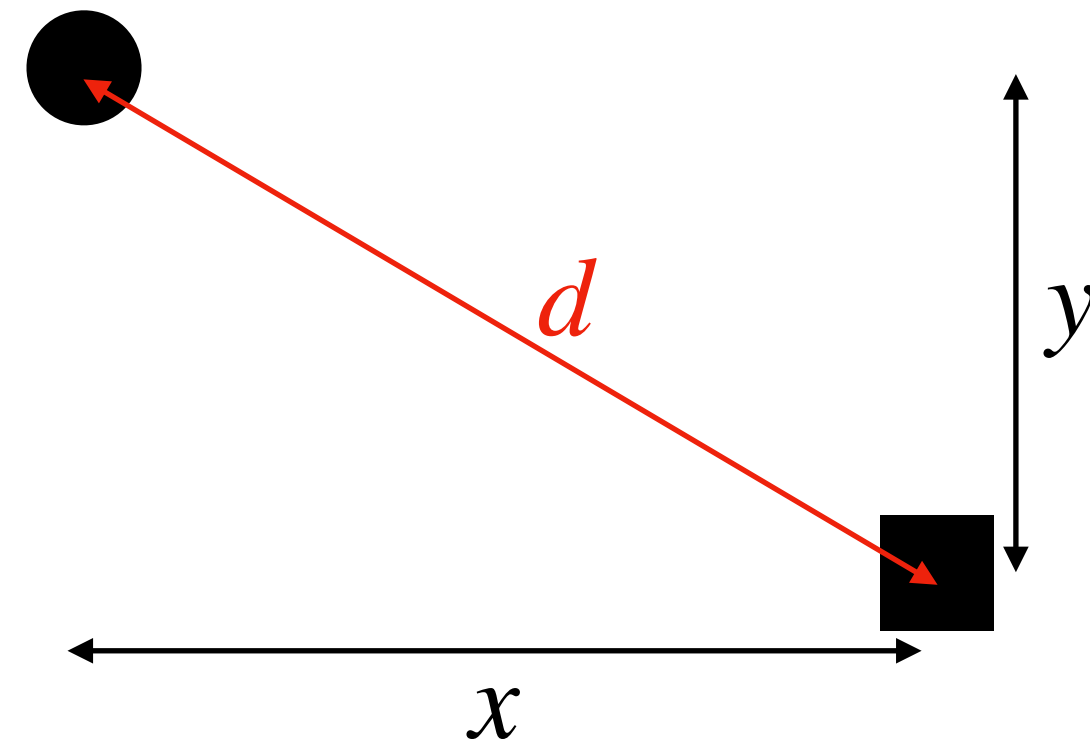


$$p(d^2) = p(x^2 + y^2) = p(x^2)p(y^2)$$

The Astronomy Error Function



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$$p(d^2) = p(x^2 + y^2) = p(x^2)p(y^2)$$

$$g(x, y, \sigma) = (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2} \frac{x^2 + y^2}{\sigma^2}\right)$$

Probabilistic Cross-Matching

The Likelihood Ratio

$$dp(r|id) = r \times e^{-r^2/2} dr.$$

$$dp(r|c) = 2\lambda r \times e^{-\lambda r^2} dr$$

$$LR(r) = dp(r|id)/dp(r|c) = \frac{1}{2\lambda} \exp\left\{\frac{r^2}{2}(2\lambda - 1)\right\}$$

de Ruiter, Willis, & Arp (1977)

$$dp_{id} = Qr \exp\left(\frac{-r^2}{2}\right) dr. \quad dp_{uo} = 2\lambda r dr$$

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Wolstencroft et al. (1986)

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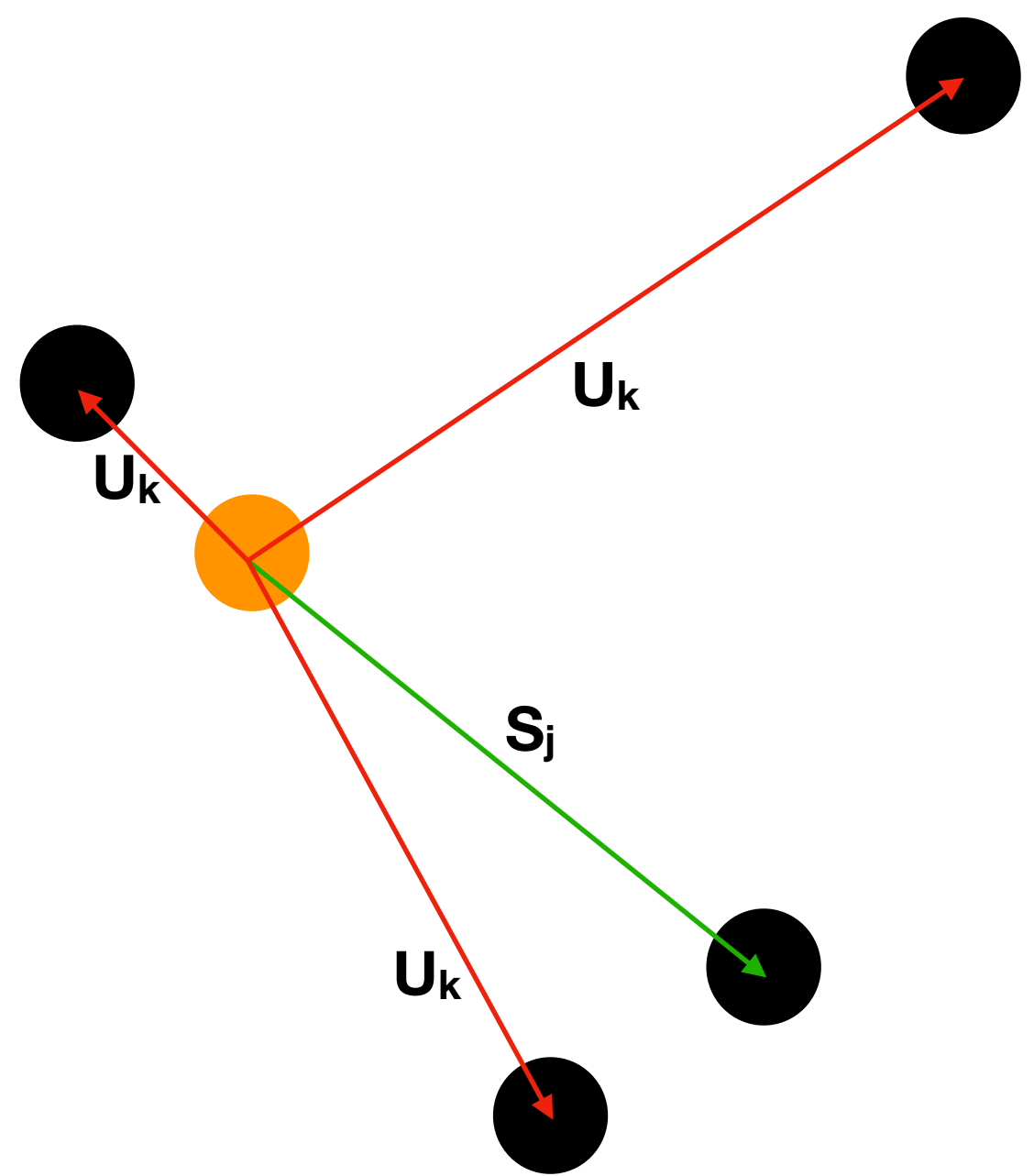
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Wolstencroft et al. (1986)

The "Reliability" – Sutherland & Saunders (1992)

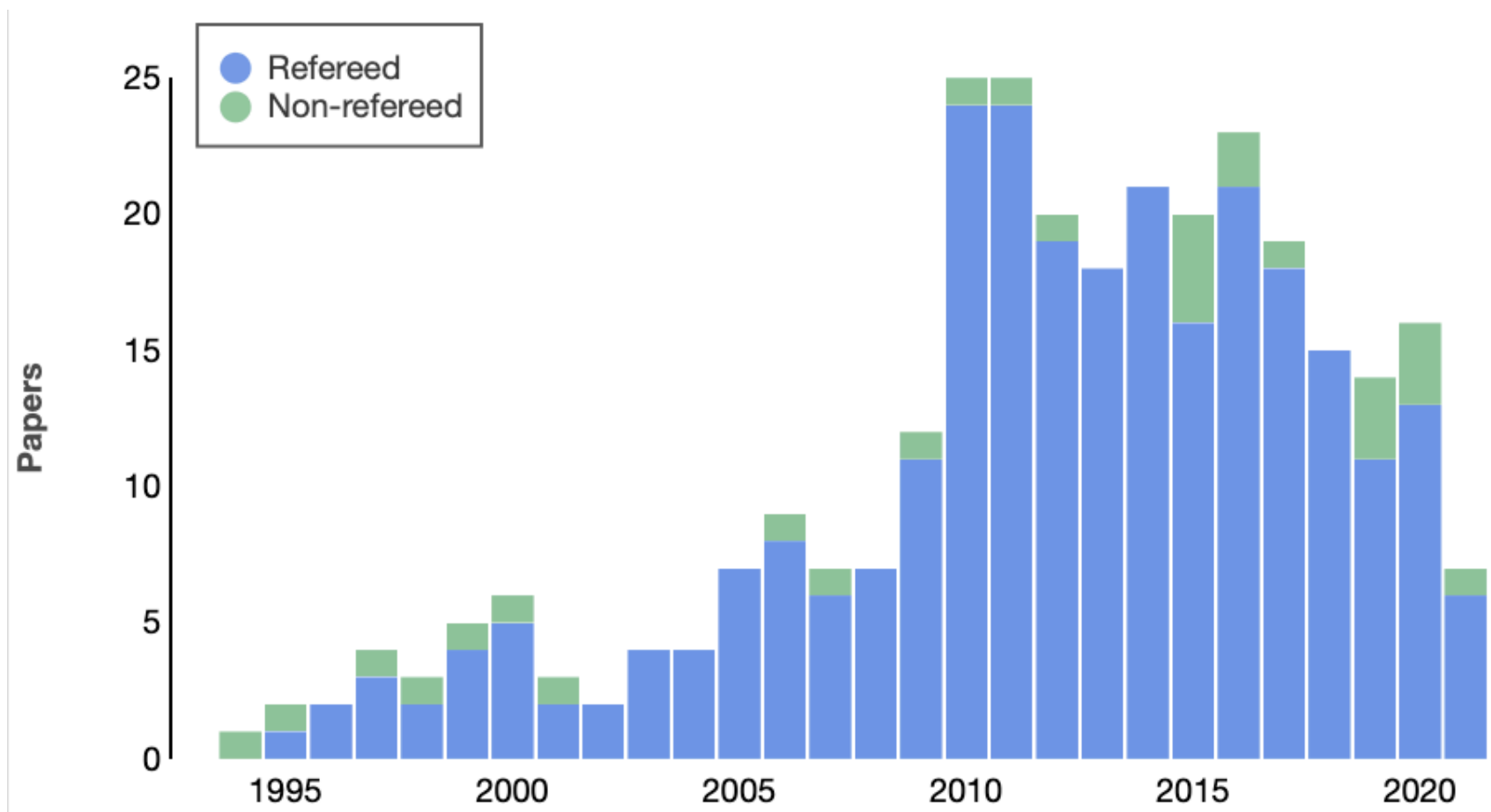
Declination / degrees



Right Ascension / degrees

$$R_j = \frac{\Pr\left[S_j \cap \left(\bigcap_{k \neq j} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]}{\sum_i \Pr\left[S_i \cap \left(\bigcap_{k \neq i} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right] + \Pr\left[(m_S > m_{lim}) \cap \left(\bigcap_k U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]} = \frac{L_j}{\sum_i L_i + (1-Q)}$$

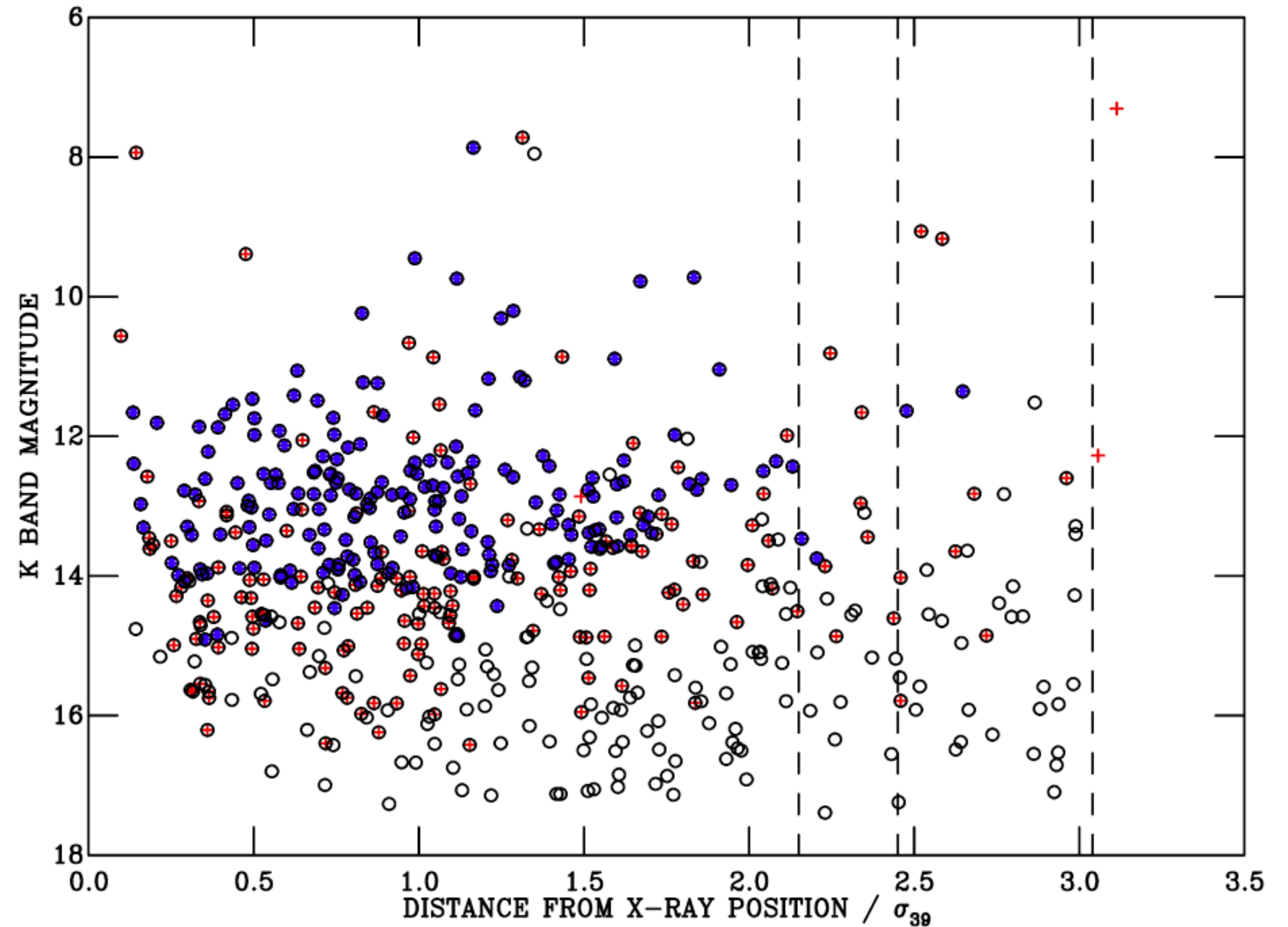
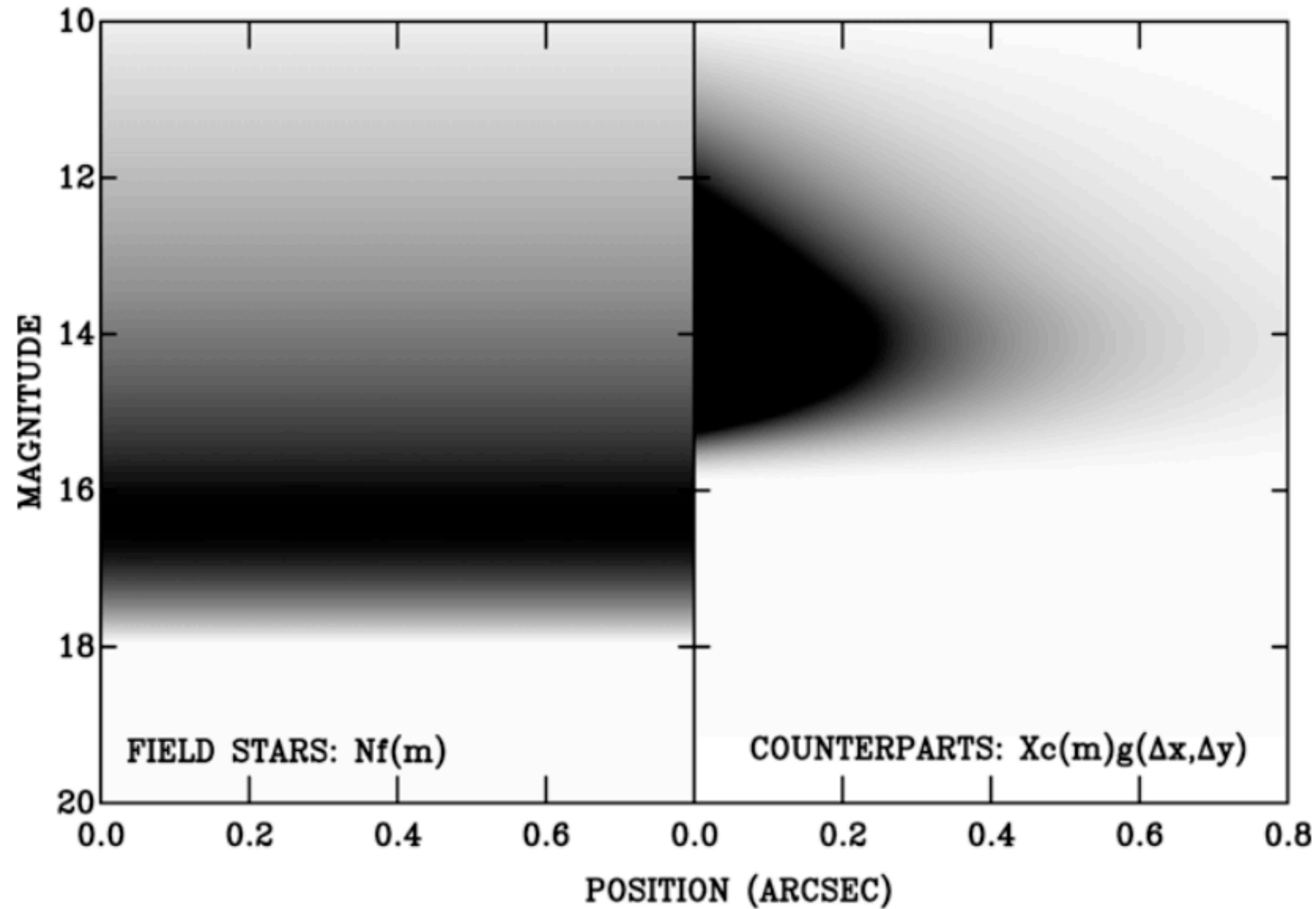
$$L = \frac{q(m, c) f(x, y)}{n(m, c)}$$



Probabilistic Cross-Matching

$$P(0) = \frac{1 - X}{1 - X + \sum_j \frac{Xc(m_j)g(\Delta x_j, \Delta y_j)}{Nf(m_j)}}$$

$$P(i) = \frac{\frac{Xc(m_i)g(\Delta x_i, \Delta y_i)}{Nf(m_i)}}{1 - X + \sum_j \frac{Xc(m_j)g(\Delta x_j, \Delta y_j)}{Nf(m_j)}}$$



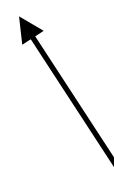
Probabilistic Cross-Matching

$$p(D|H) = \int p(\mathbf{m}|H) \prod_{i=1}^n p_i(\mathbf{x}_i|\mathbf{m}, H) d^3 m$$

$$p(D|K) = \prod_{i=1}^n \left[\int p(\mathbf{m}_i|K) p_i(\mathbf{x}_i|\mathbf{m}_i, K) d^3 m_i \right]$$

$$B(H, K|D') = \frac{\int p(\boldsymbol{\eta}|H) \prod_{i=1}^n p_i(\mathbf{g}_i|\boldsymbol{\eta}, H) d^r \boldsymbol{\eta}}{\prod_{i=1}^n \left[\int p(\boldsymbol{\eta}_i|K) p_i(\mathbf{g}_i|\boldsymbol{\eta}_i, K) d^r \boldsymbol{\eta}_i \right]}$$

Budavári & Szalay (2008)



Includes SED model fitting to all sources

Probabilistic Cross-Matching

Nearest neighbour or brightest neighbour: one-to-one, either astrometry OR photometry

Likelihood ratio: one-to-one matches, mostly just astrometry (e.g., Wolstencroft et al. 1986)

Reliability: One-to-many matches, uses photometry from one dataset (e.g. Naylor et al. 2013)

Budavári & Szalay (2008): one-to-one-to-one-to... matches, include SED fitting

e.g. Pineau et al. (2017): many-to-many-to-many-to... matches, no photometry implemented

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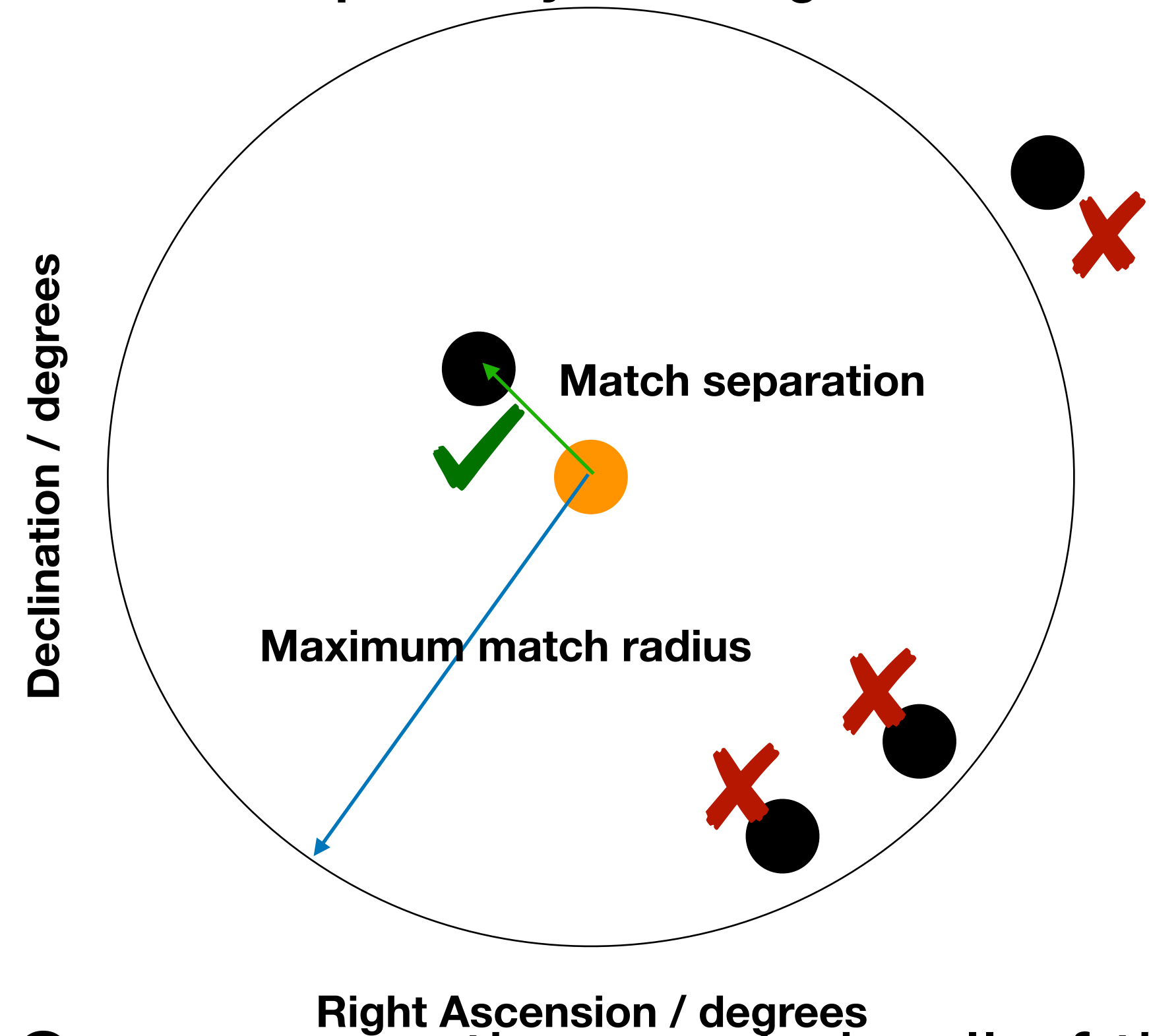
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One assumption made in all of these works: positional errors of sources are Gaussian!

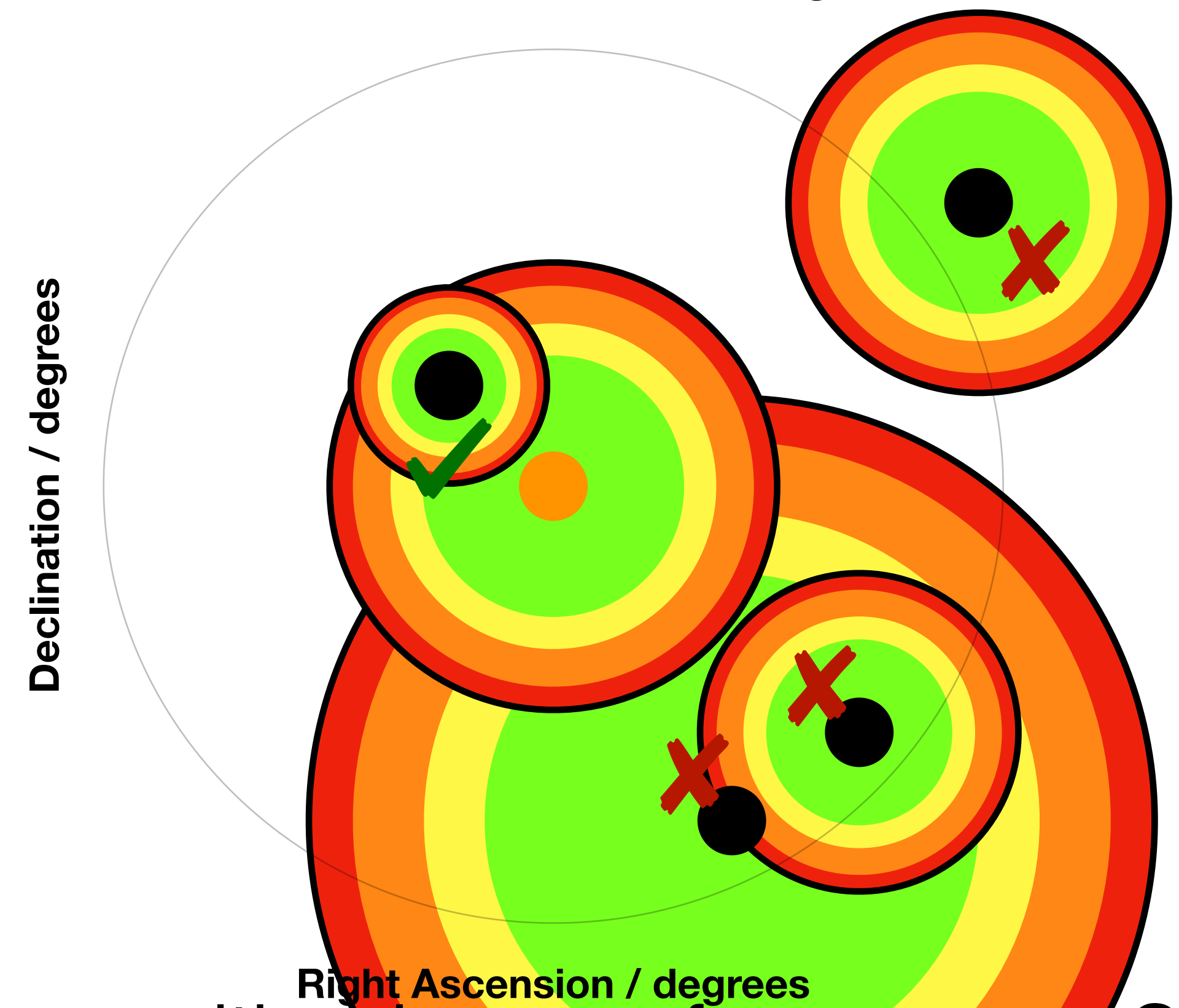
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Probabilistic Cross-Matching

Nearest neighbour/
proximity matching



Probabilistic matching



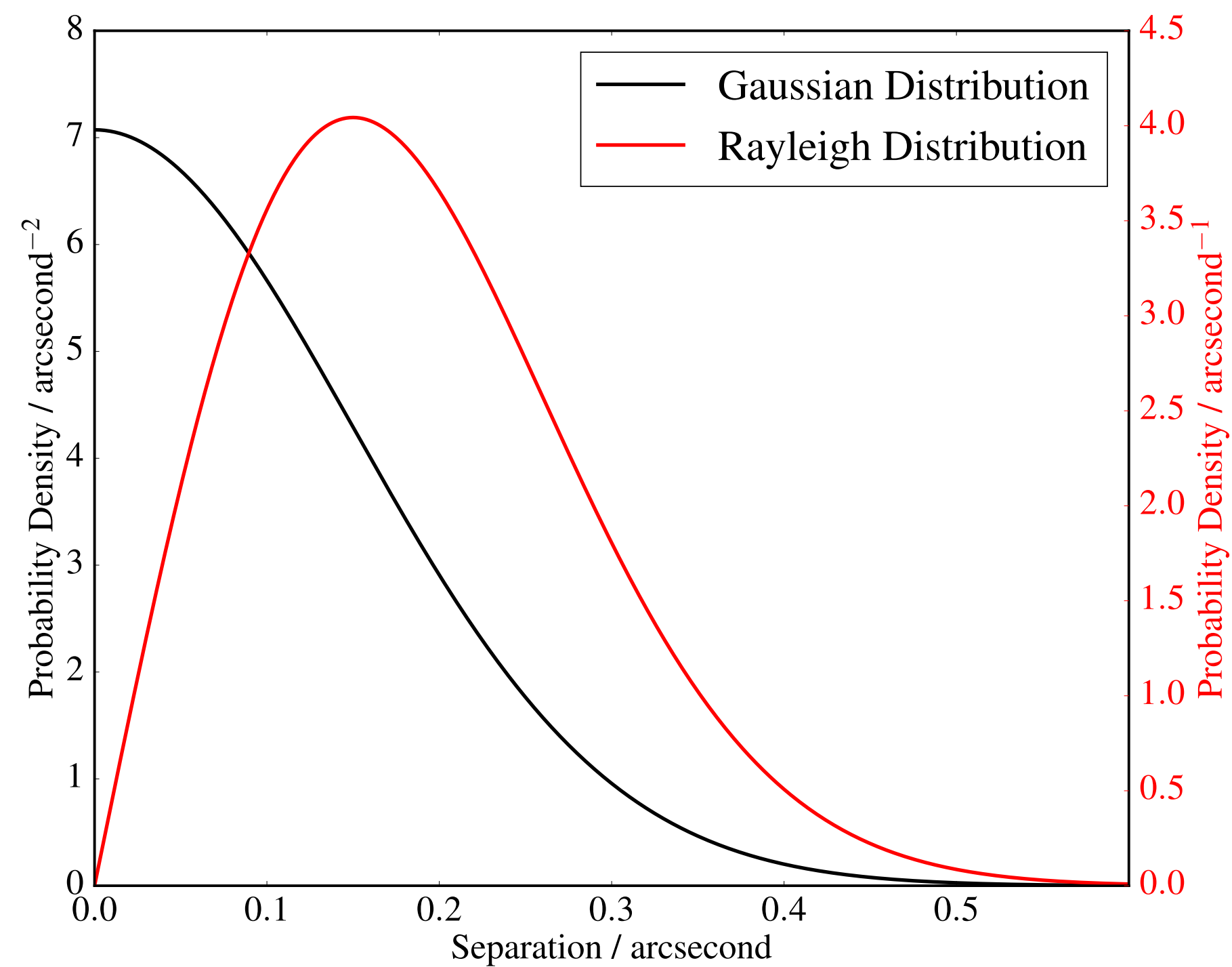
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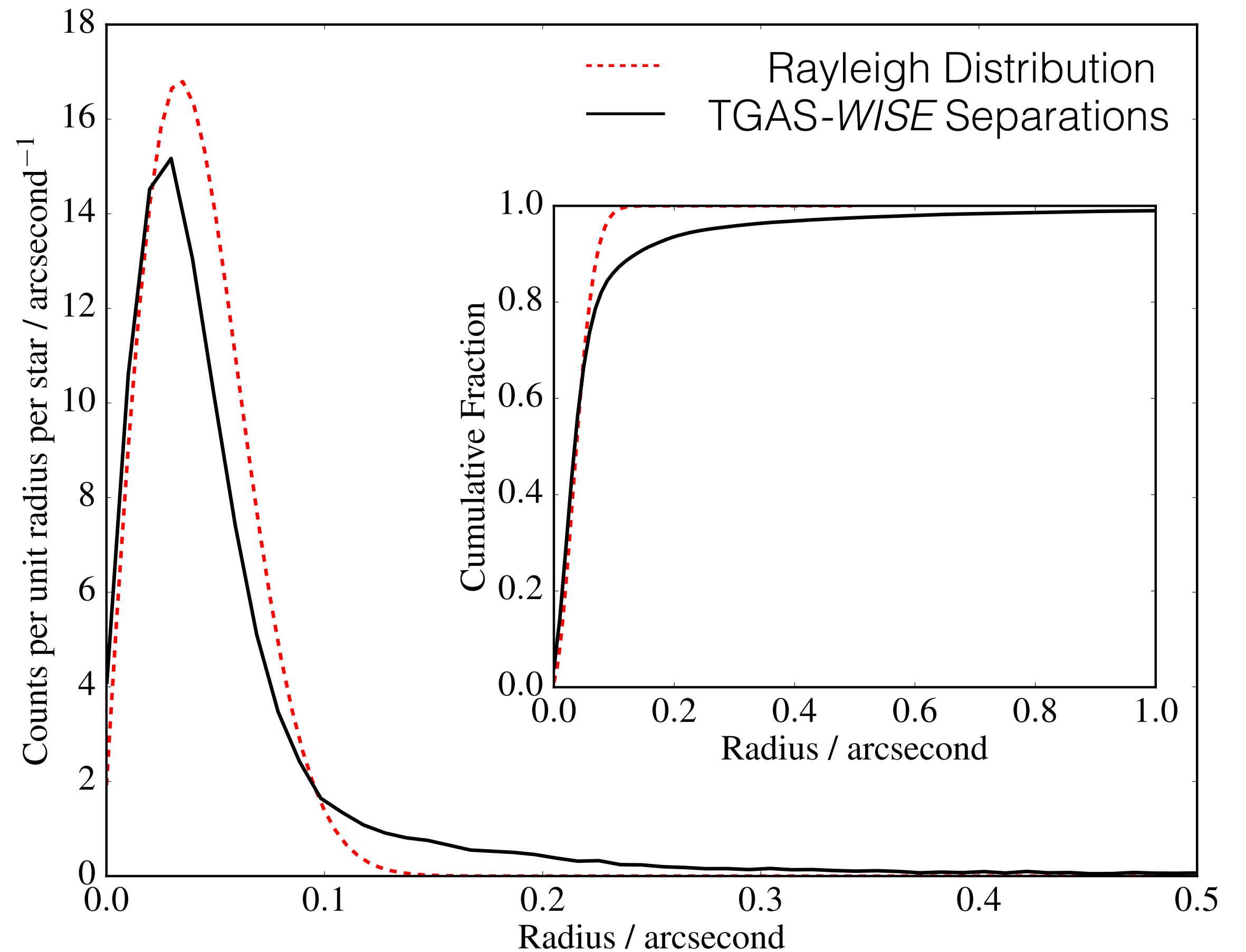
The *Astrometric Uncertainty* Function

$$g(x, y, \sigma) = (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2} \frac{x^2 + y^2}{\sigma^2}\right)$$

$$g(r, \sigma) = \frac{r}{\sigma^2} \exp\left(-\frac{1}{2} \frac{r^2}{\sigma^2}\right)$$



Wilson & Naylor (2017)
WISE - Wright et al. (2010)

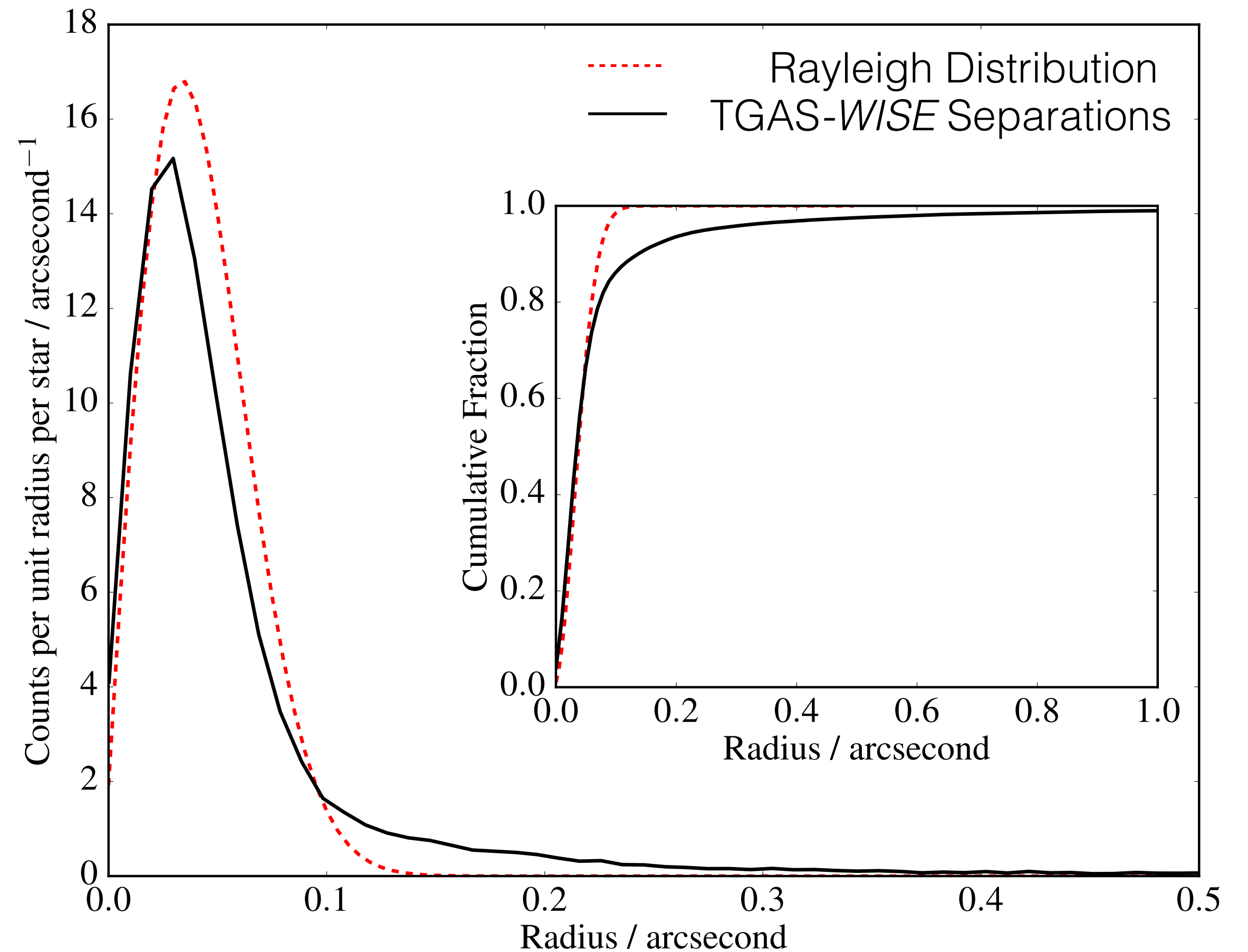


TGAS - Michalik, Lindegren, & Hobbs (2015)
Gaia - Gaia Collaboration, Brown A. G. A., et al. (2016)

The *Astrometric Uncertainty* Function

Reasons for large separations:

- 1) proper motions (e.g. AllWISE Supplement 6.4, Cutri et al. 2012) — no, TGAS provided for all sources
- 2) false matches — no, 0.1% chance of random match within 0.5 arcseconds
- 3) What else could it be?

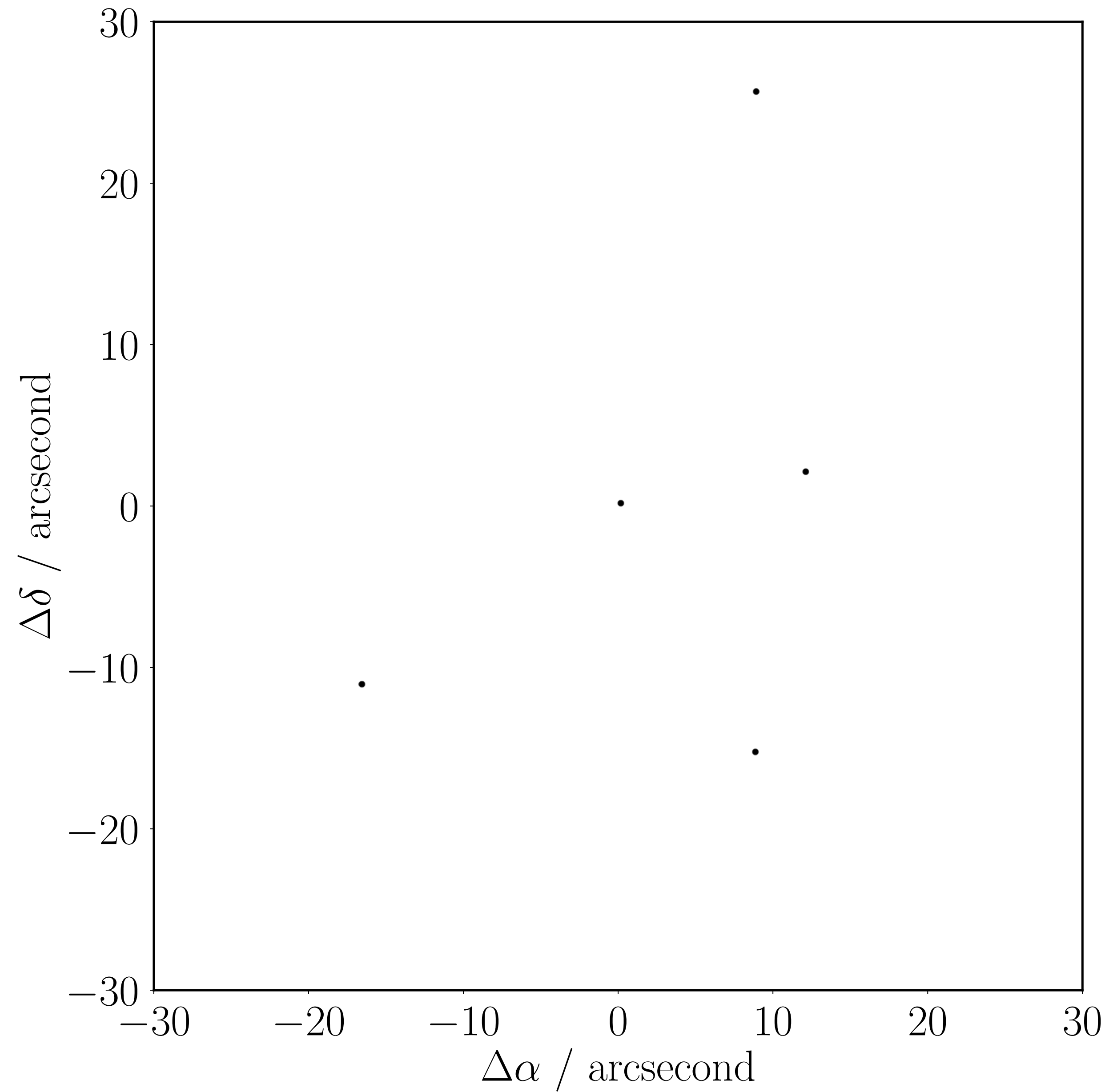


Wilson & Naylor (2017)
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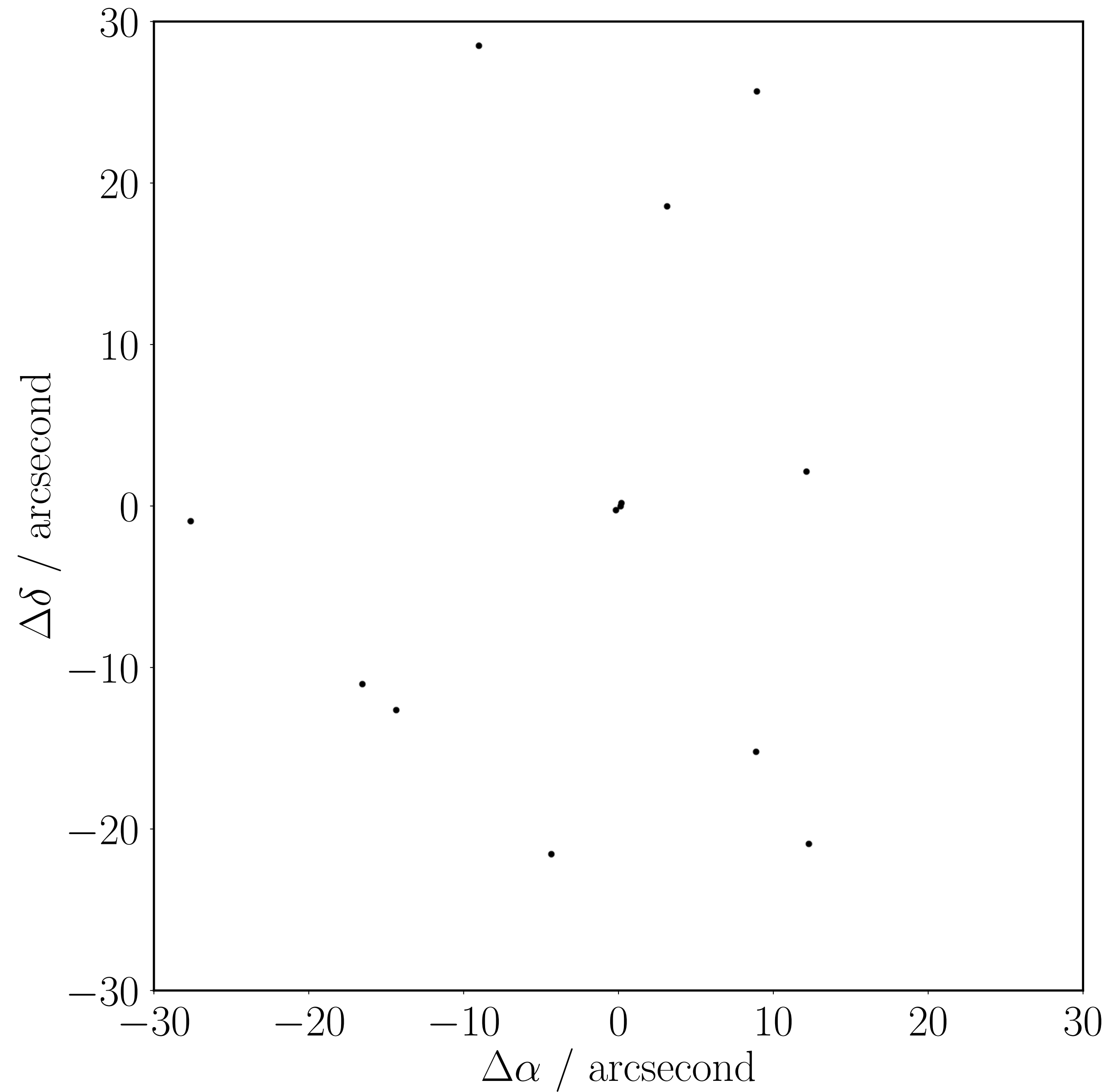
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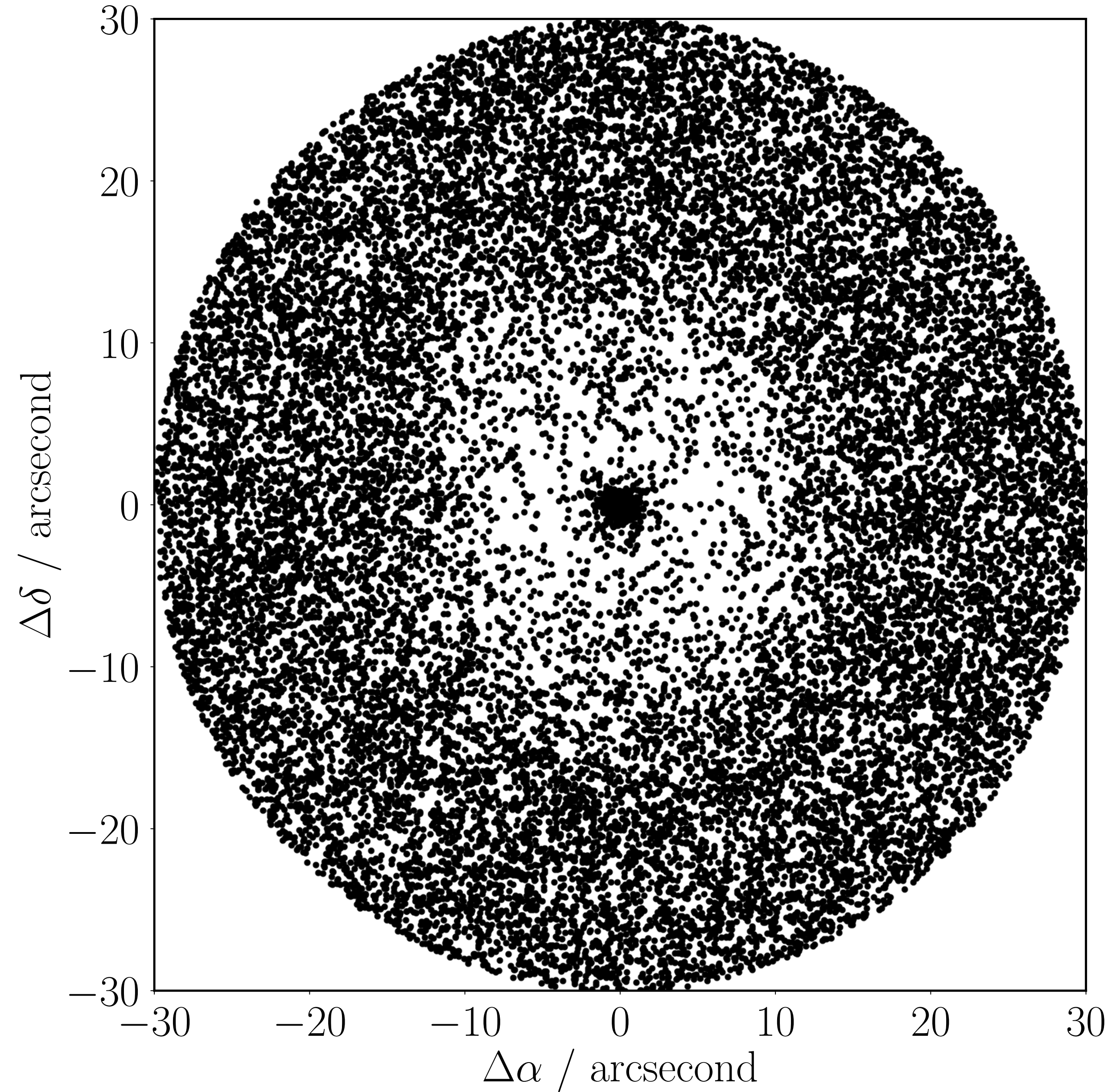
The AUF: Crowding



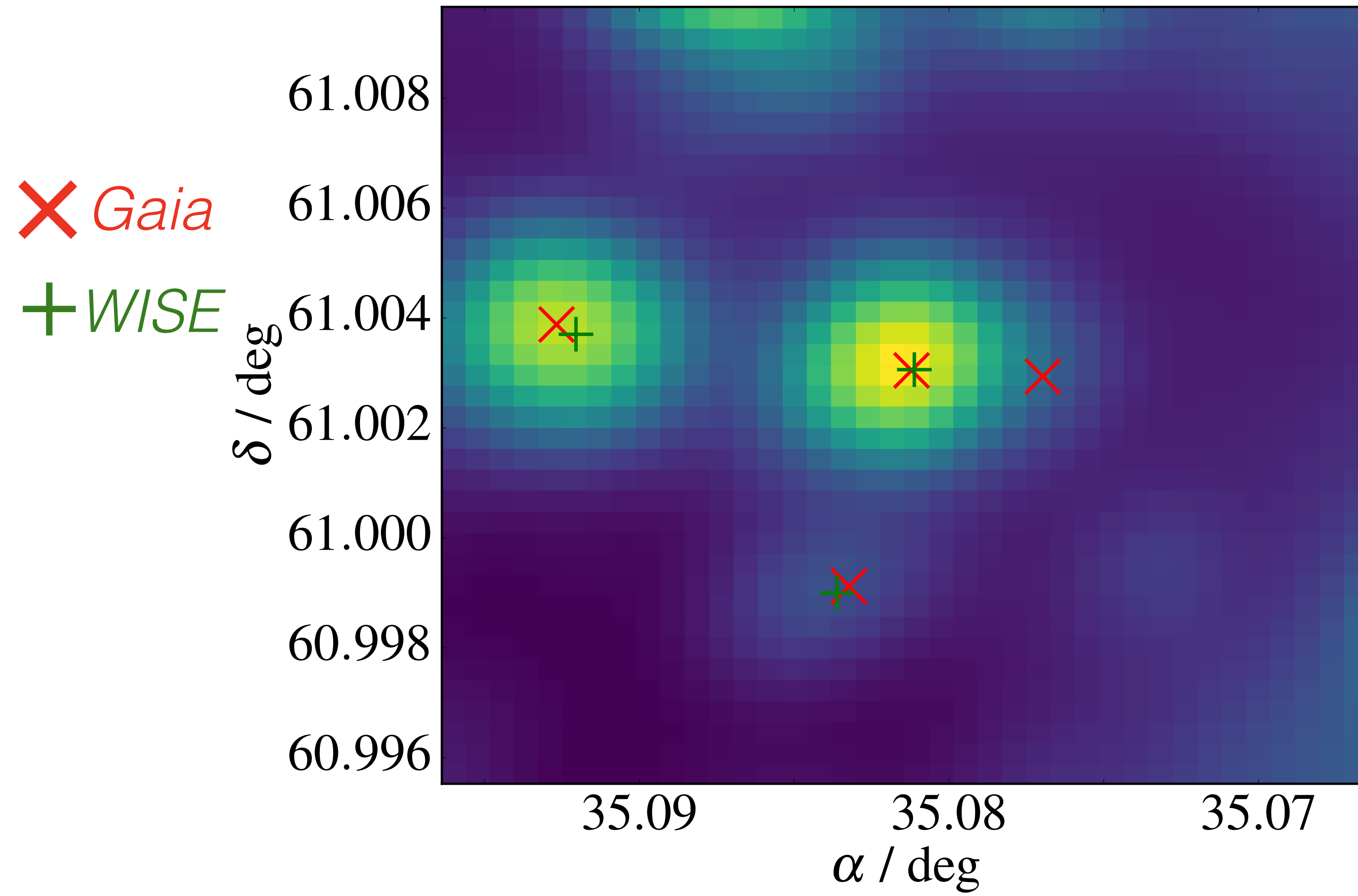
The AUF: Crowding



The AUF: Crowding



Resolving *Gaia*-*WISE* Blends



Wilson & Naylor (2018b)

WISE - Wright et al. (2010)

Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

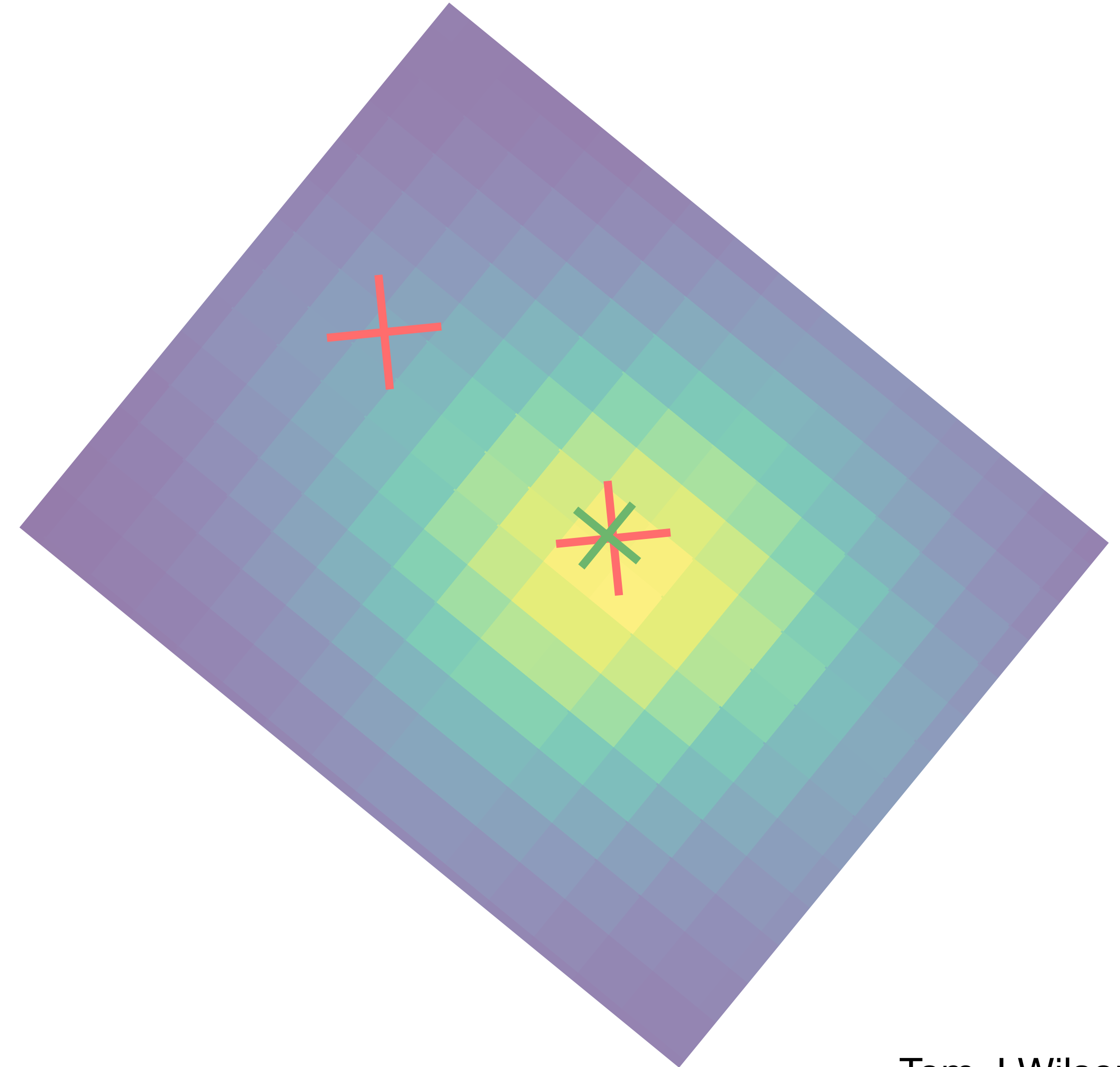
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The AUF: Perturbation

● Pure *WISE* position



✱ *Gaia* position



Wilson & Naylor (2017)

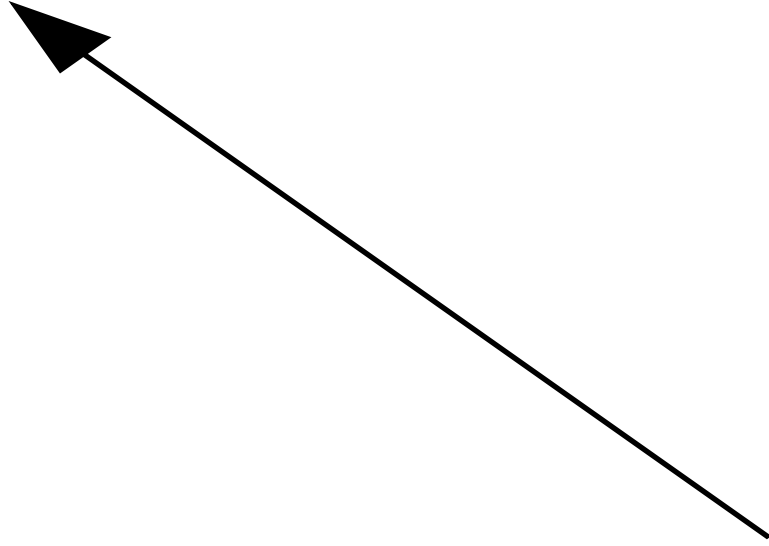
Wilson & Naylor (2018b)

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The AUF: Perturbation

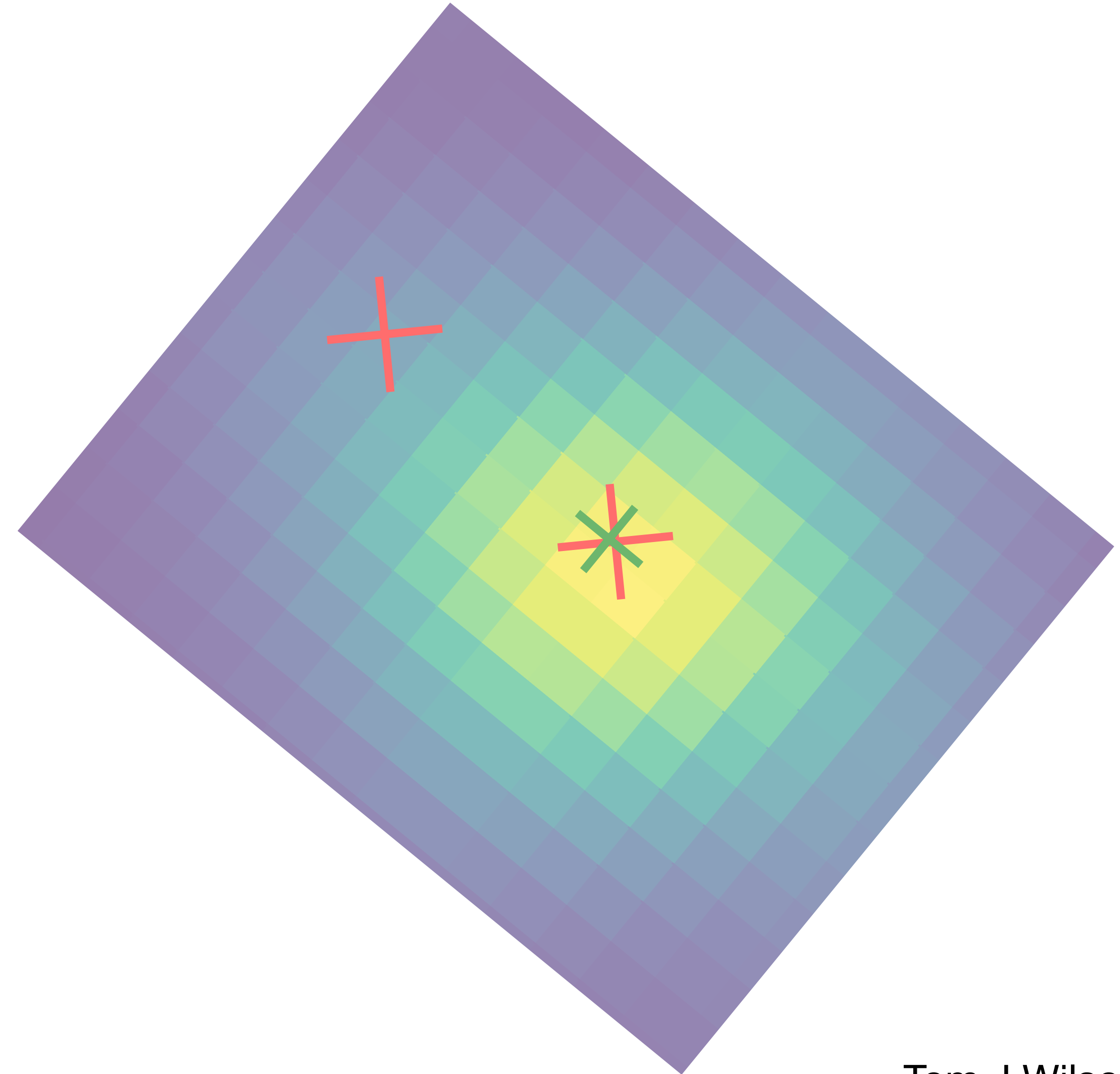
To *WISE* contaminant



Pure *WISE* position



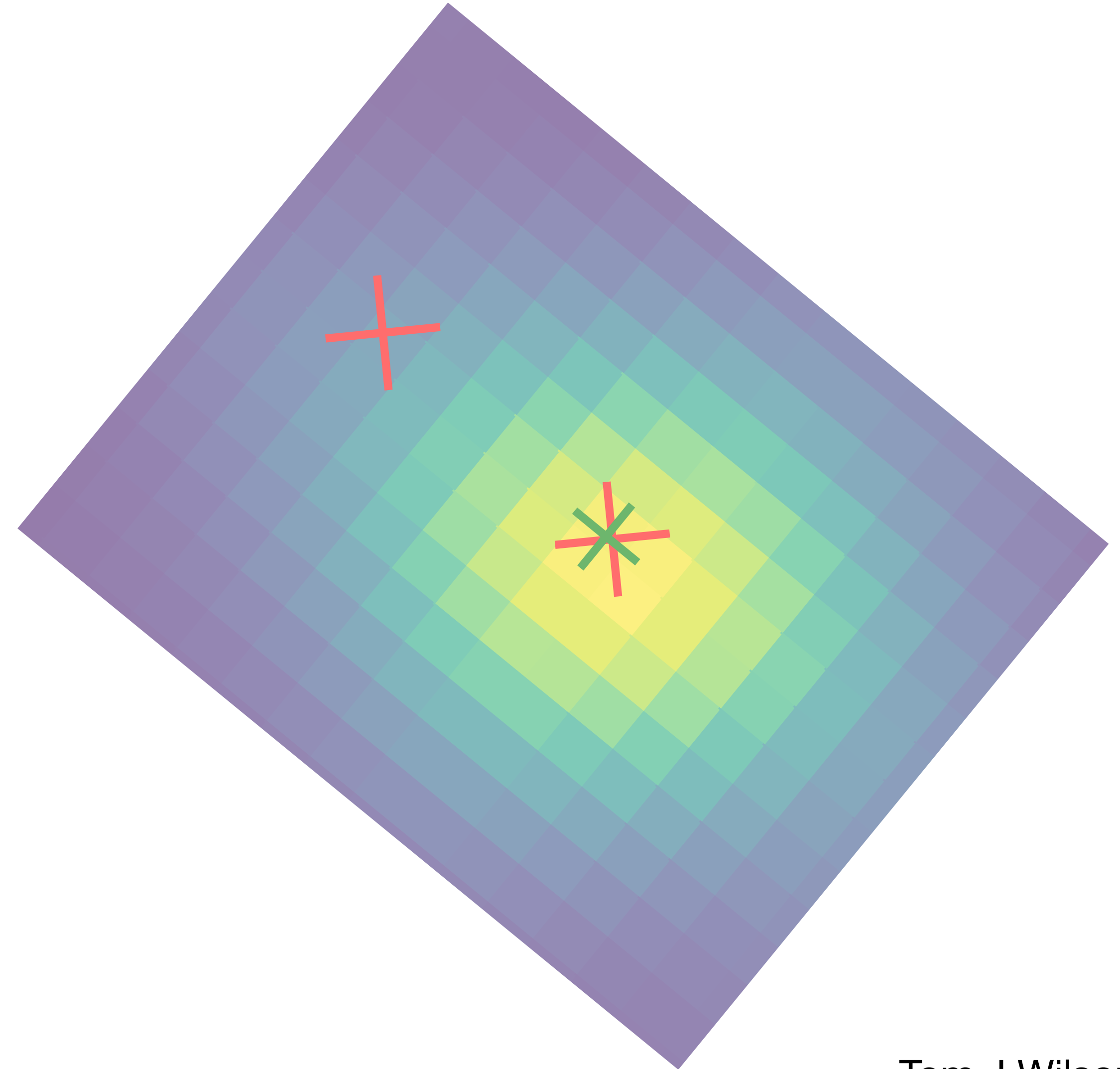
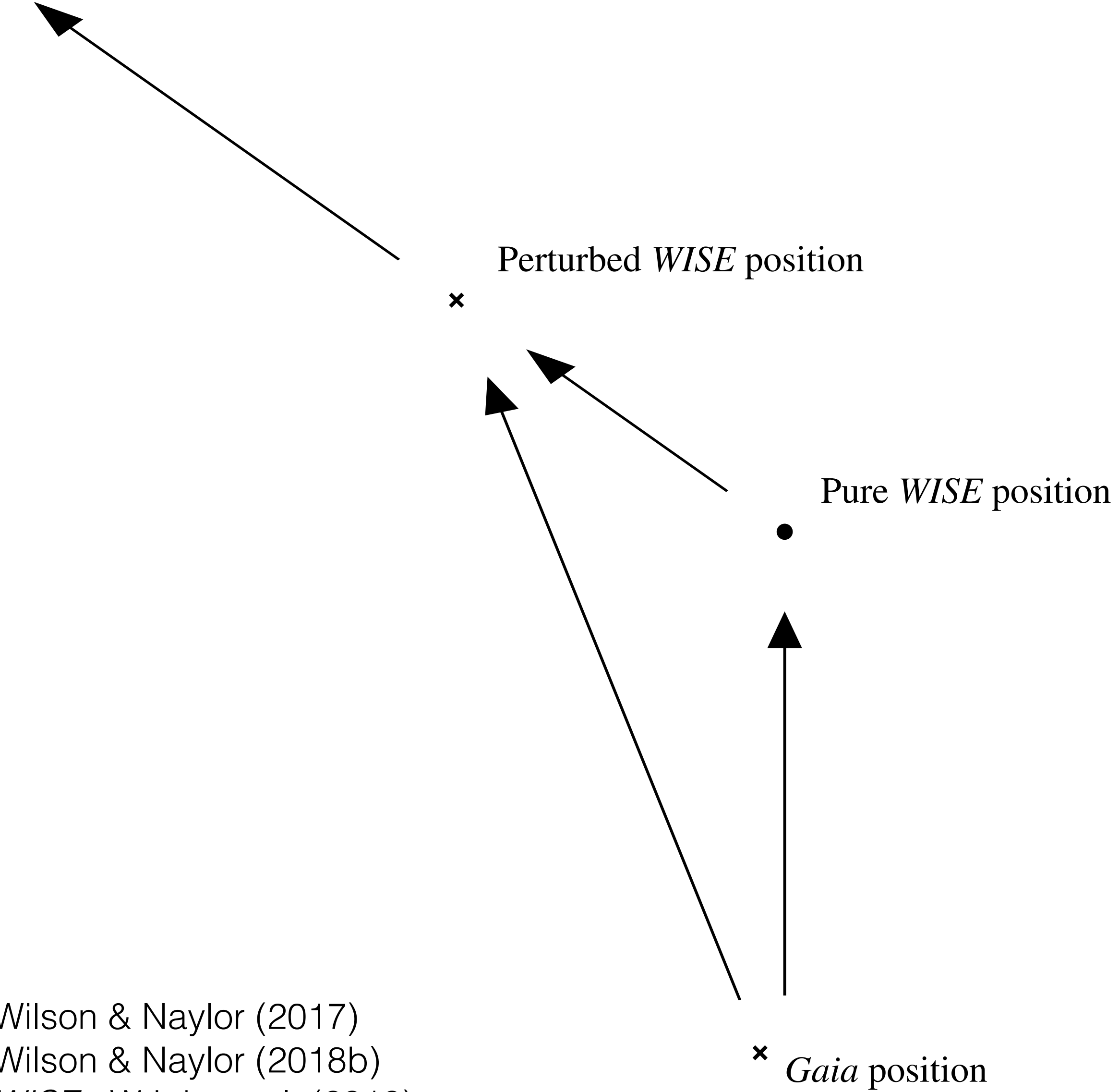
× *Gaia* position



Wilson & Naylor (2017)
Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

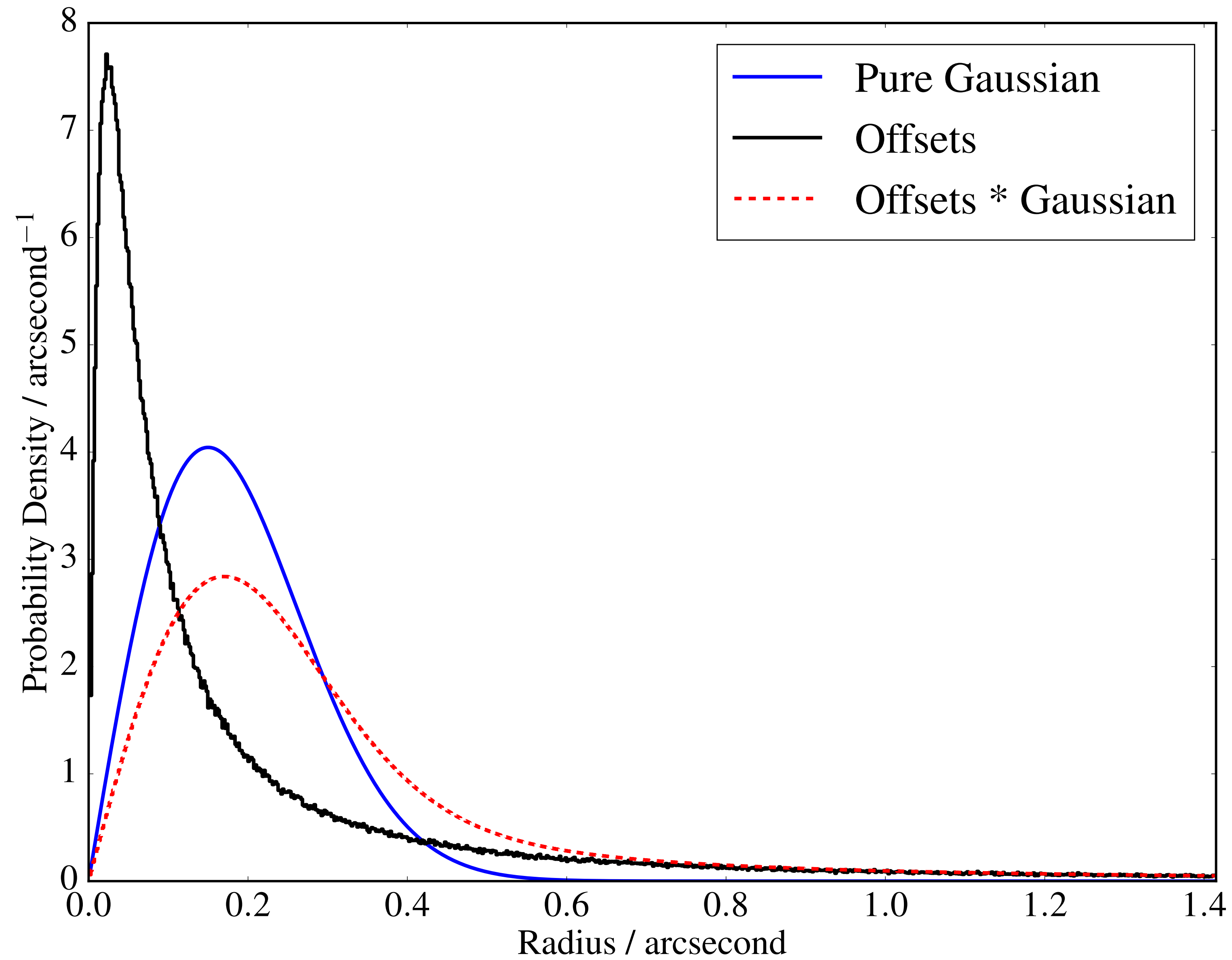
The AUF: Perturbation

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Wilson & Naylor (2017)
Wilson & Naylor (2018b)
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The AUF: Perturbation



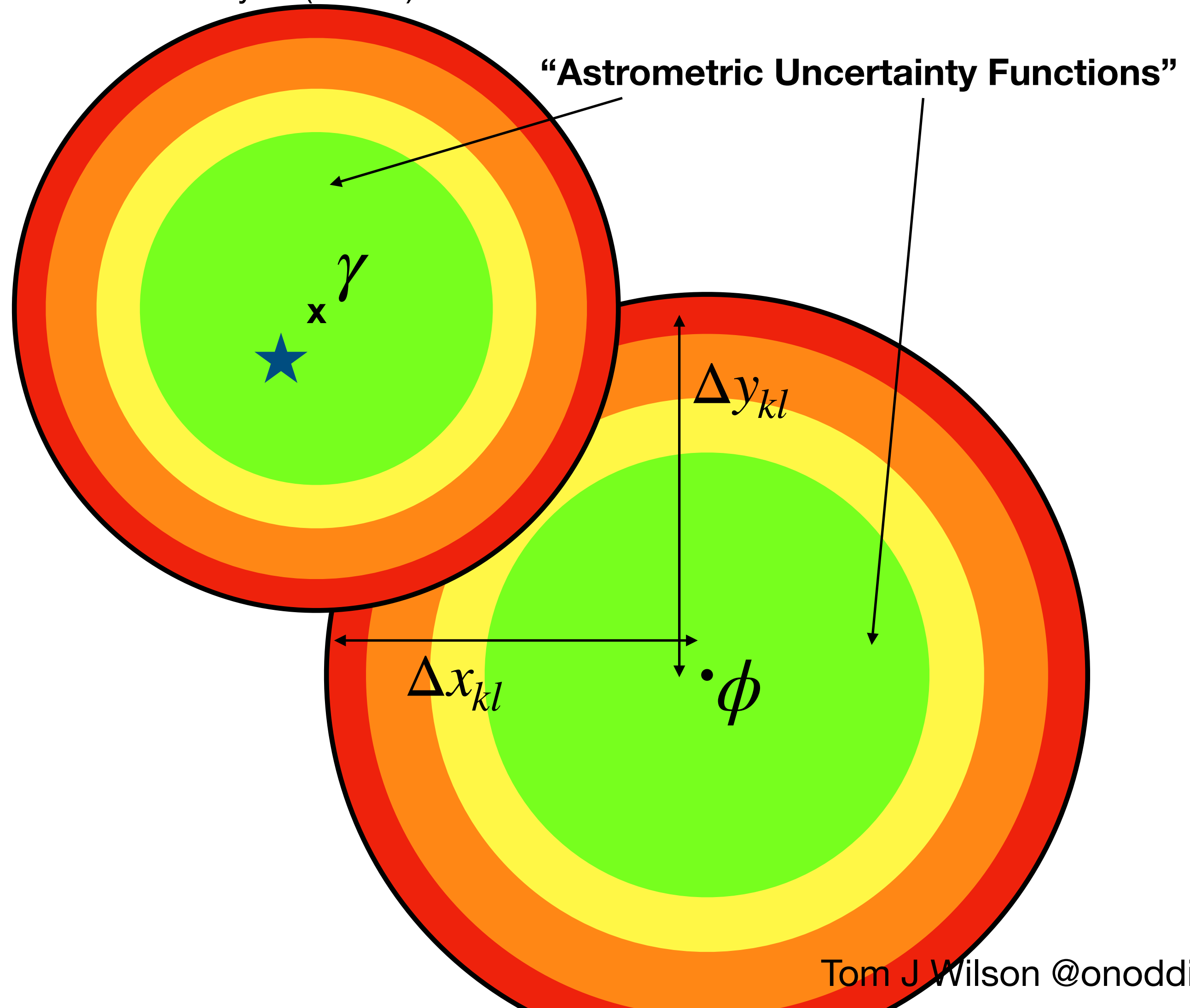
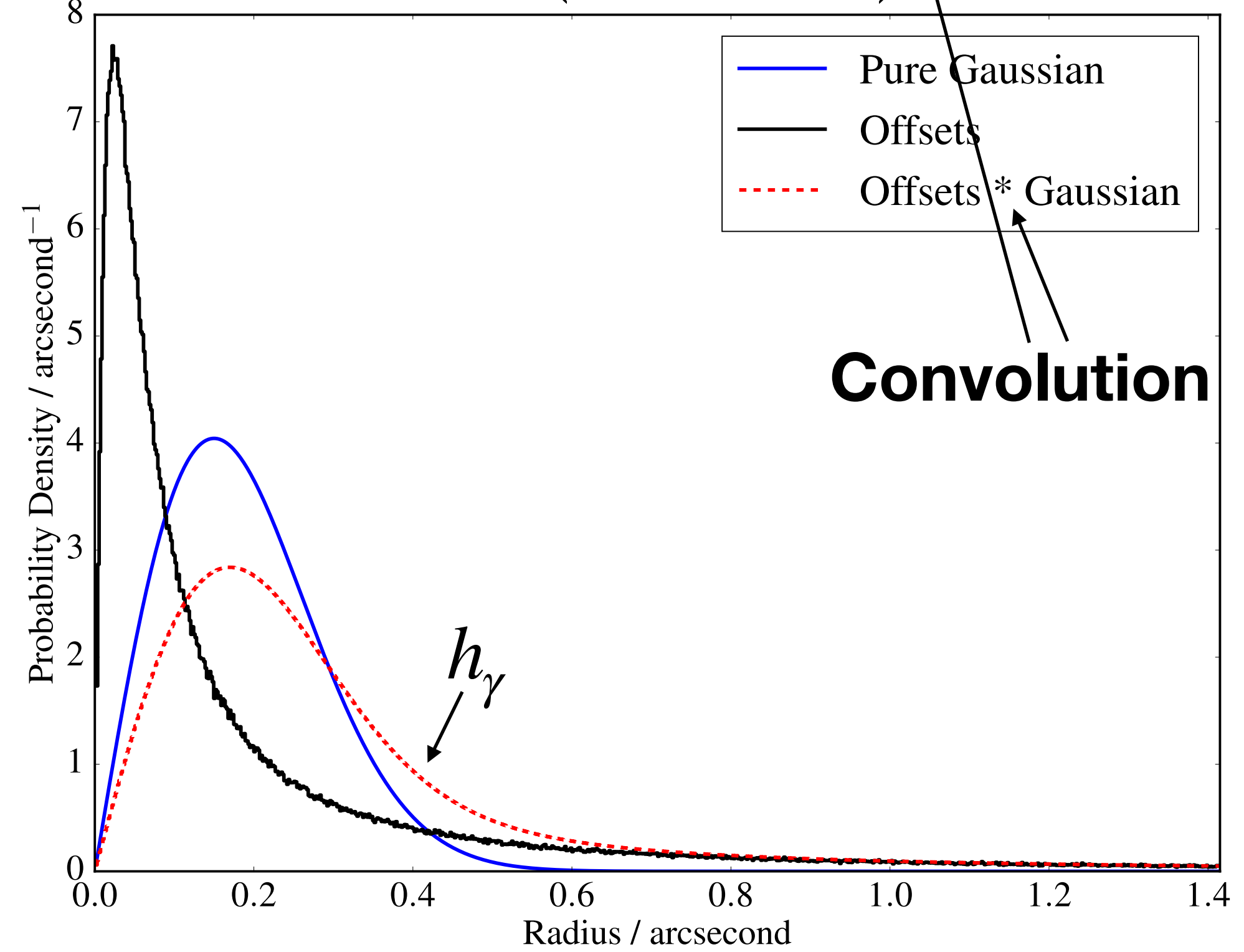
Separation Likelihood Function

$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0) dx_0 dy_0$$

Wilson & Naylor (2018a)

$$= N_c \times (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl})$$

$$g(\Delta x, \Delta y, \sigma) = (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2} \frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right) \text{ where } \sigma^2 = \sigma_\gamma^2 + \sigma_\phi^2$$



Wilson & Naylor (2018b)

Tom J Wilson @onoddil

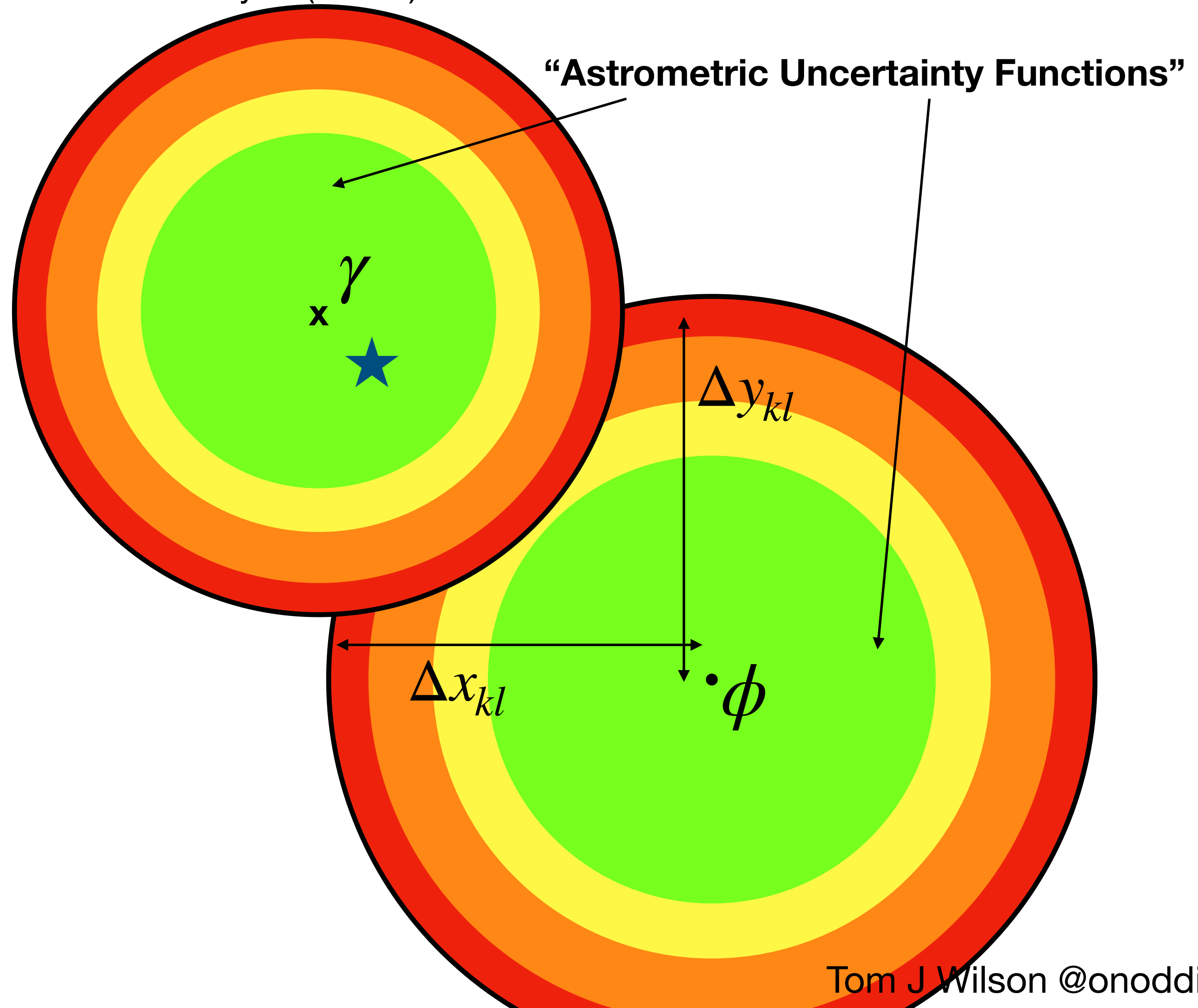
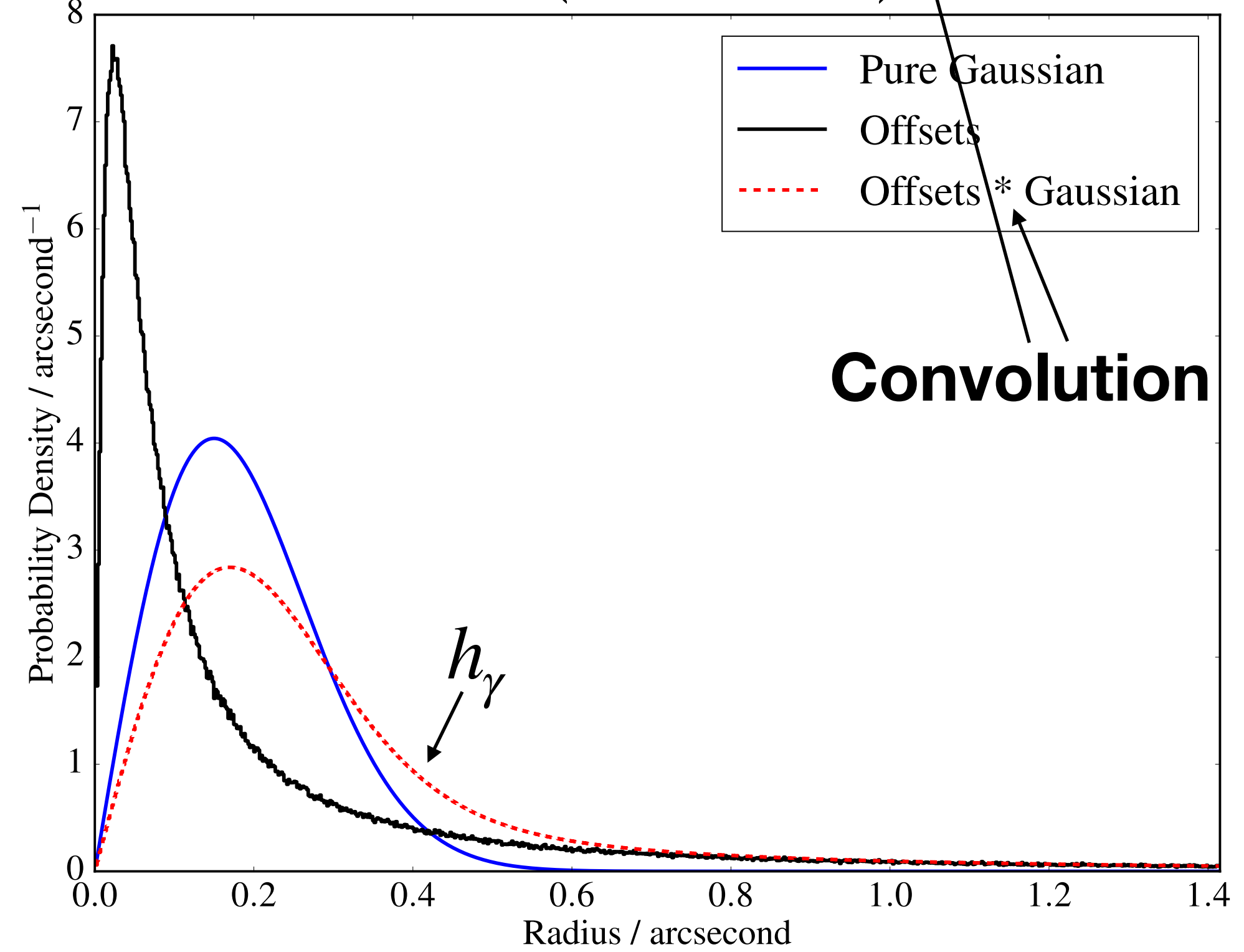
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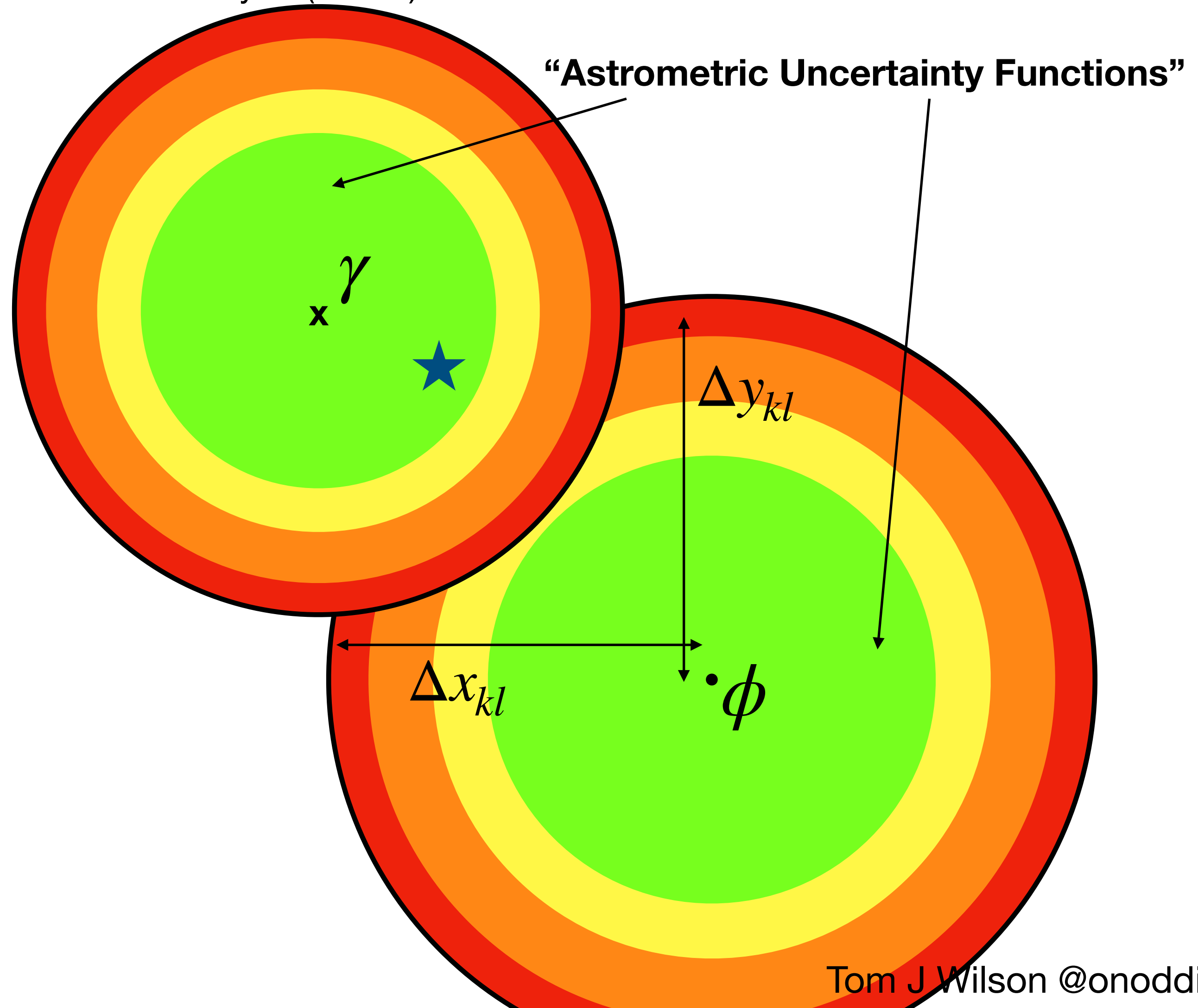
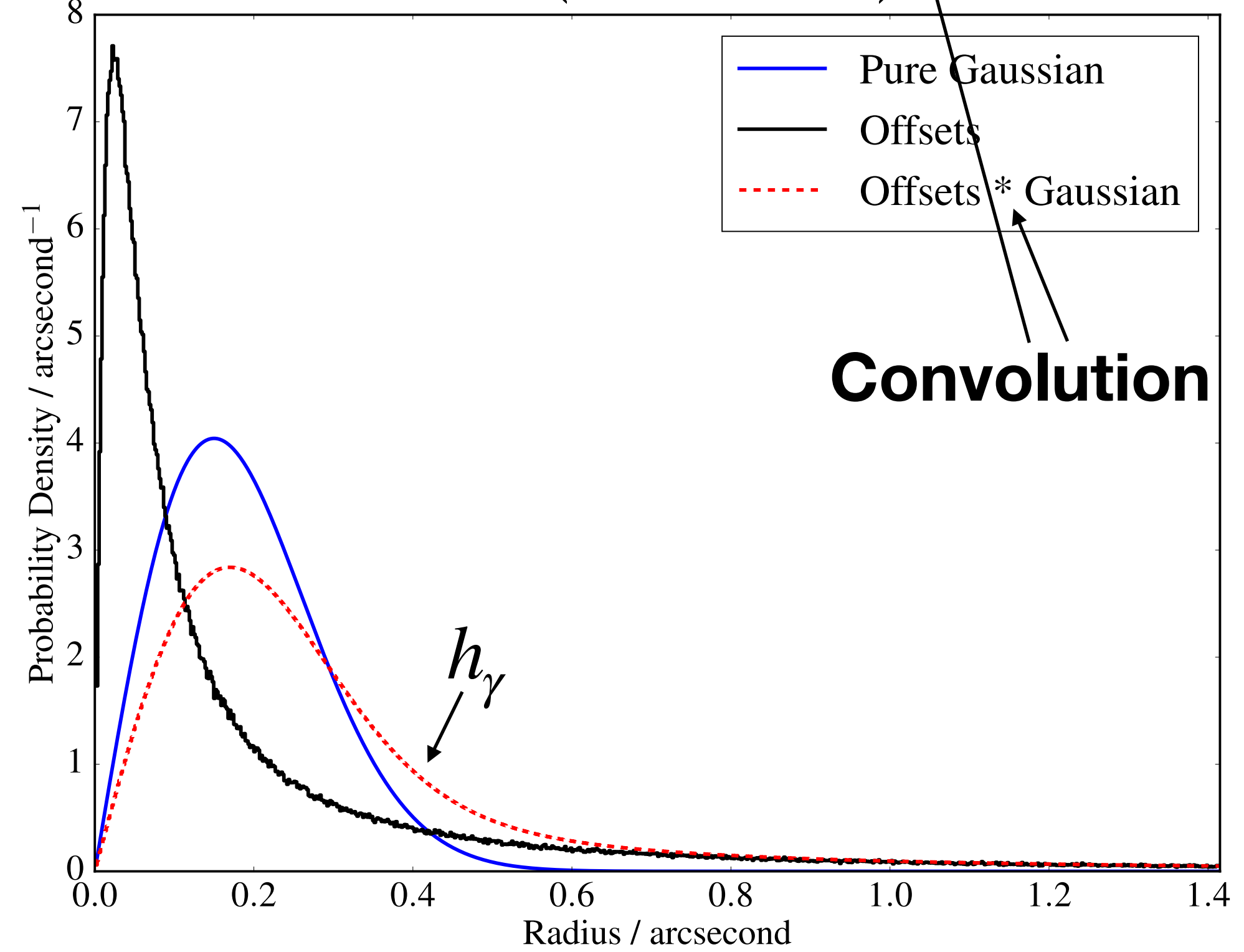
Separation Likelihood Function

$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0) dx_0 dy_0$$

Wilson & Naylor (2018a)

$$= N_c \times (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl})$$

$$g(\Delta x, \Delta y, \sigma) = (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2} \frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right) \text{ where } \sigma^2 = \sigma_\gamma^2 + \sigma_\phi^2$$



Wilson & Naylor (2018b)

Tom J Wilson @onoddil

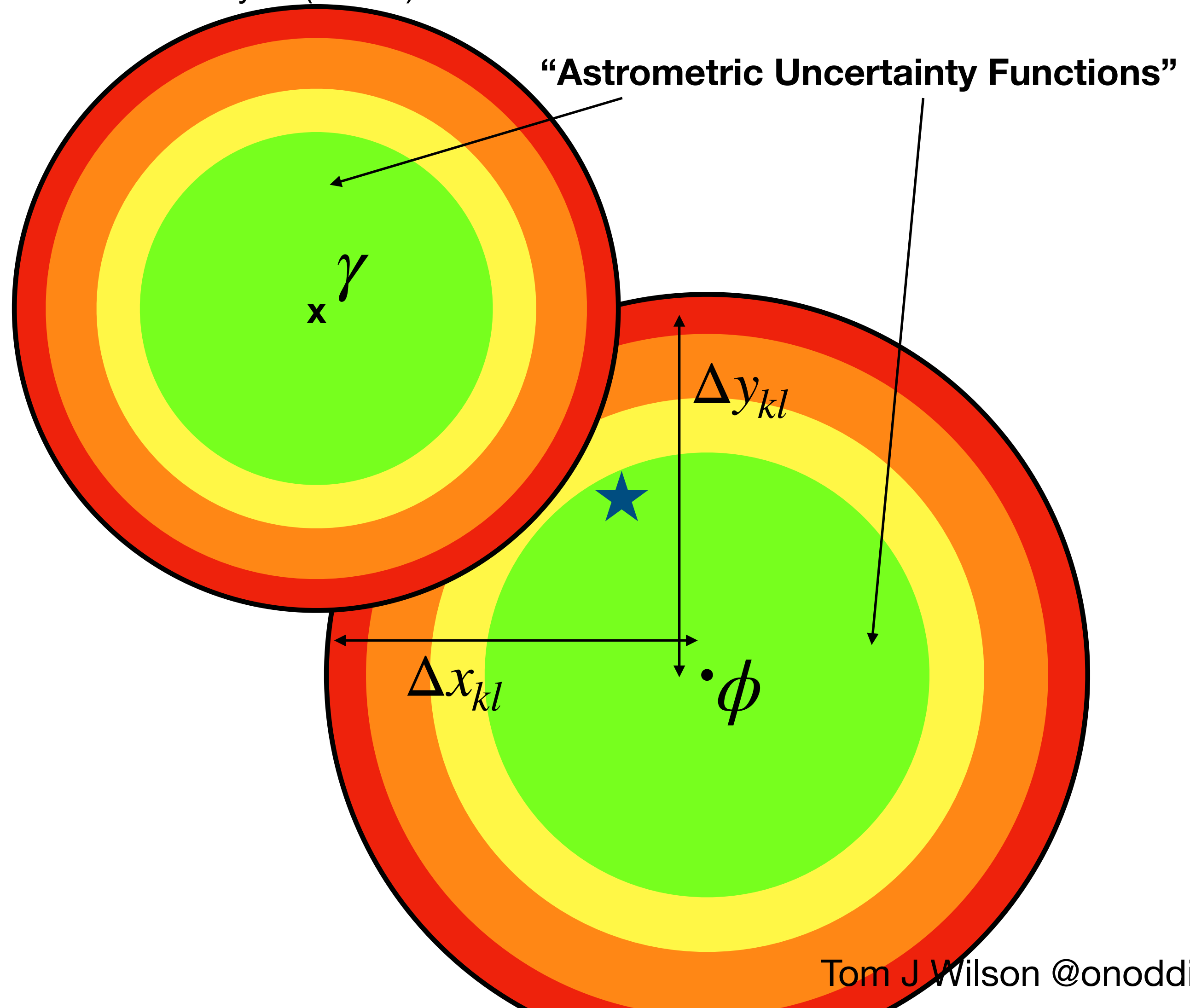
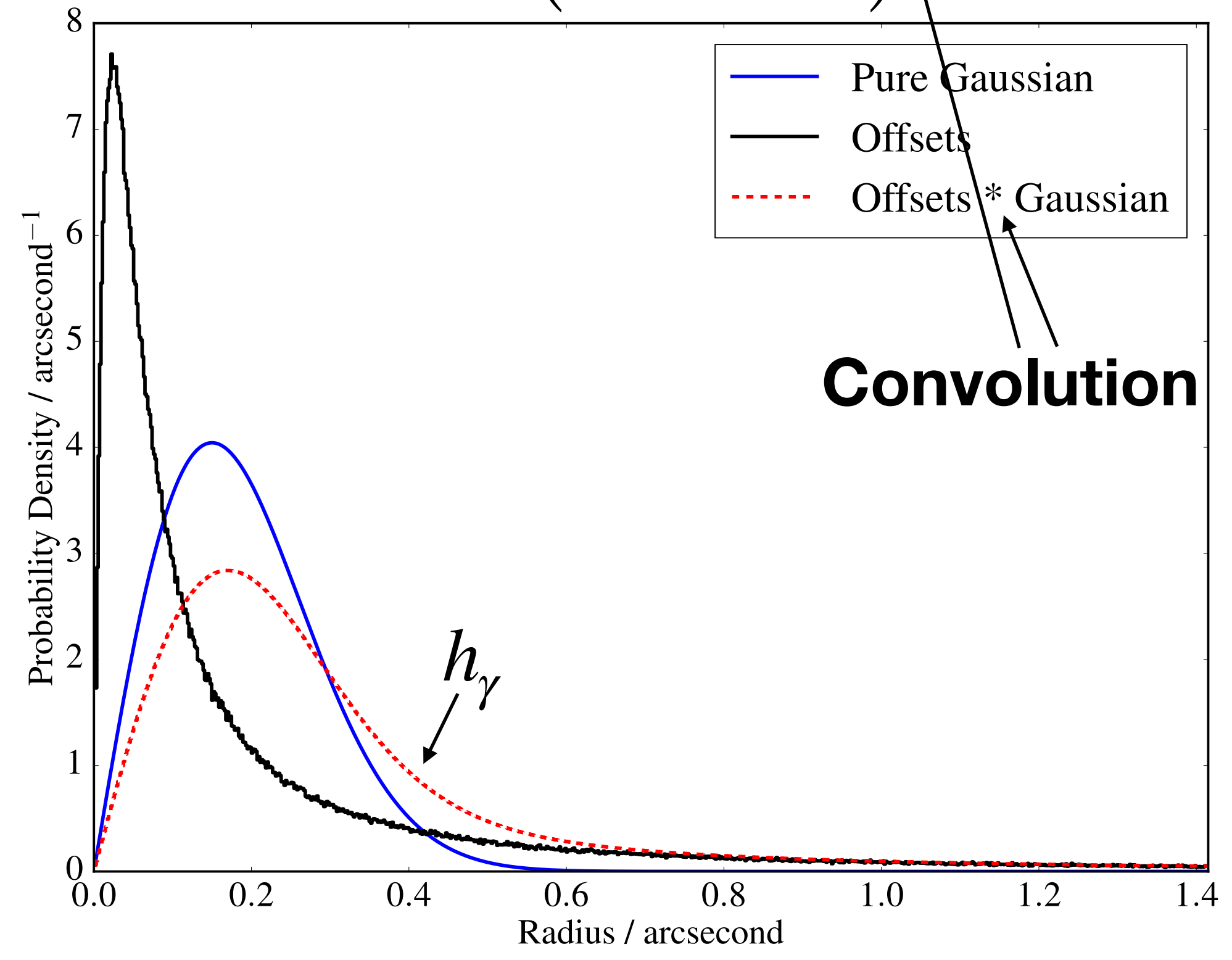
Separation Likelihood Function

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Wilson & Naylor (2018b)

Tom J Wilson @onoddil

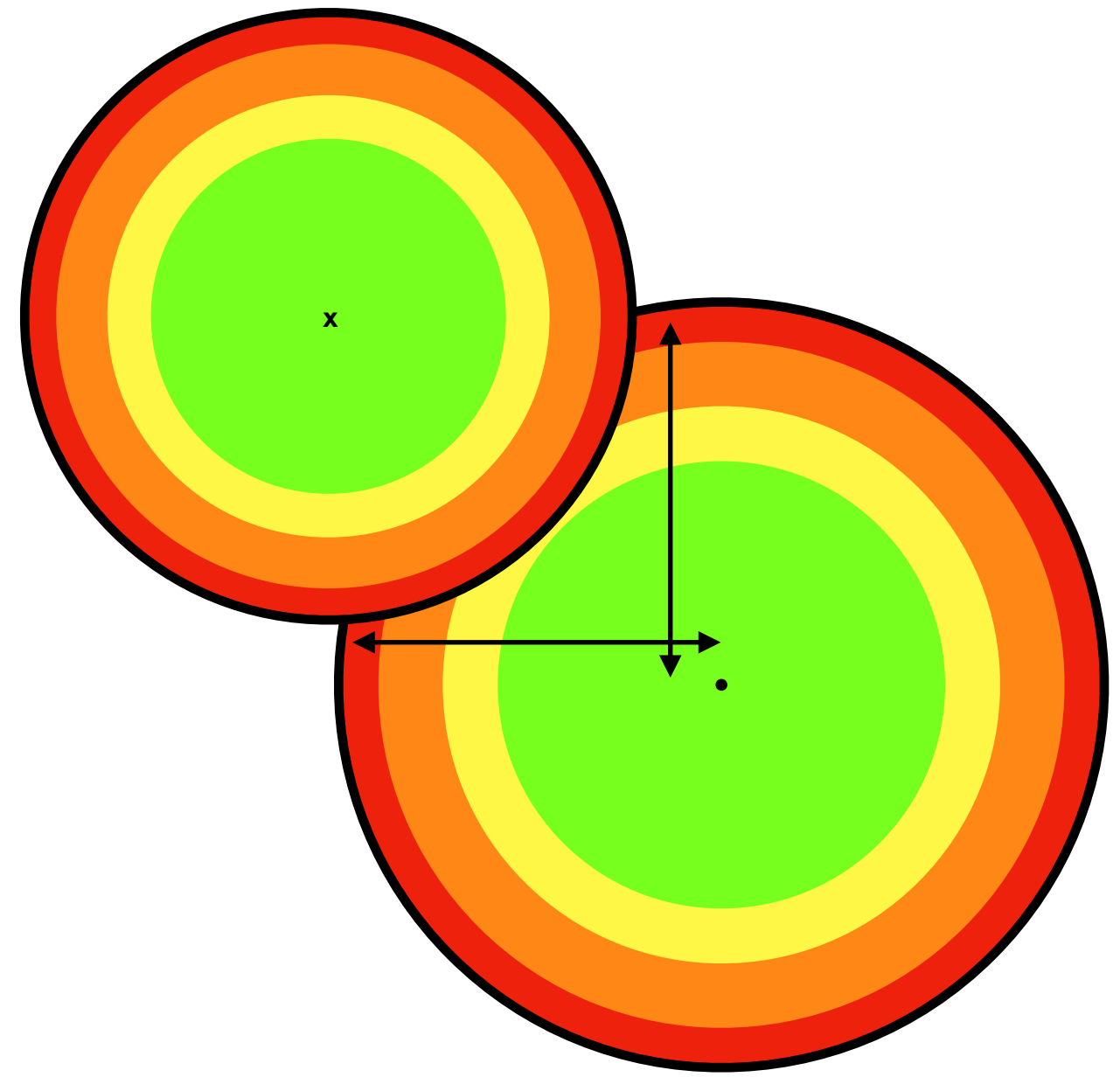
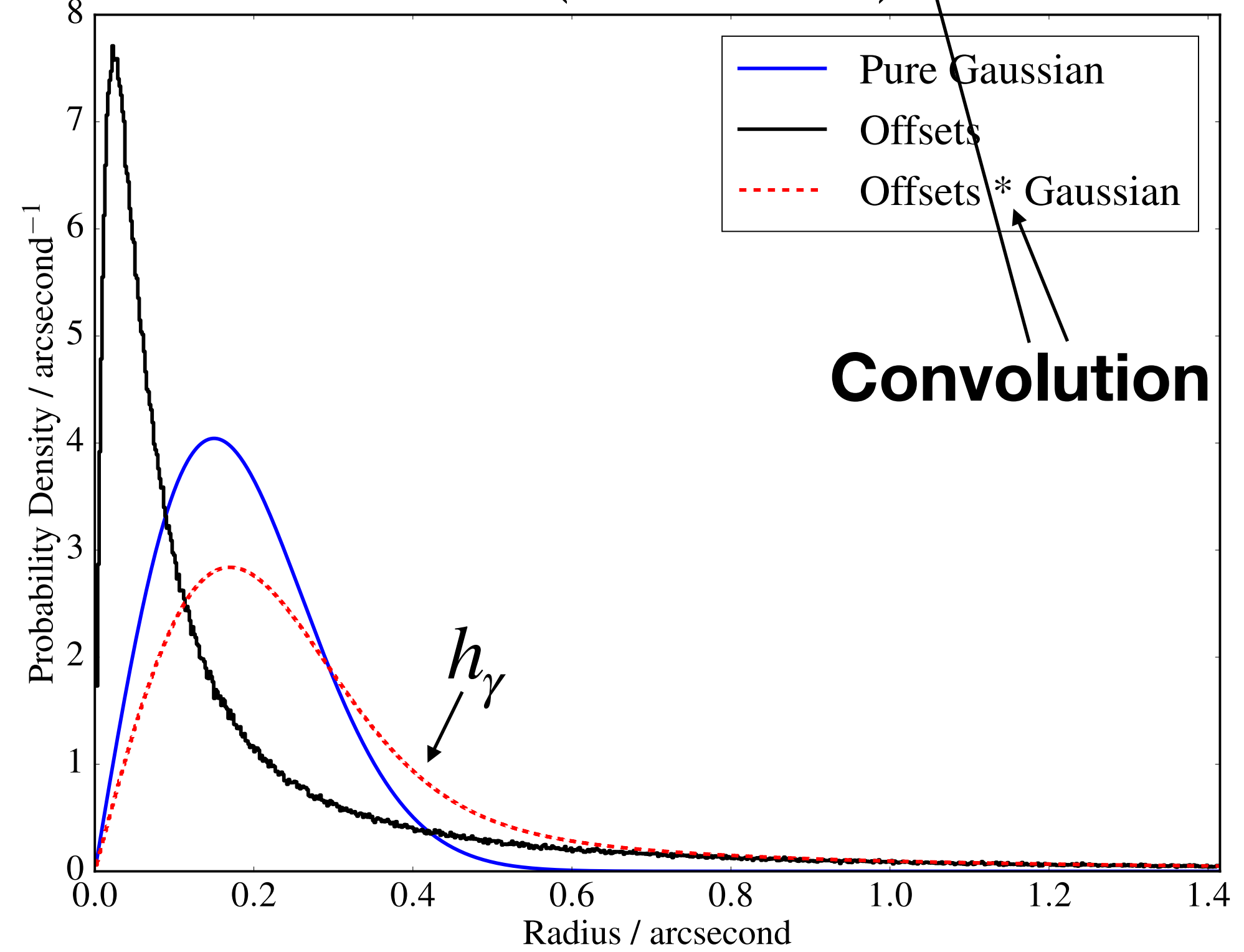
Separation Likelihood Function

$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0) dx_0 dy_0$$

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**“Were the succession of stars endless... there could be absolutely no point, in all that background, at which would not exist a star.”
— Edgar Allan Poe, Eureka (1848)**

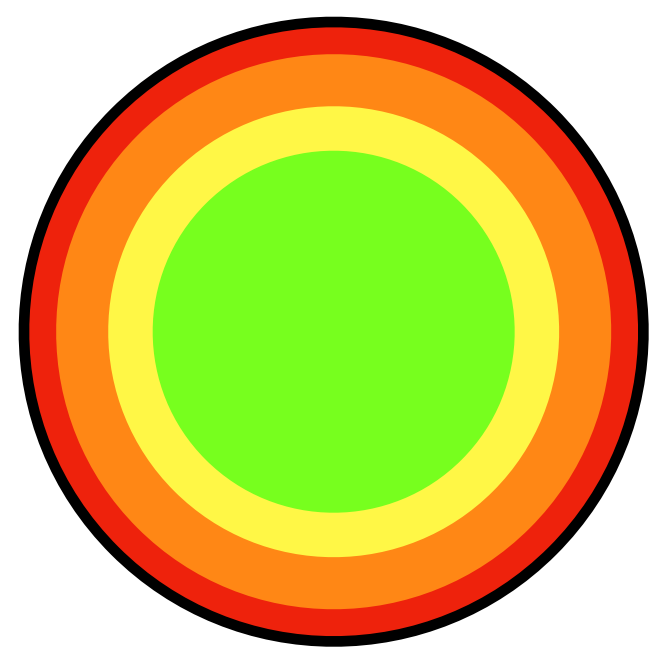
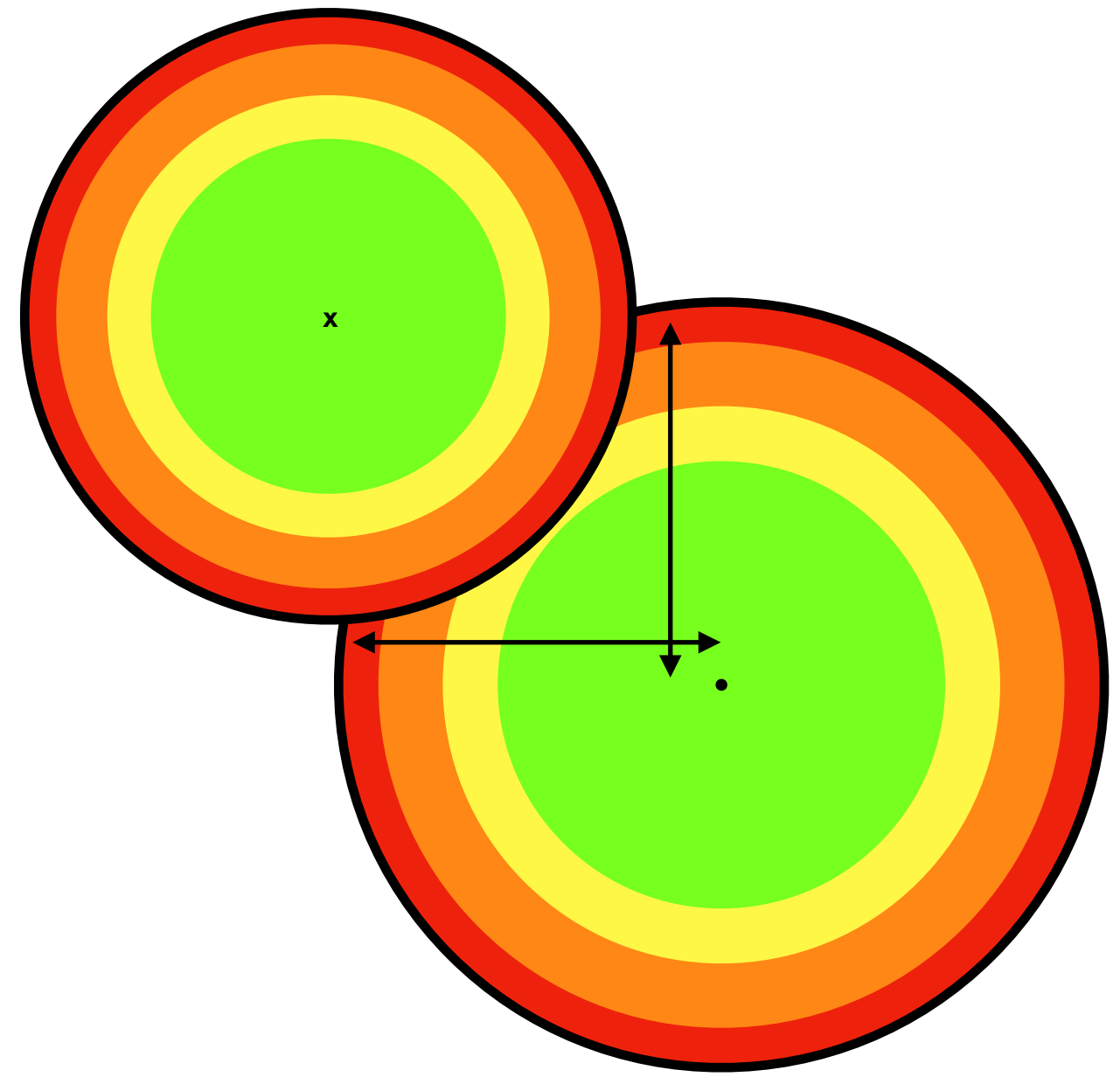
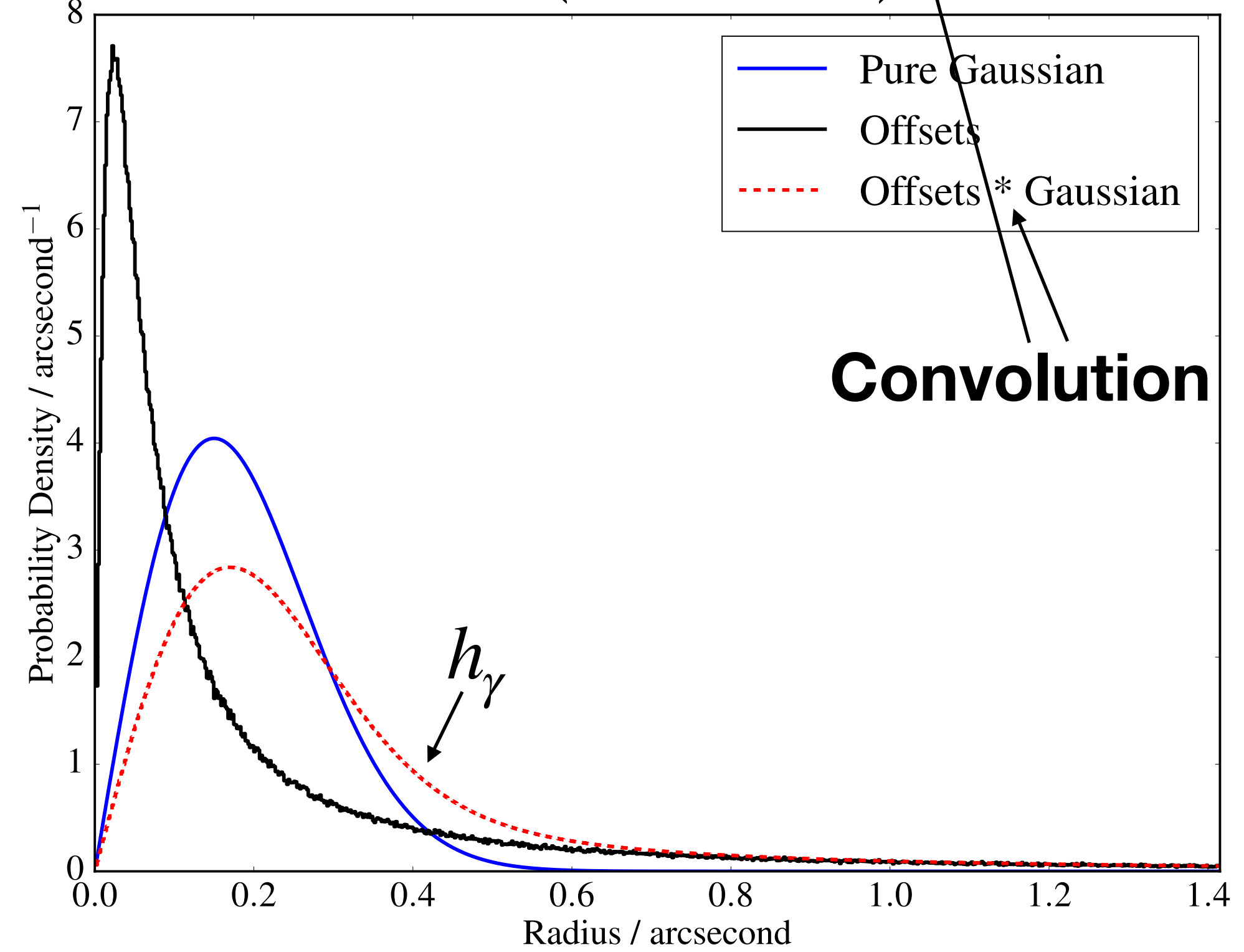
Separation Likelihood Function

$$g(x_k, y_k, x_l, y_l) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0) p(x_0, y_0) dx_0 dy_0$$

Wilson & Naylor (2018a)

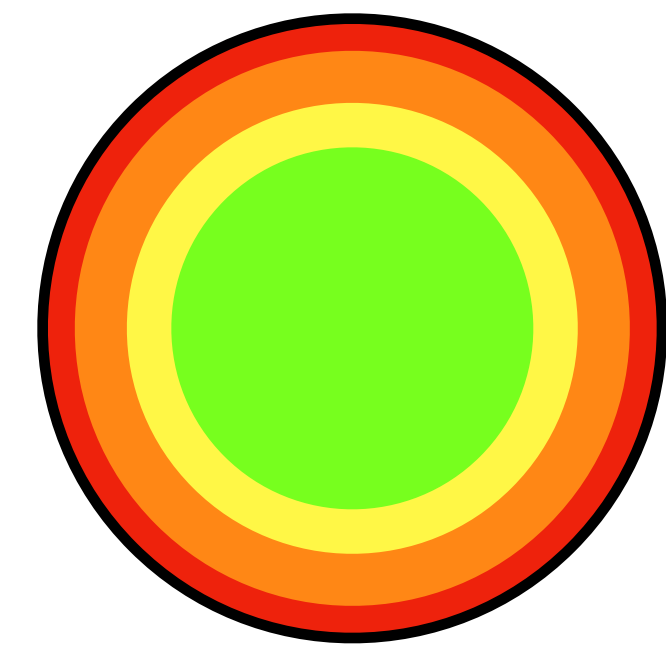
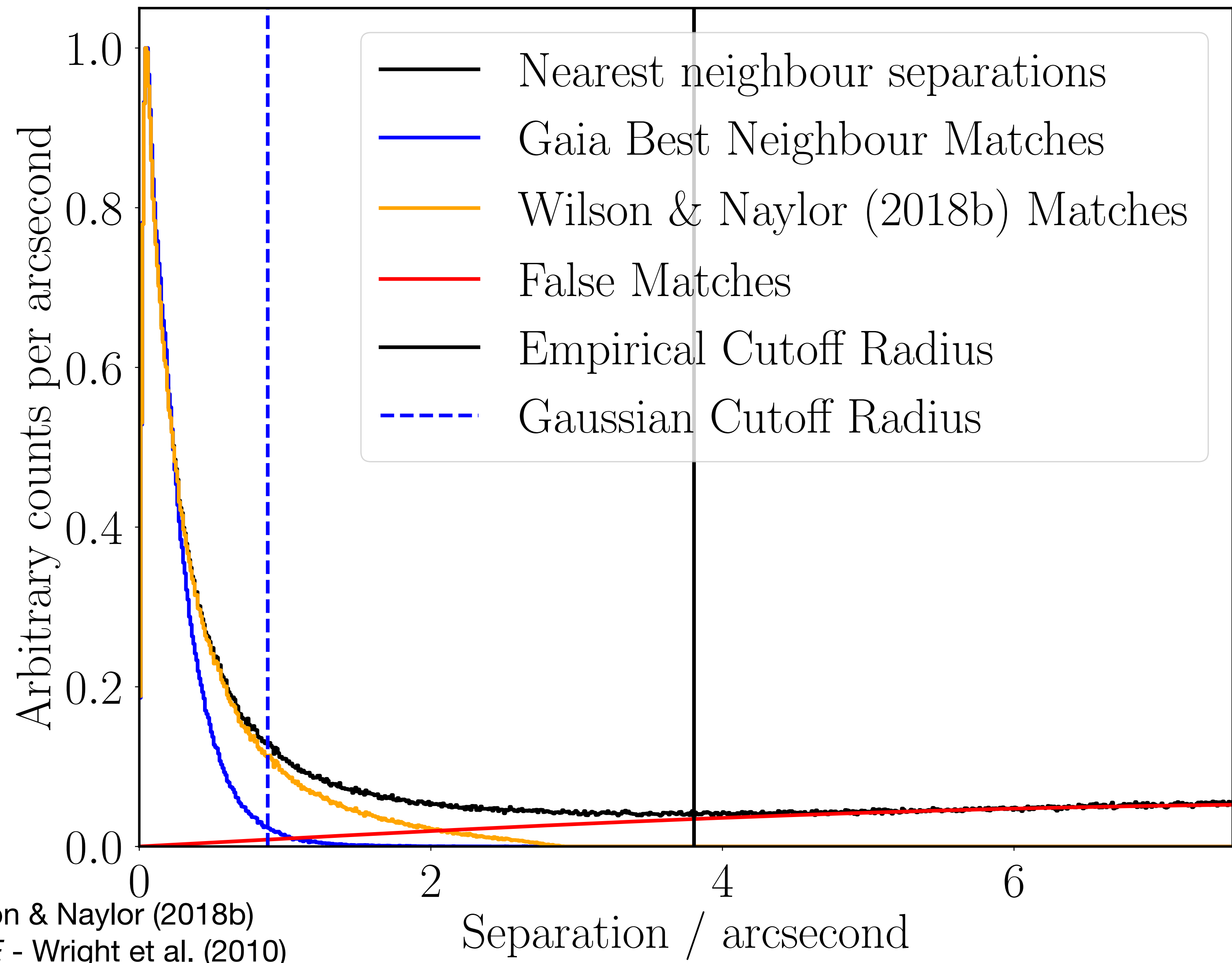
$$= N_c \times (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl})$$

$$g(\Delta x, \Delta y, \sigma) = (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2} \frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right) \text{ where } \sigma^2 = \sigma_\gamma^2 + \sigma_\phi^2$$



The AUF does not need to, and in fact quite often should not, be Gaussian!

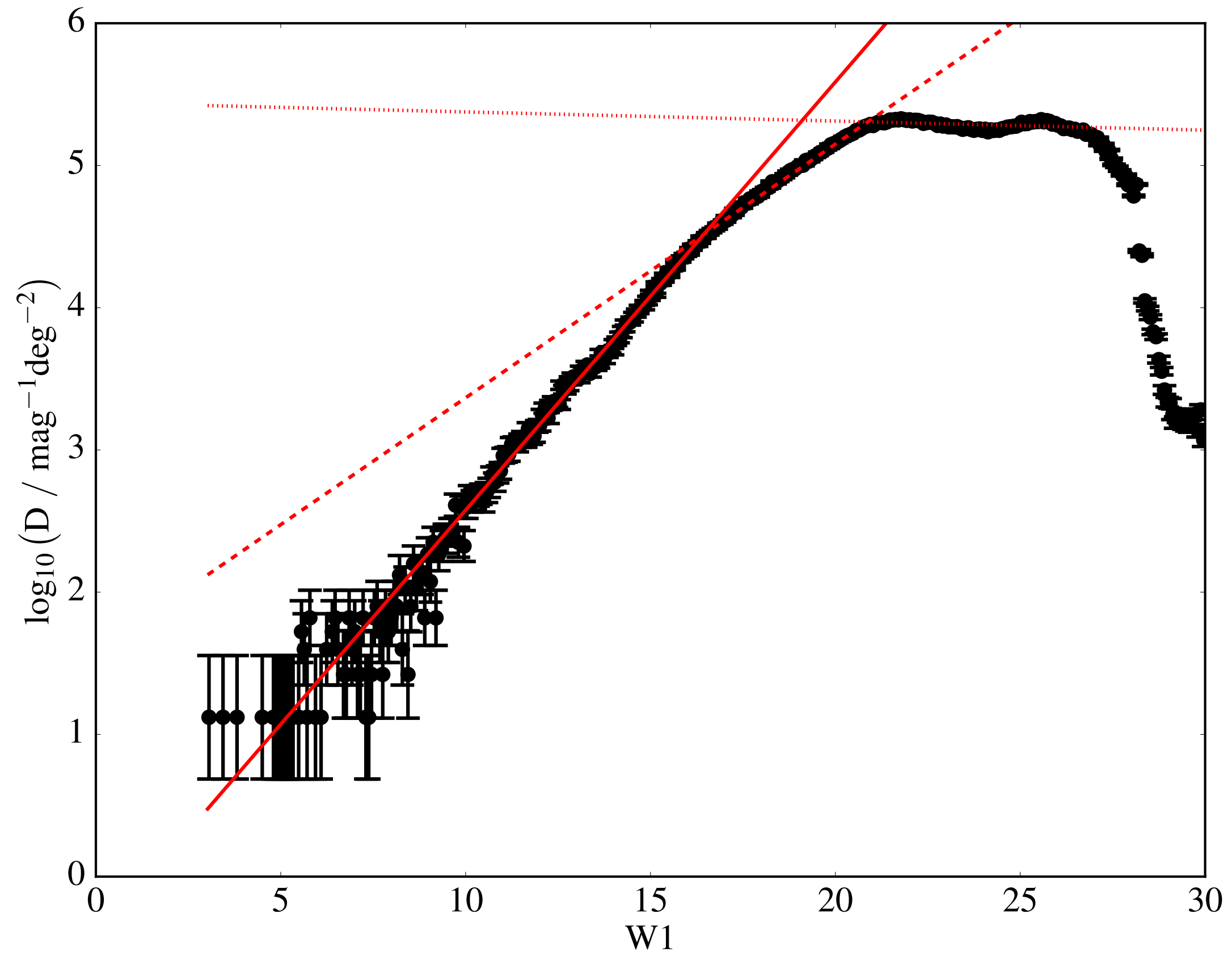
Match Separations



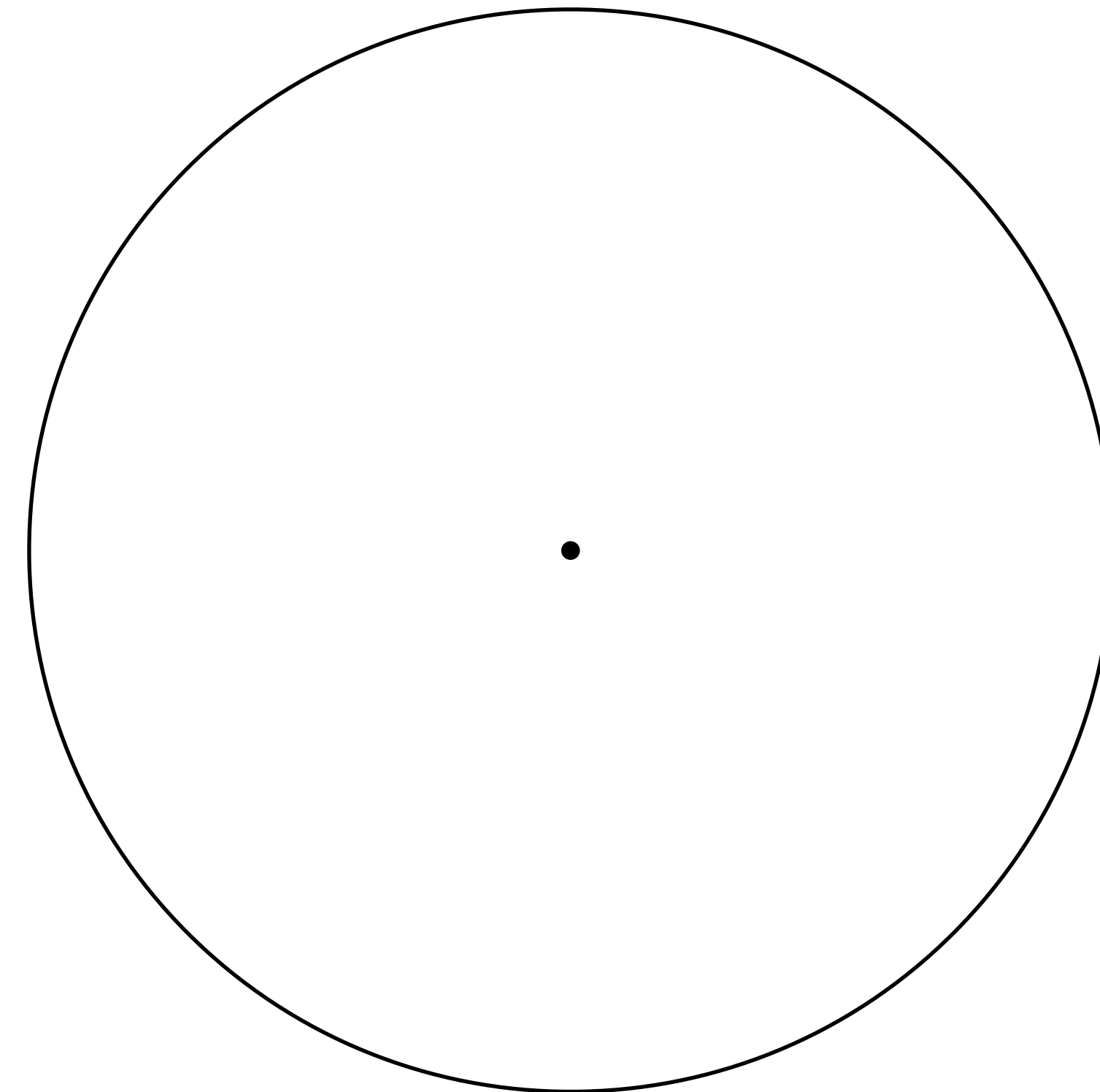
The AUF does not need to, and in fact quite often should *not*, be Gaussian!

Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia matches - Marrese et al. (2019)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

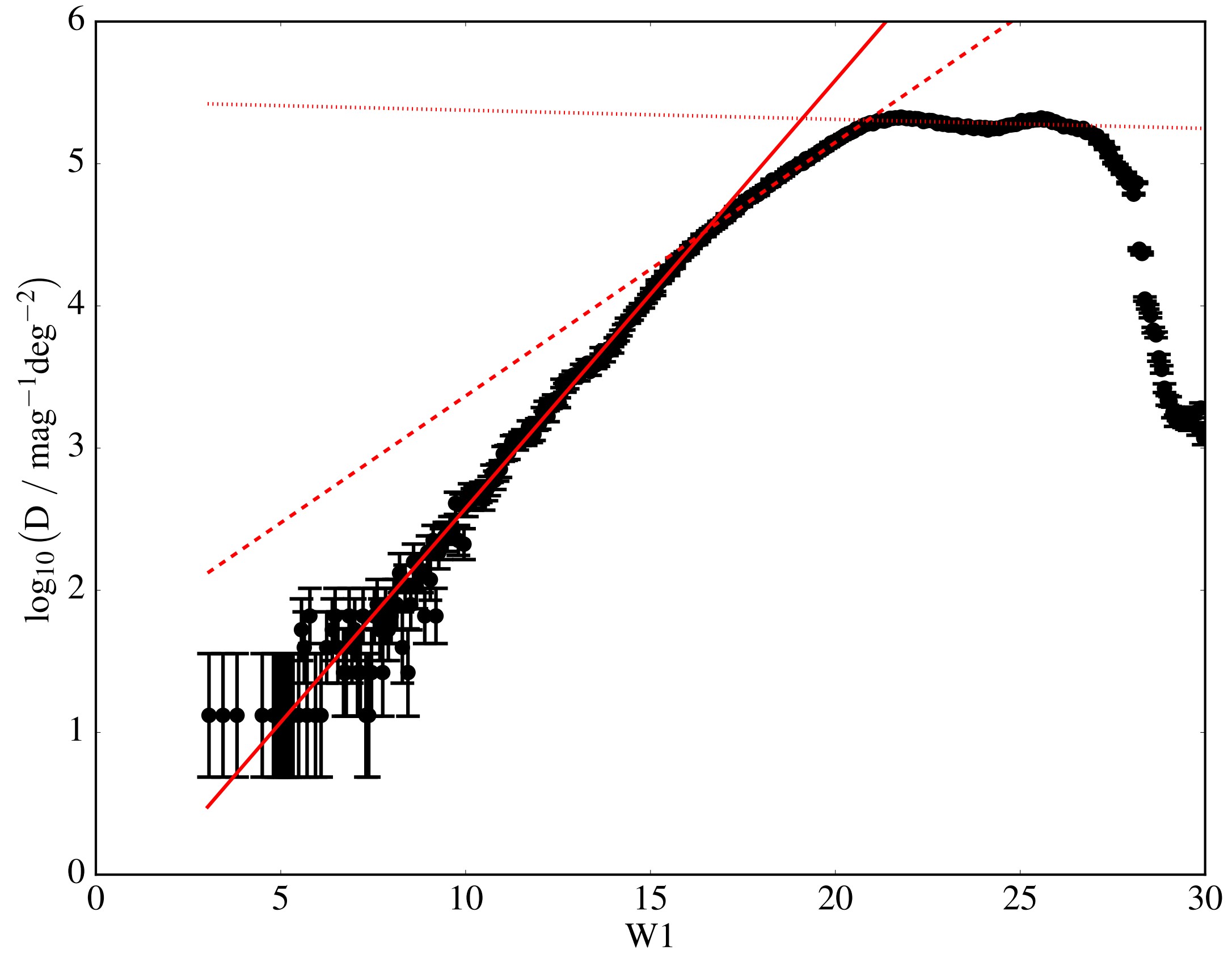
Building Empirical AUFs



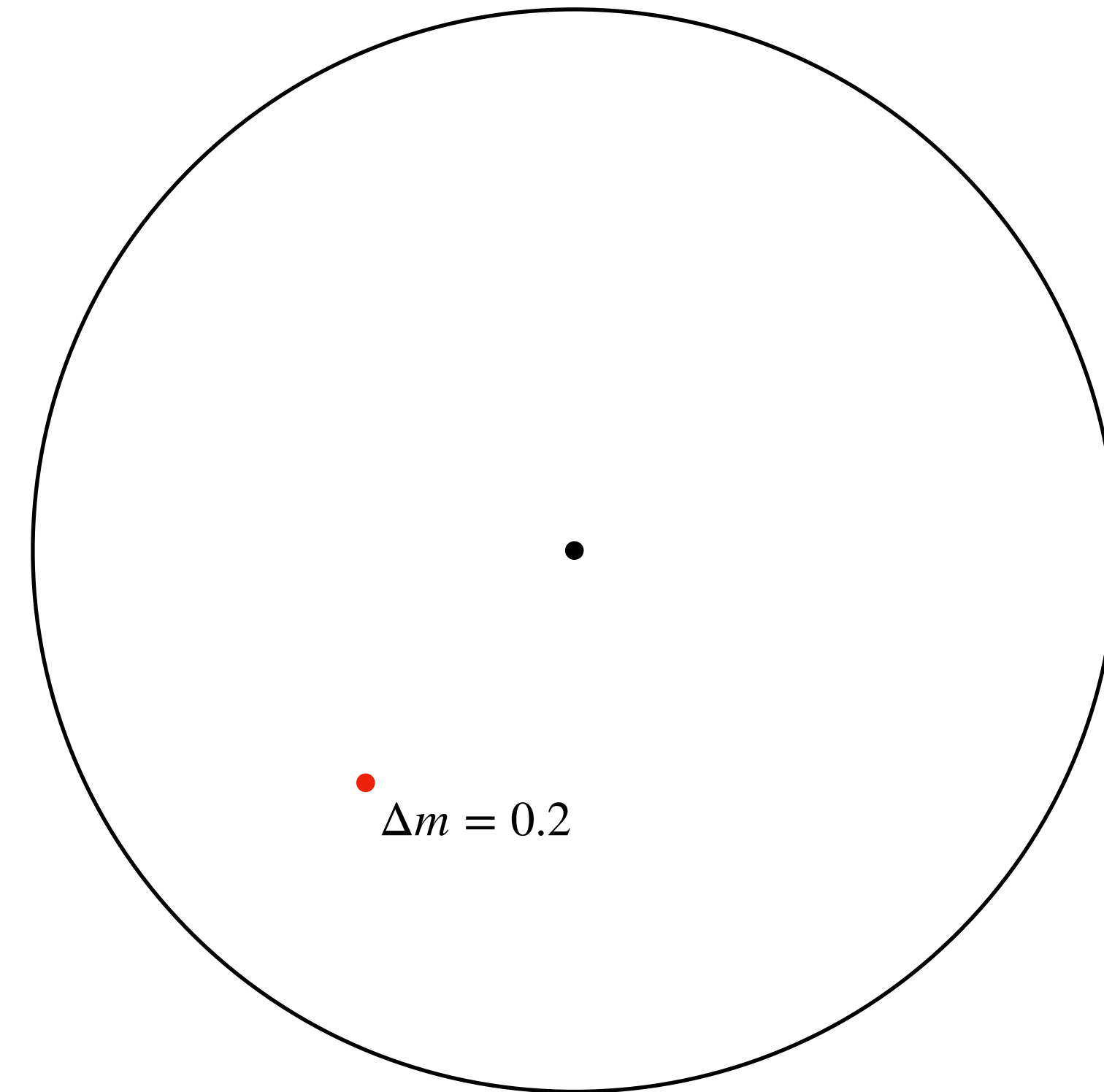
PSF radius ~ 1.2 FWHM (Rayleigh criterion)



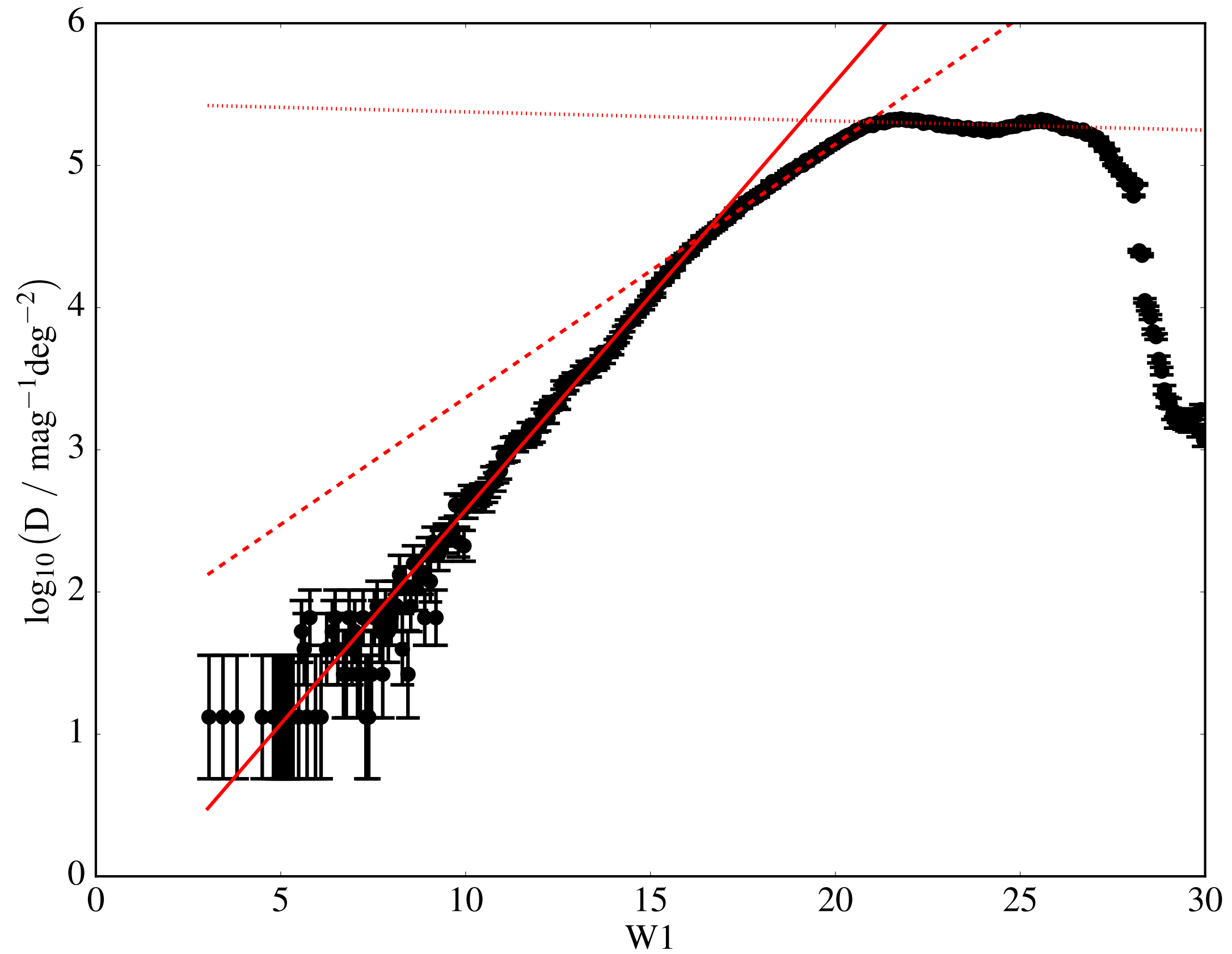
Building Empirical AUFs



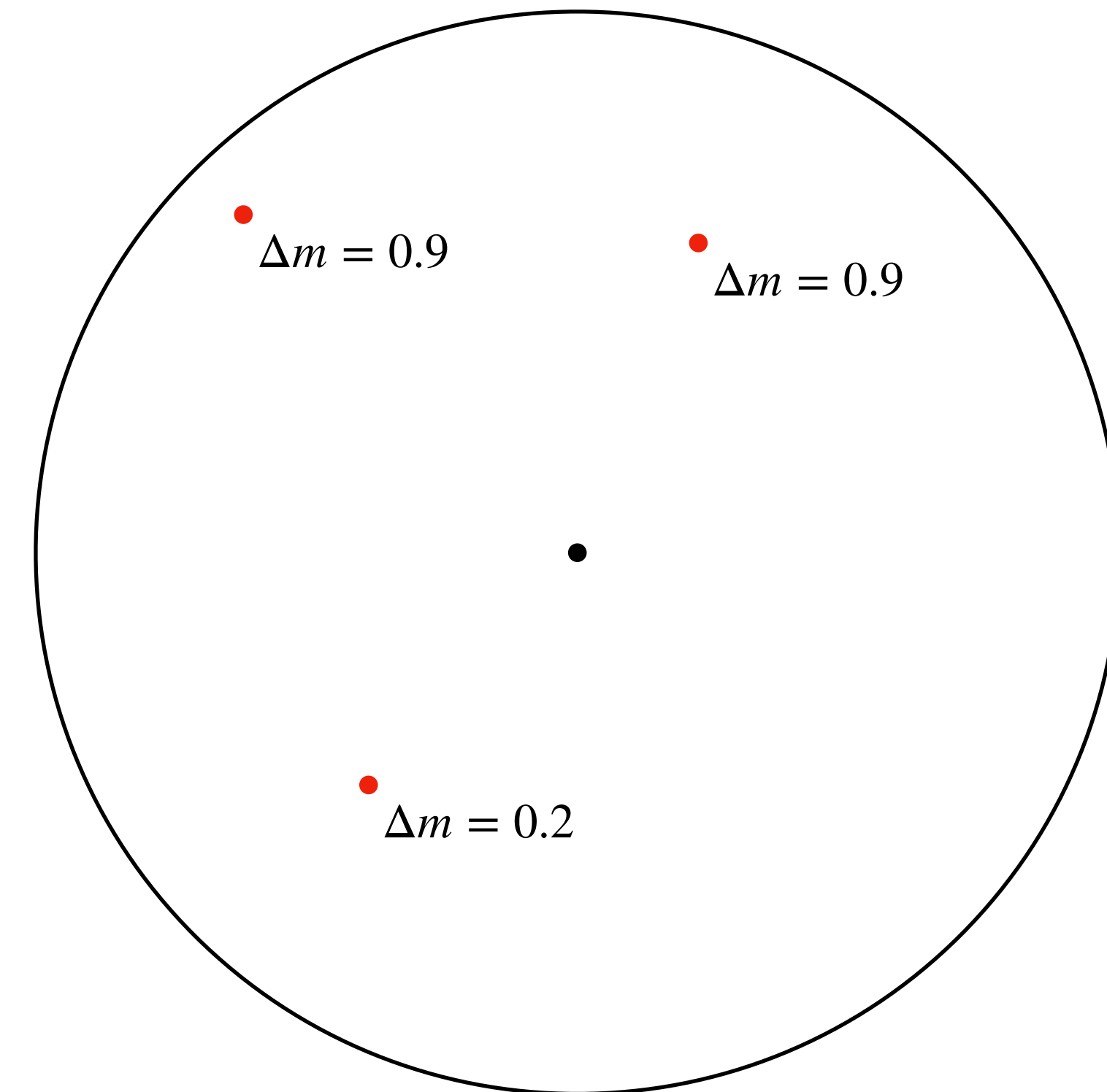
PSF radius ~ 1.2 FWHM (Rayleigh criterion)



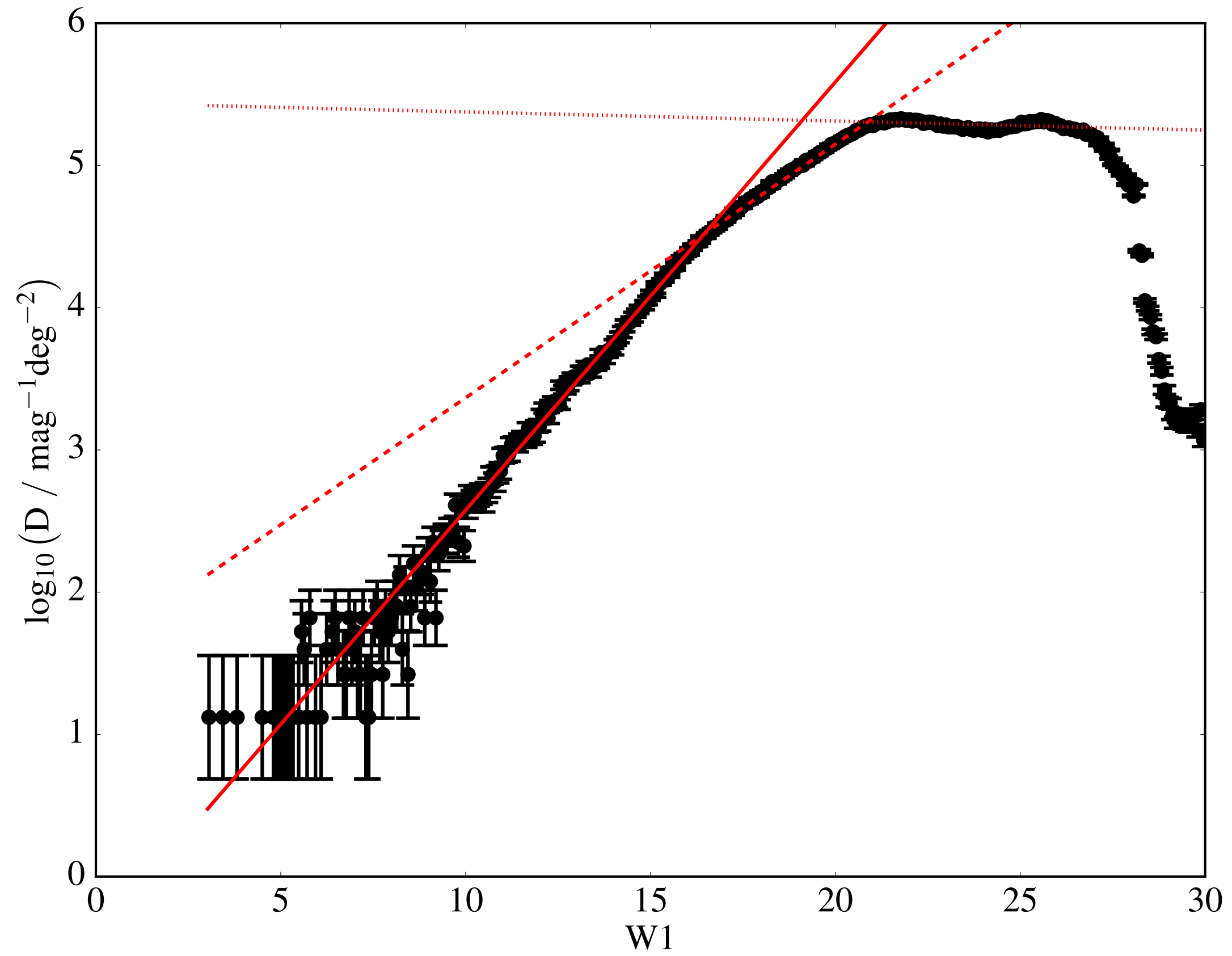
Building Empirical AUFs



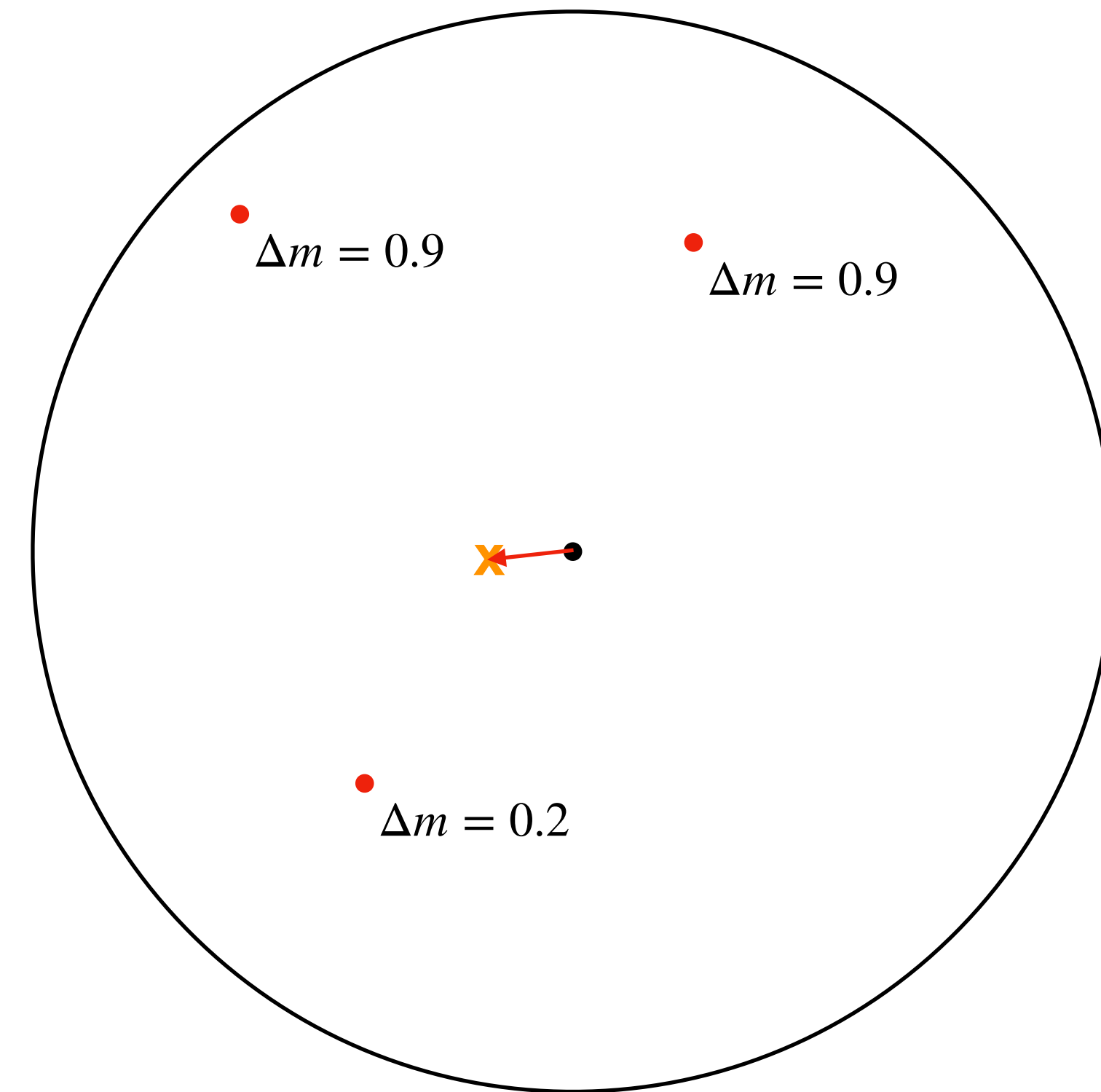
PSF radius ~ 1.2 FWHM (Rayleigh criterion)



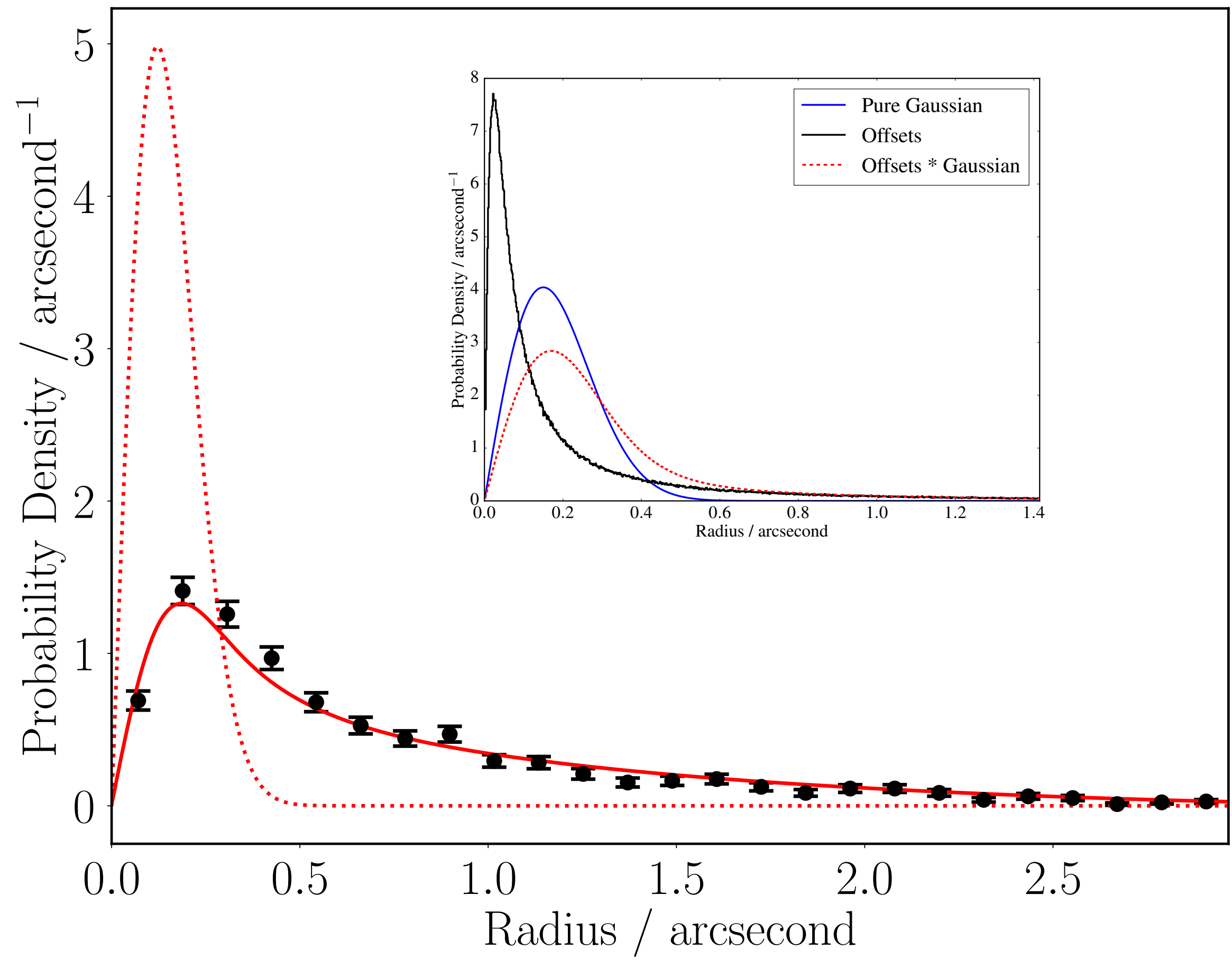
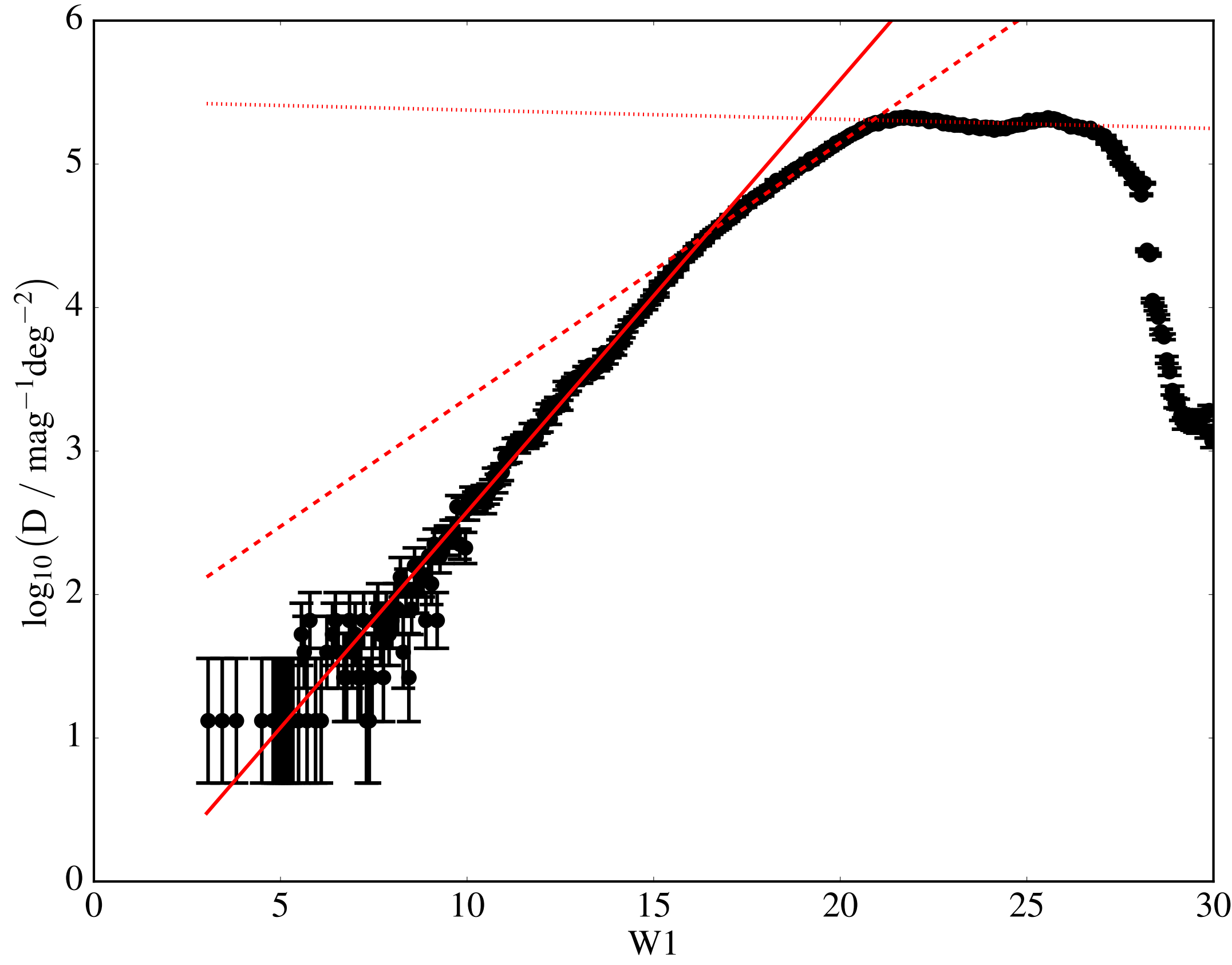
Building Empirical AUFs



PSF radius ~ 1.2 FWHM (Rayleigh criterion)

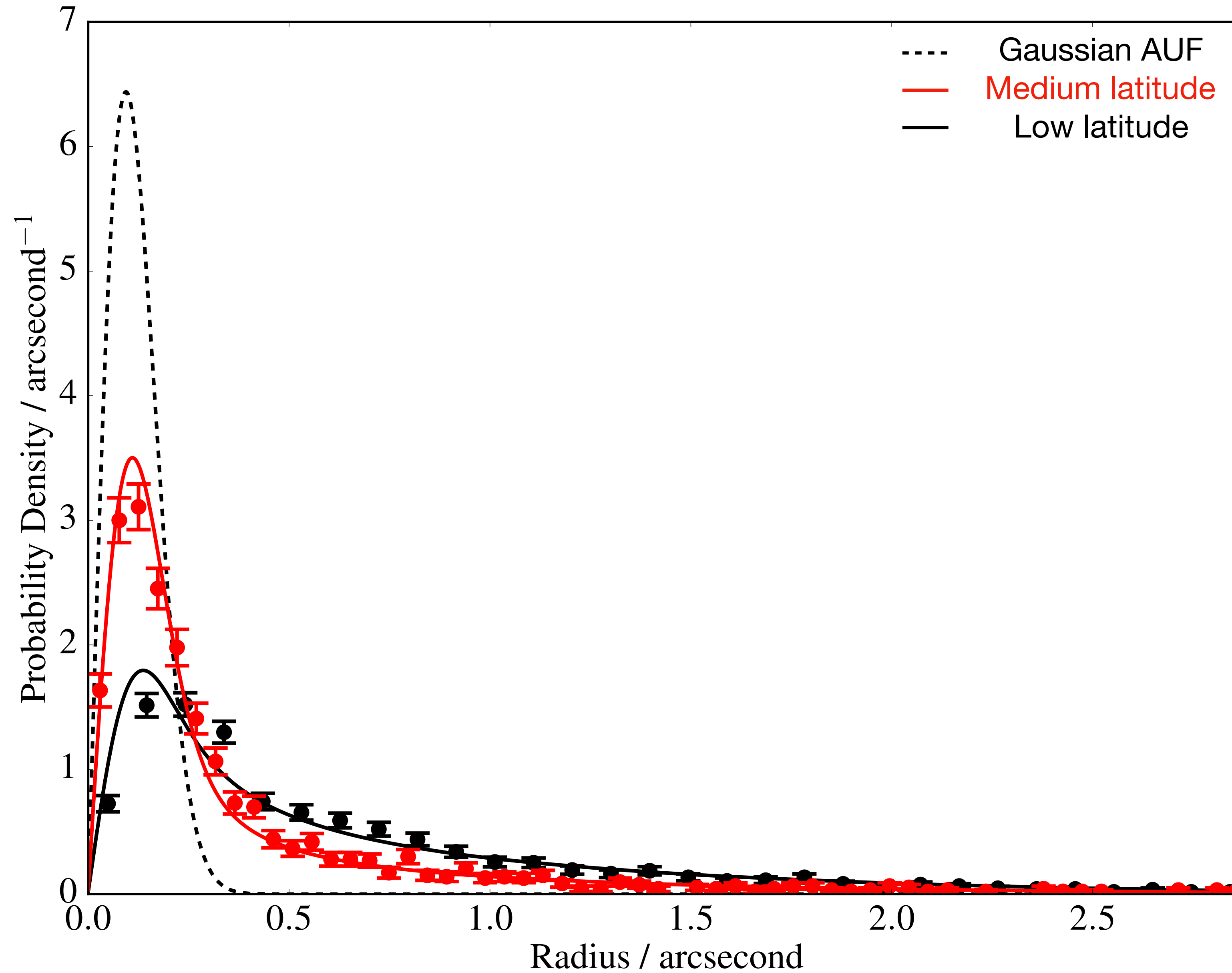


Building Empirical AUFs

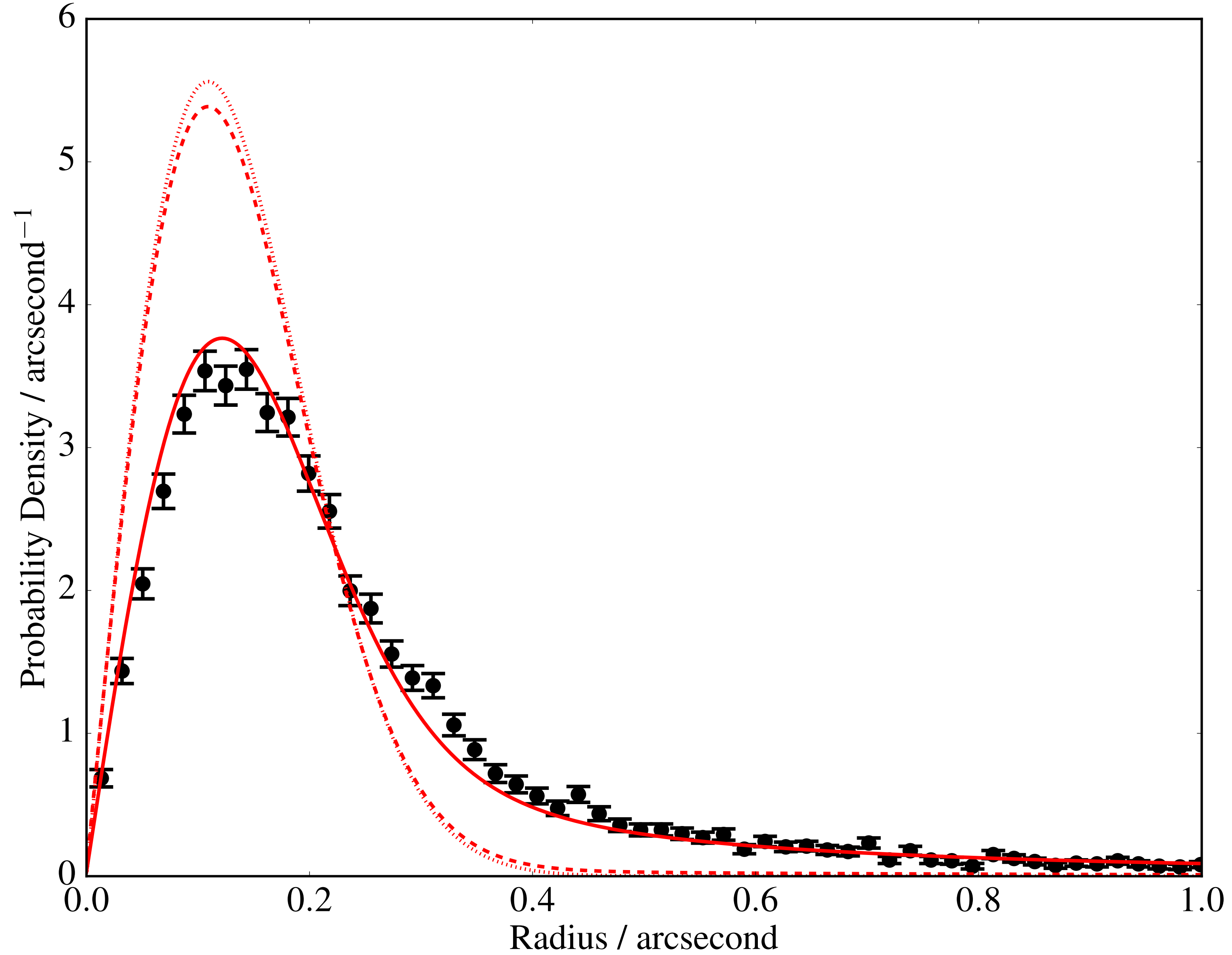


Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)

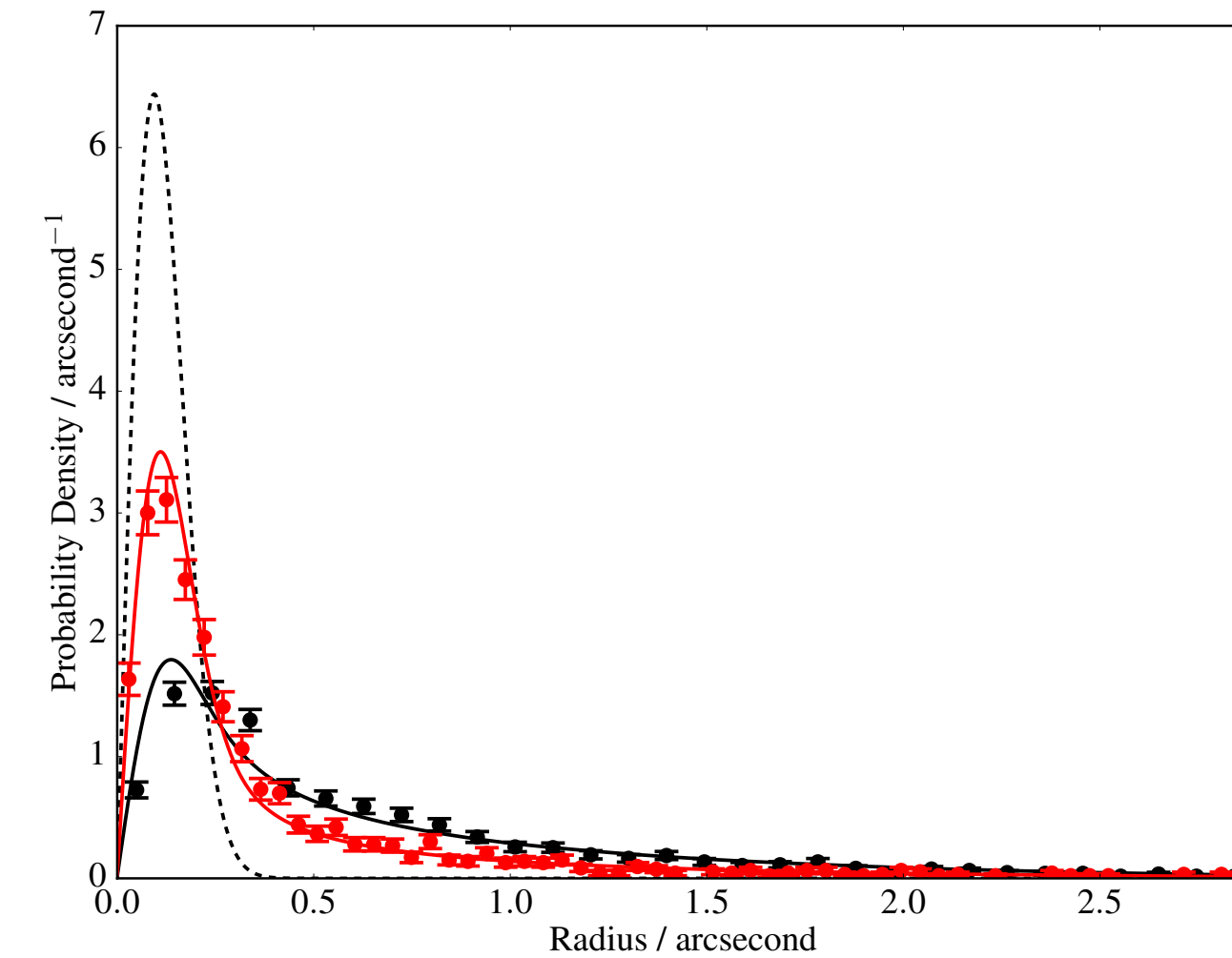
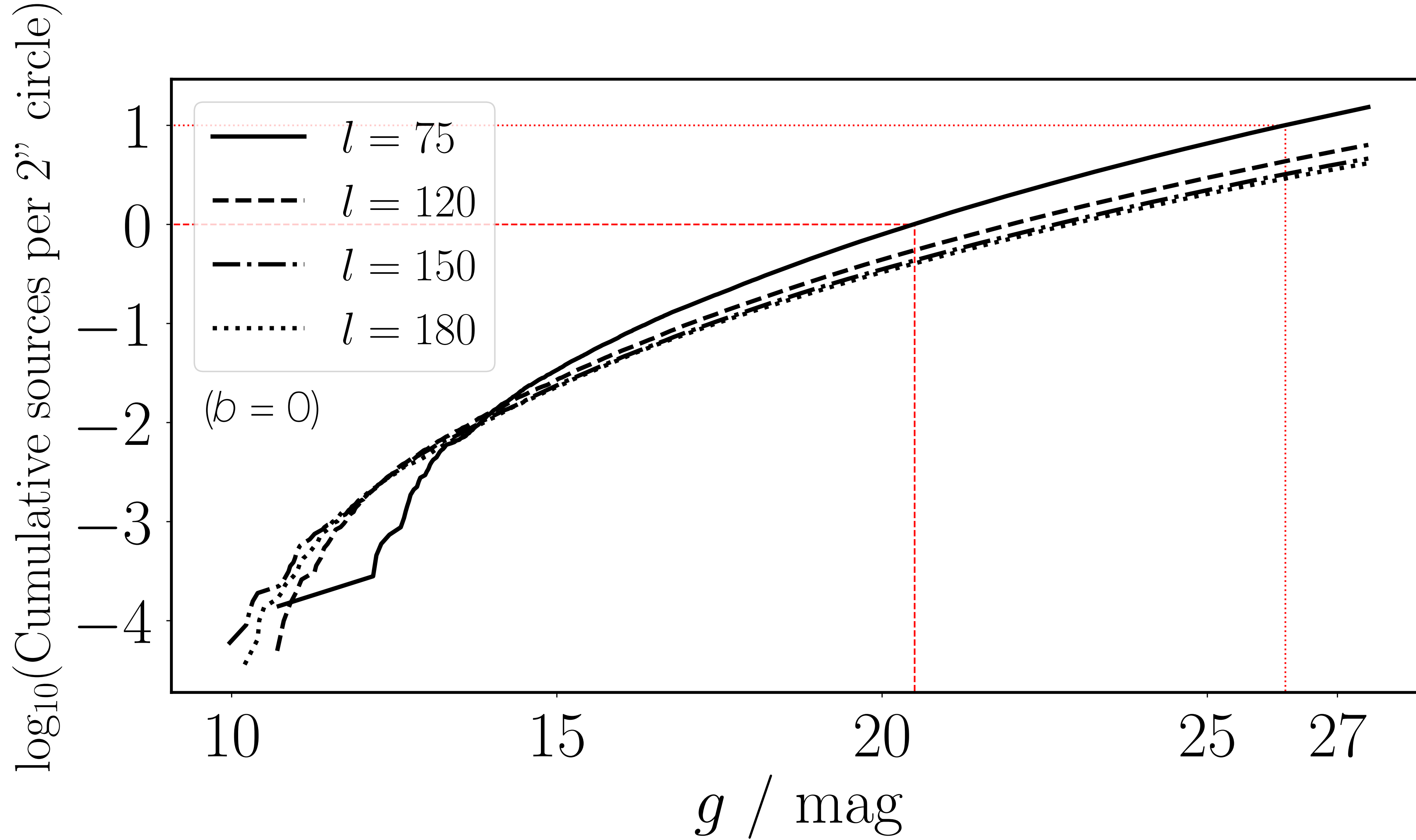
Crowding Normalisation



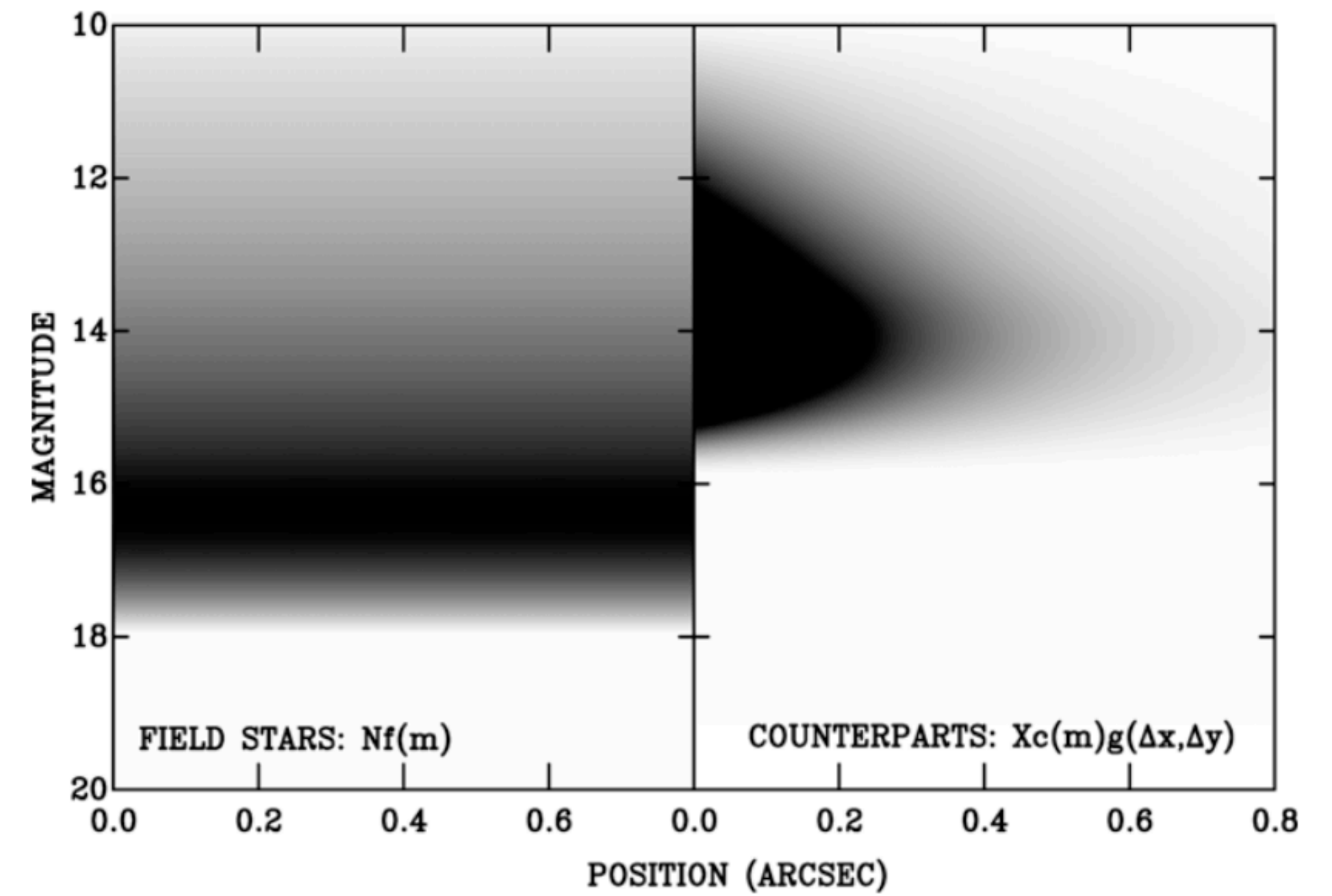
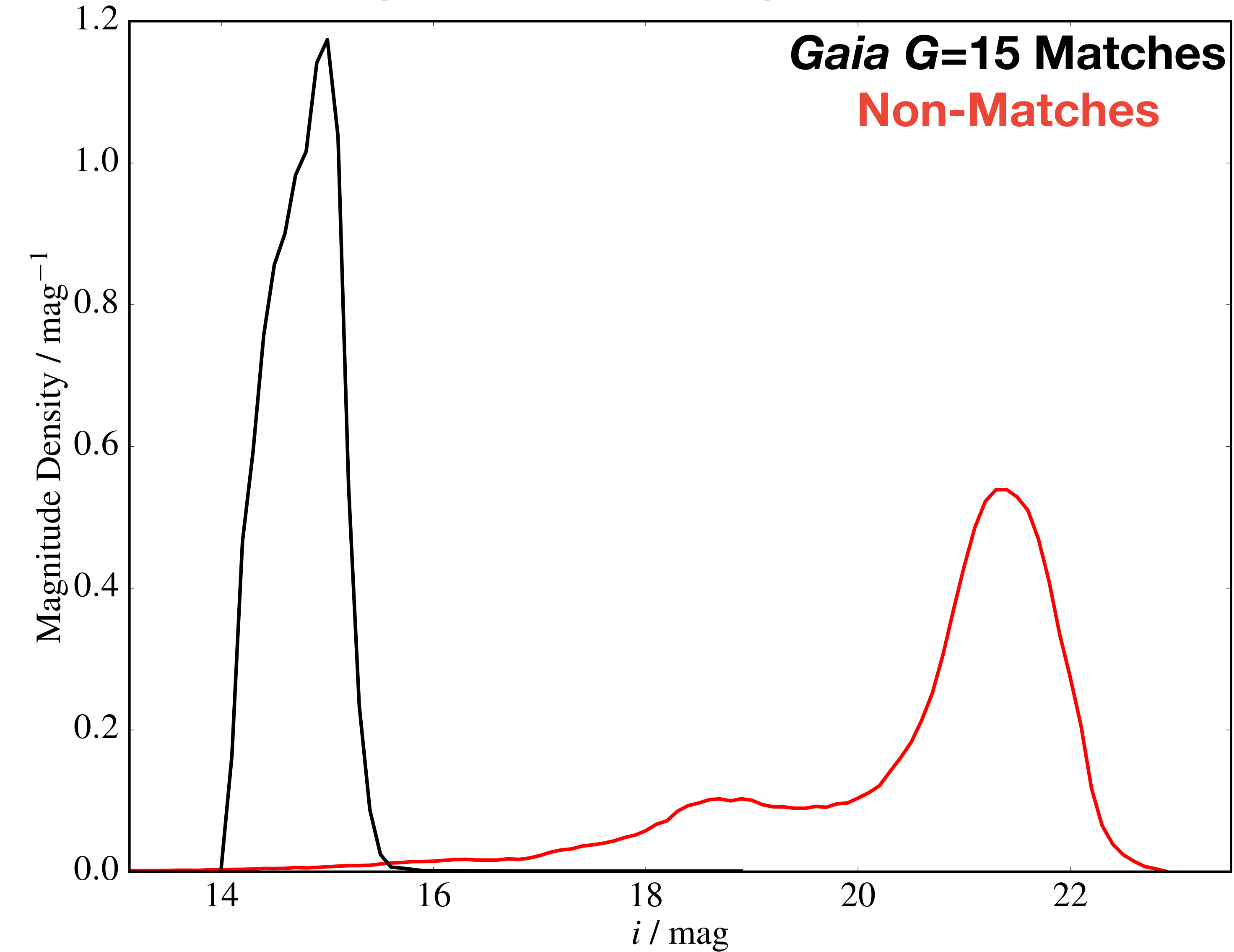
Extra-galactic Effects of Crowding



Vera C. Rubin Observatory's LSST



Including the Magnitude Information



Naylor, Broos, & Feigelson (2013)

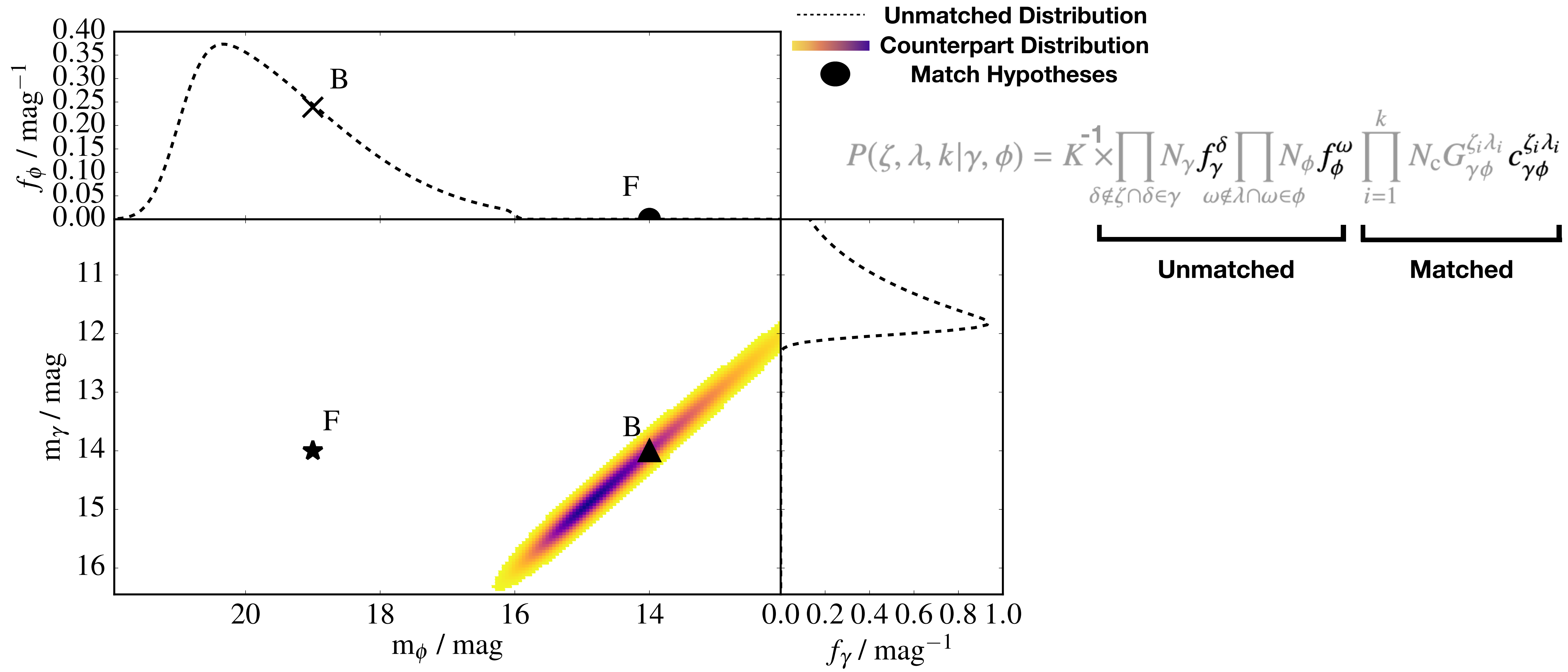
Wilson & Naylor (2018a)

IPHAS - Barentsen et al. (2014)

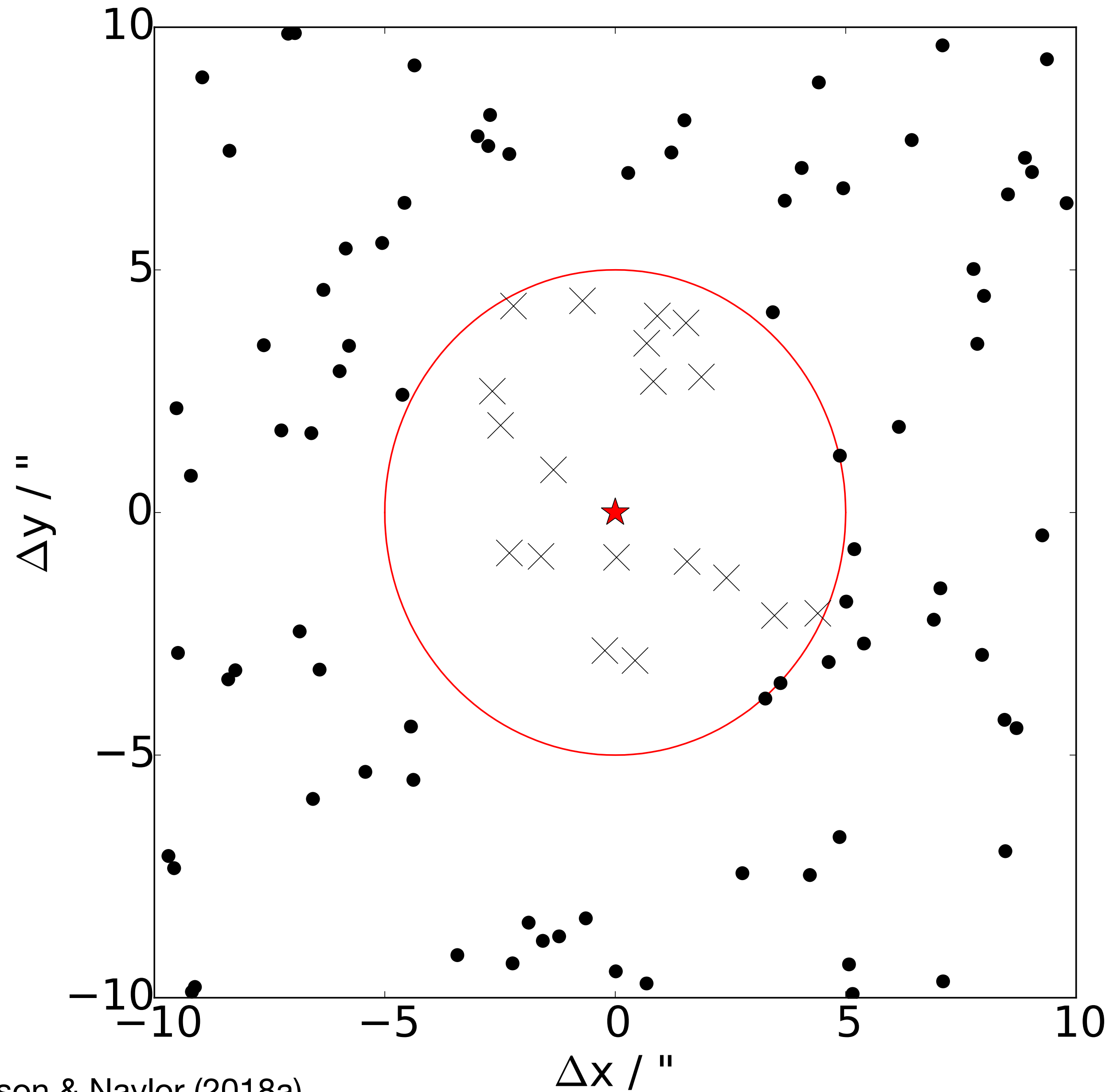
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

Tom J Wilson @onoddil

Including the Magnitude Information

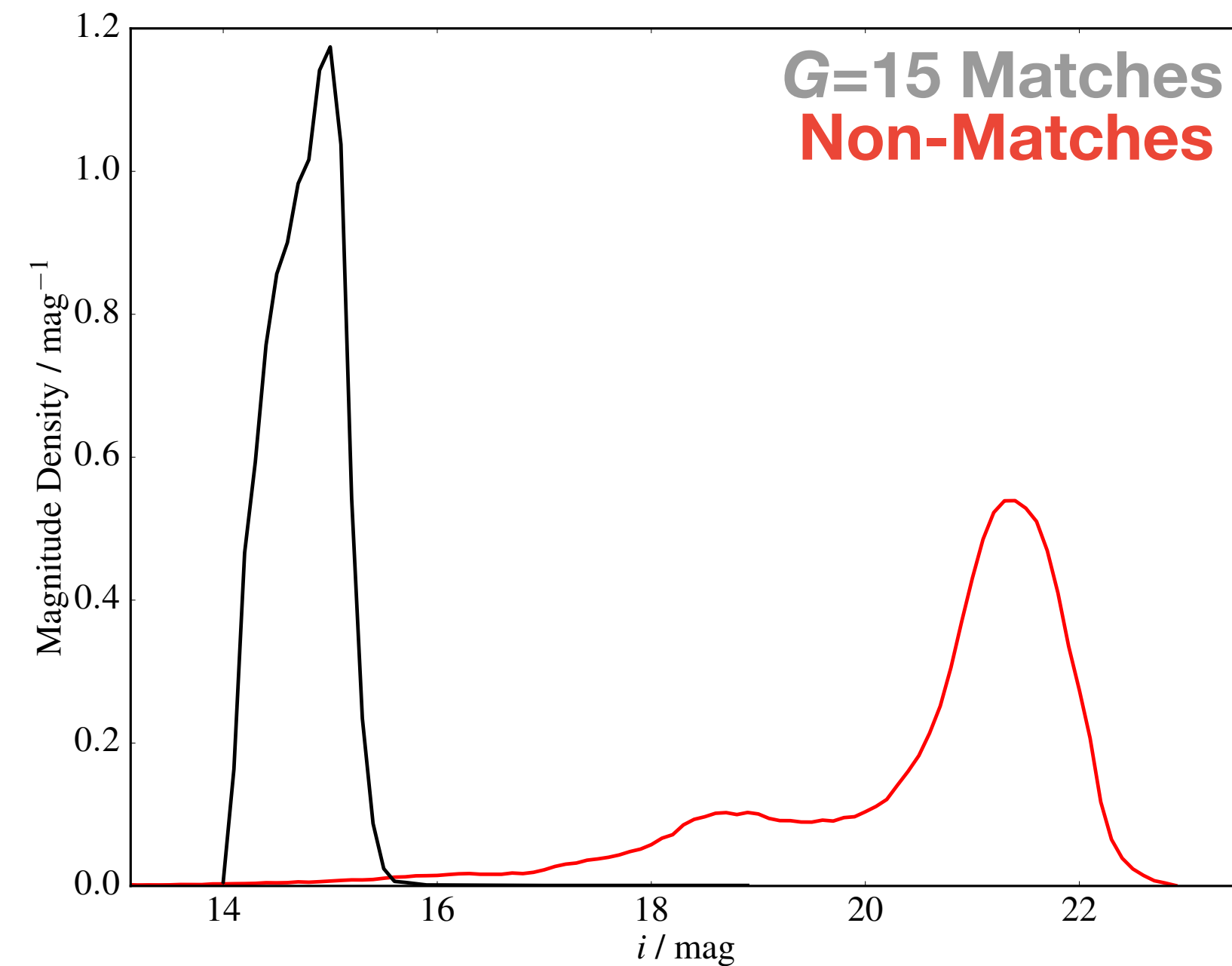


The “Field Star” Distribution



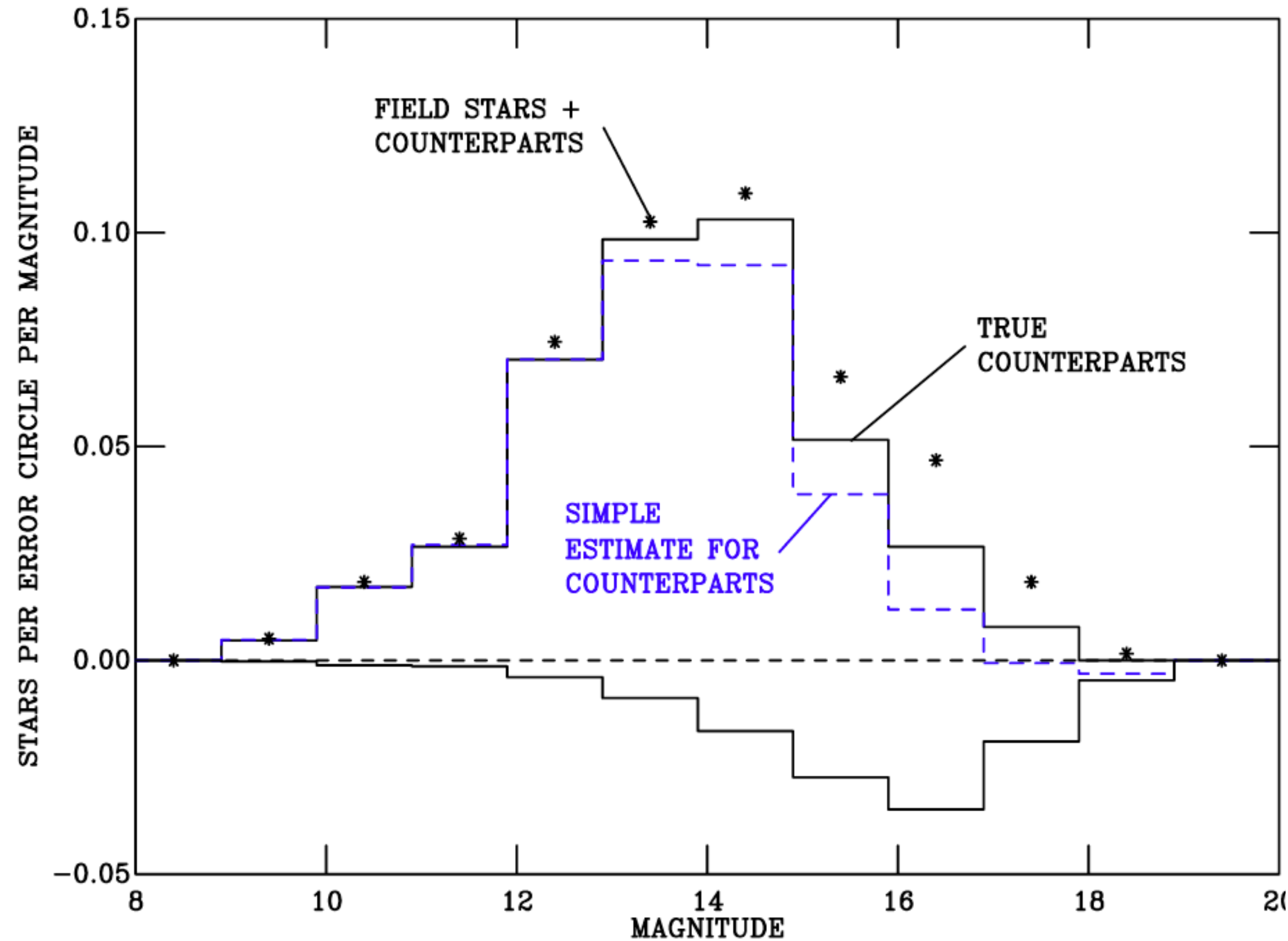
Wilson & Naylor (2018a)

$$P(\zeta, \lambda, k | \gamma, \phi) = K^{-1} \times \underbrace{\prod_{\delta \notin \zeta \cap \delta \in \gamma} N_{\gamma} f_{\gamma}^{\delta}}_{\text{Unmatched}} \underbrace{\prod_{\omega \notin \lambda \cap \omega \in \phi} N_{\phi} f_{\phi}^{\omega}}_{\text{Matched}} \prod_{i=1}^k N_c G_{\gamma\phi}^{\zeta_i \lambda_i} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

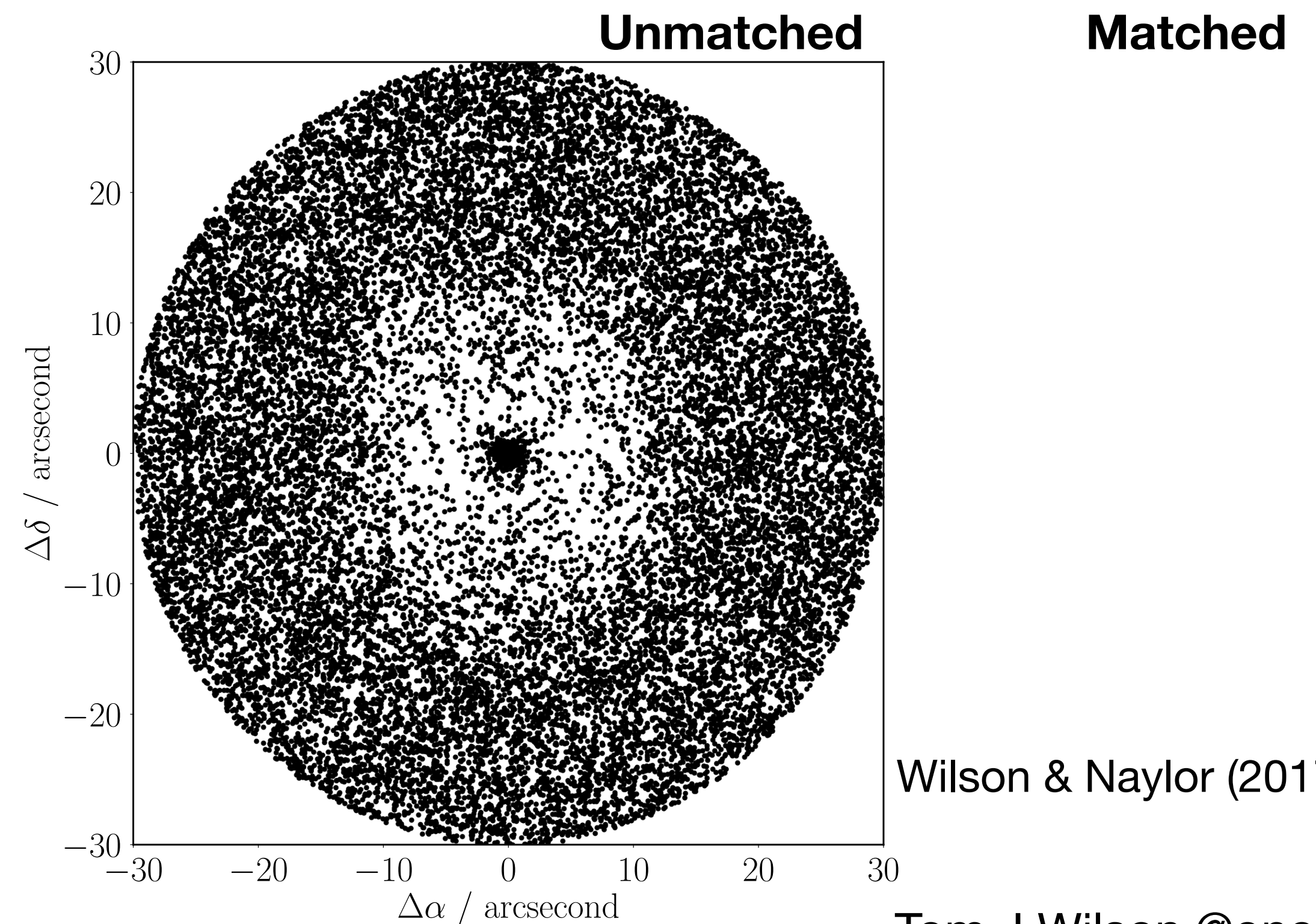


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The Counterpart Source Distribution

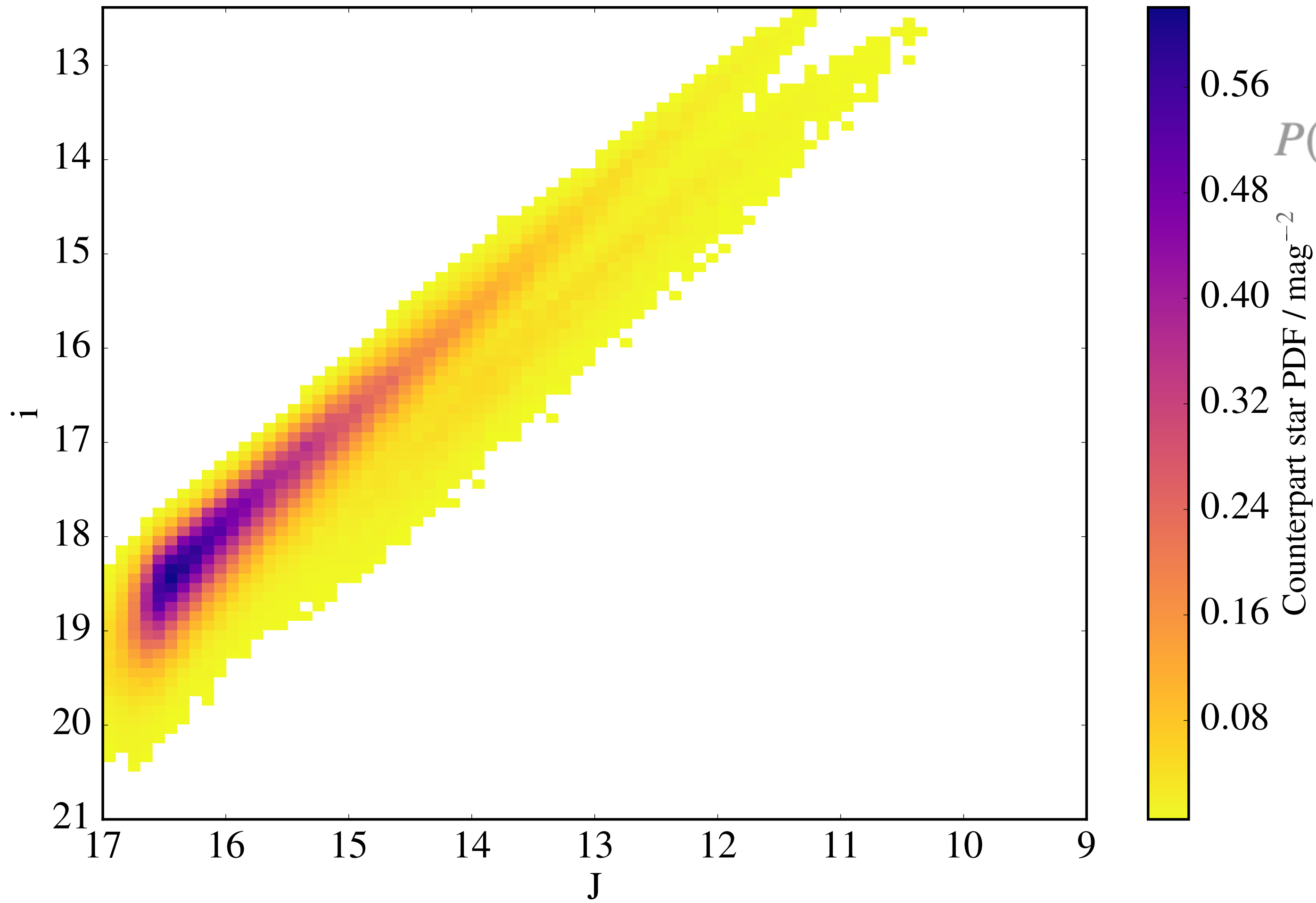


$$P(\zeta, \lambda, k | \gamma, \phi) = K^{-1} \times \underbrace{\prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta}_{\text{Unmatched}} \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \underbrace{\prod_{i=1}^k N_c G_{\gamma\phi}^{\zeta_i \lambda_i}}_{\text{Matched}} \underbrace{C_{\gamma\phi}^{\zeta_i \lambda_i}}_{\text{Matched}}$$



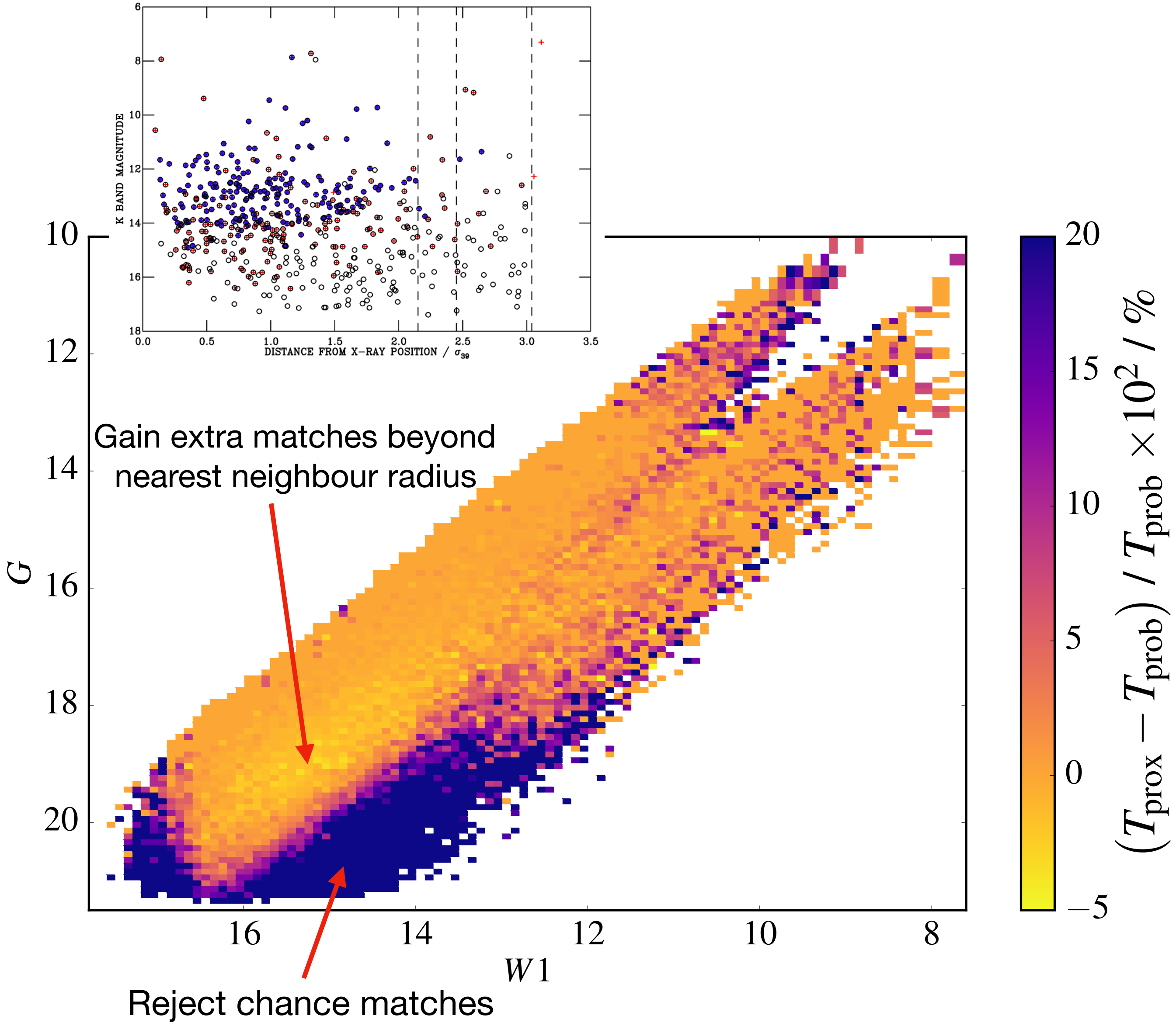
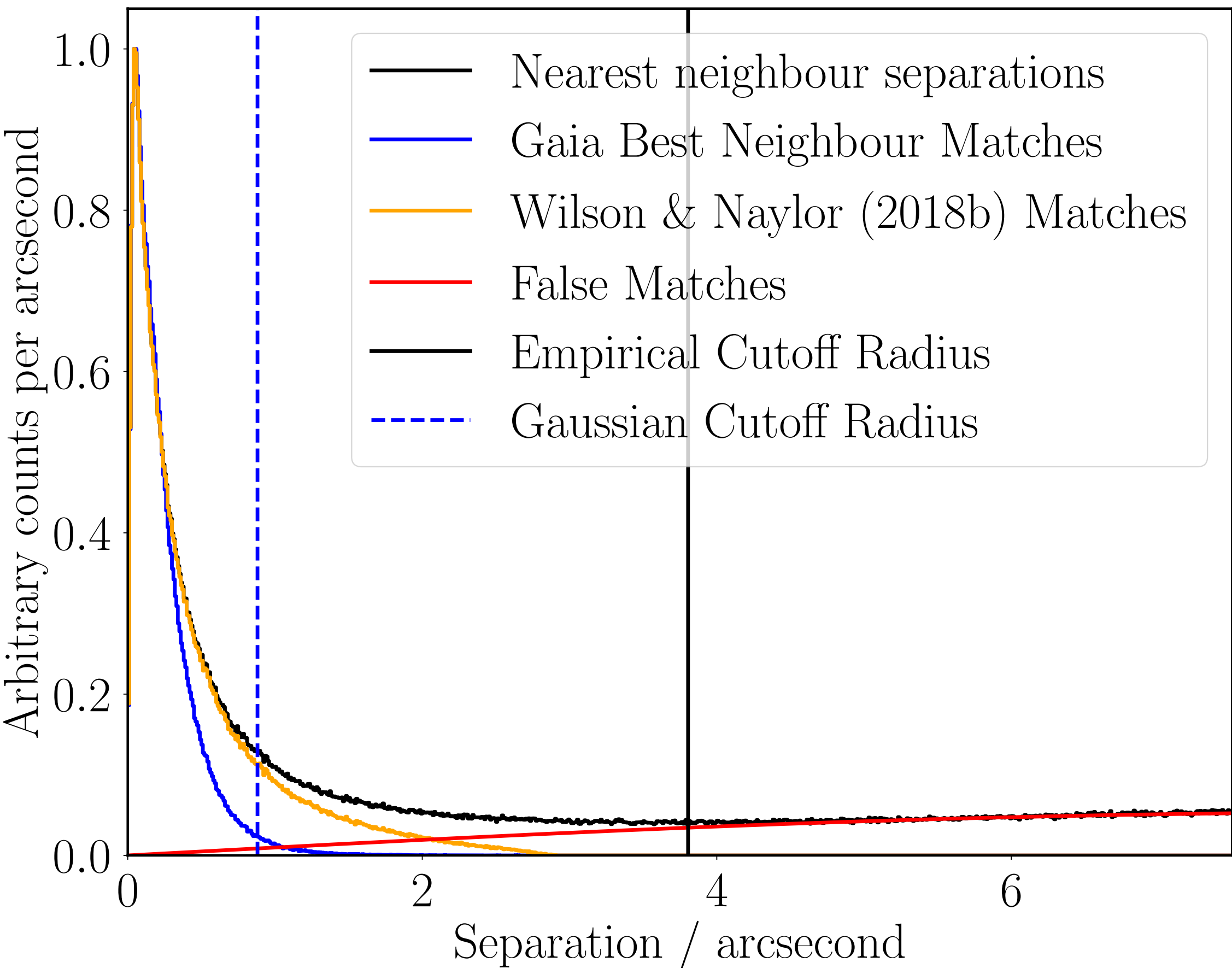
Wilson & Naylor (2017)

The Counterpart Source Distribution



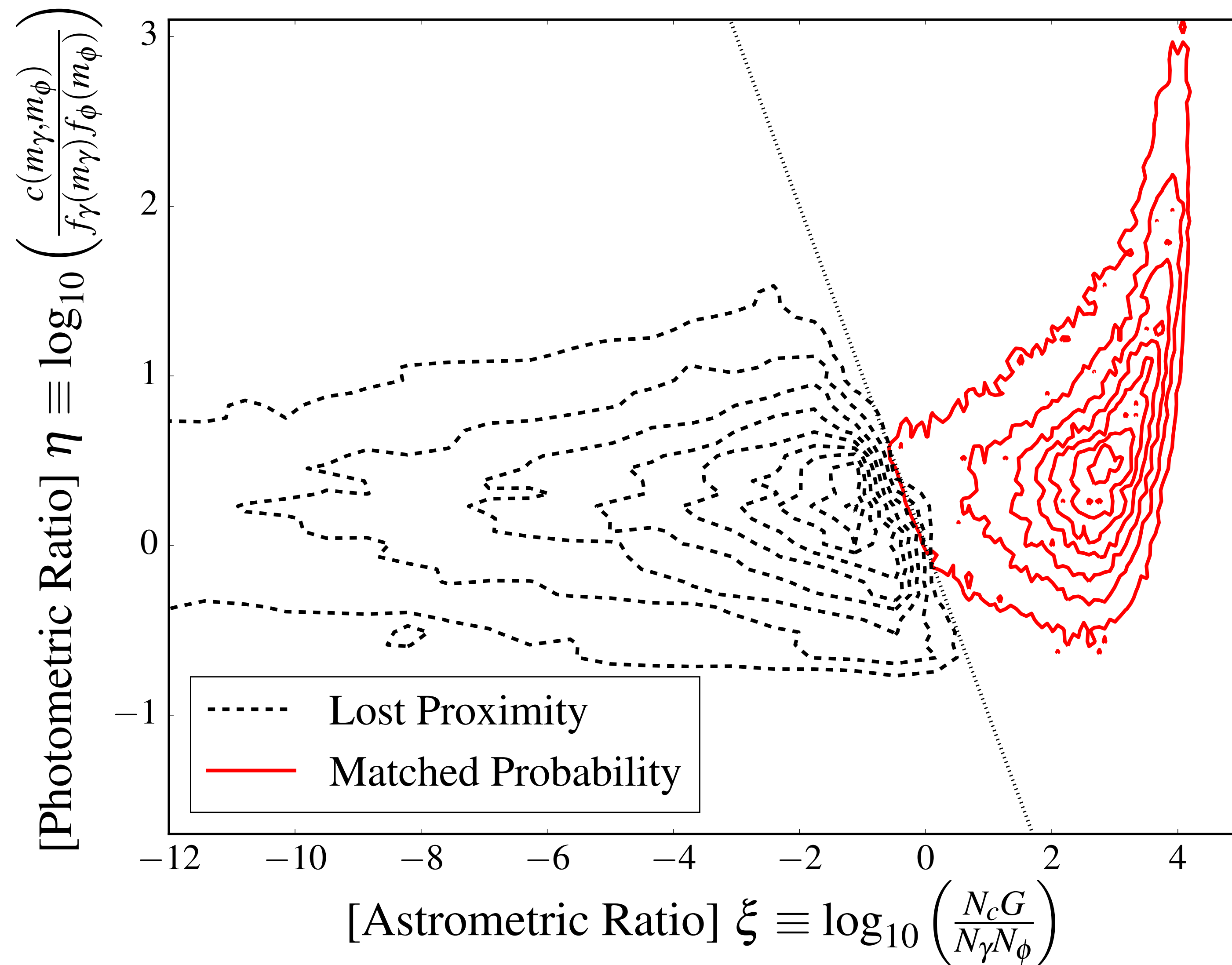
$$P(\zeta, \lambda, k | \gamma, \phi) = K^{-1} \times \underbrace{\prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega}_{\text{Unmatched}} \underbrace{\prod_{i=1}^k N_c G_{\gamma\phi}^{\zeta_i \lambda_i} C_{\gamma\phi}^{\zeta_i \lambda_i}}_{\text{Matched}}$$

Comparing Match Distributions

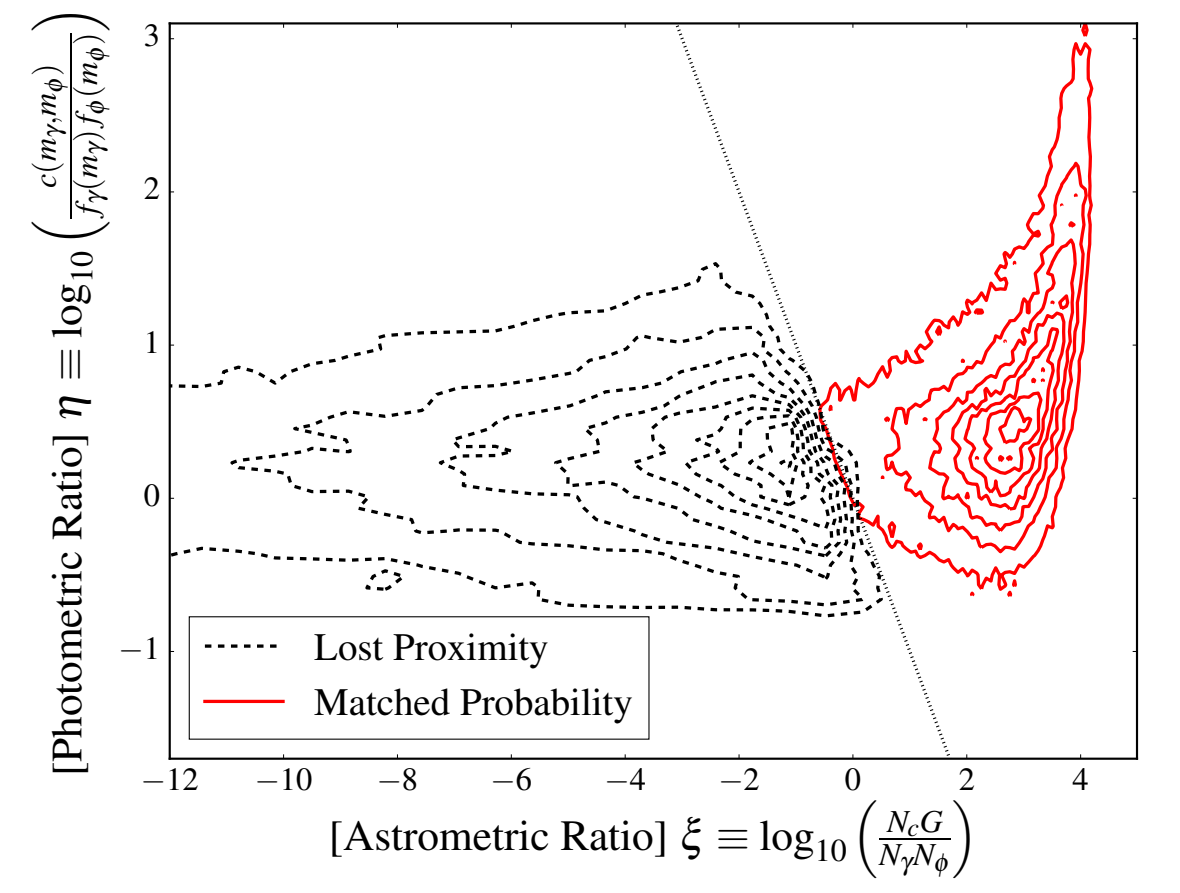
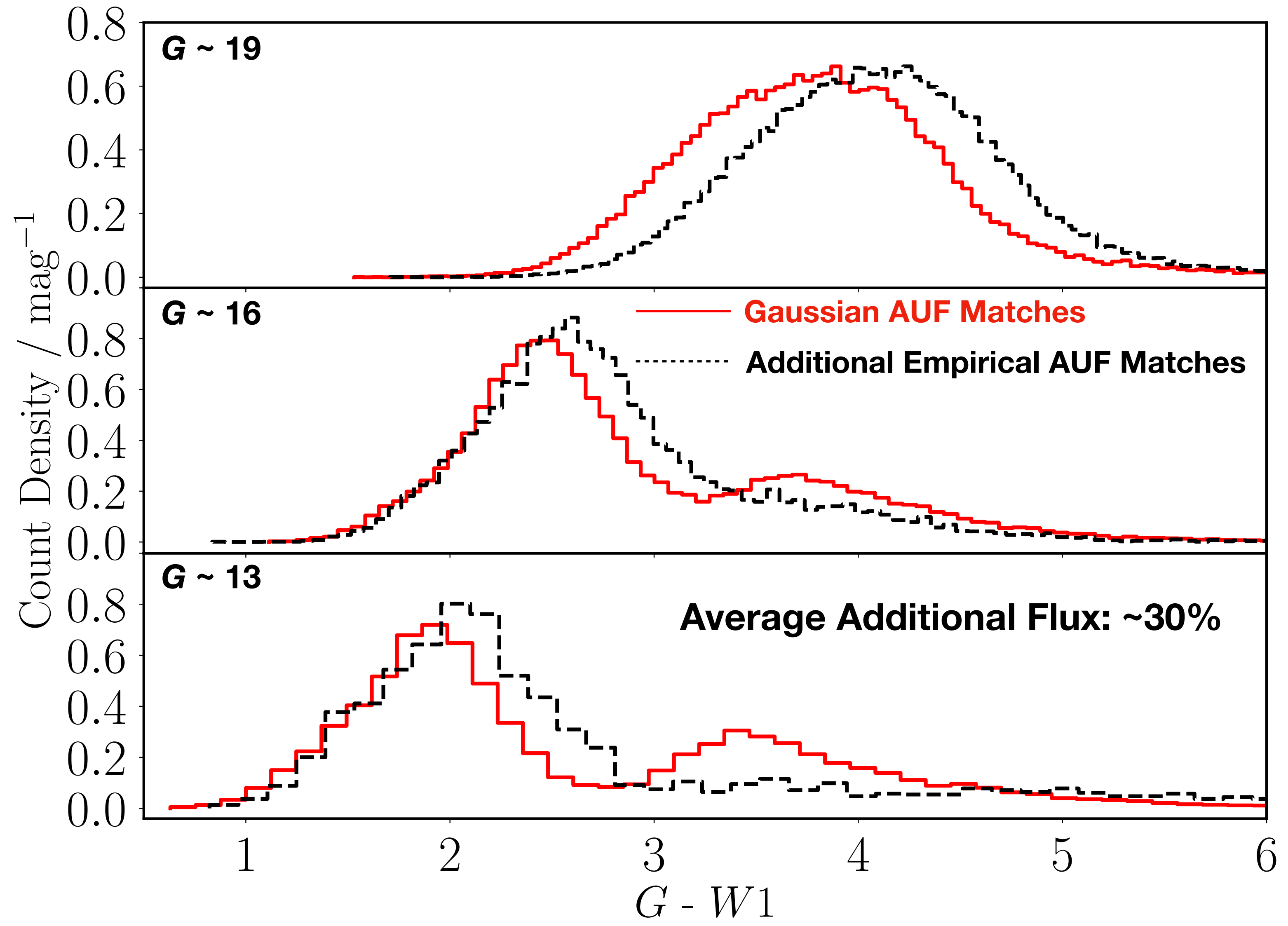


Wilson & Naylor (2018b)
 WISE - Wright et al. (2010)
 Gaia matches - Marrese et al. (2019)
 Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

Lost Gaussian-Only Matches

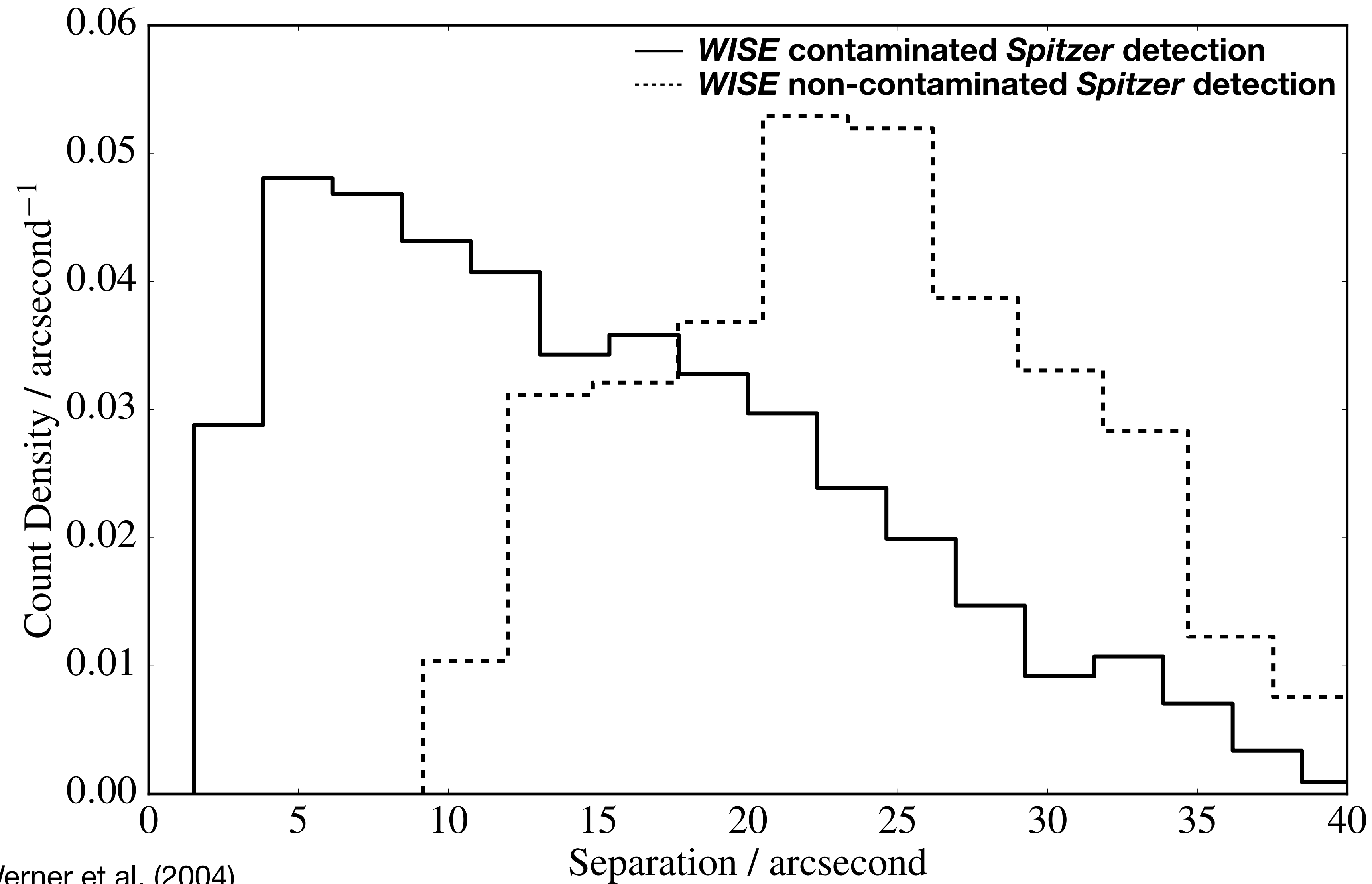


Perturbation-Colour Correlation



“Extra flux” has an impact on derived proper motions and parallaxes, and IR excesses!

Resolving Contaminants



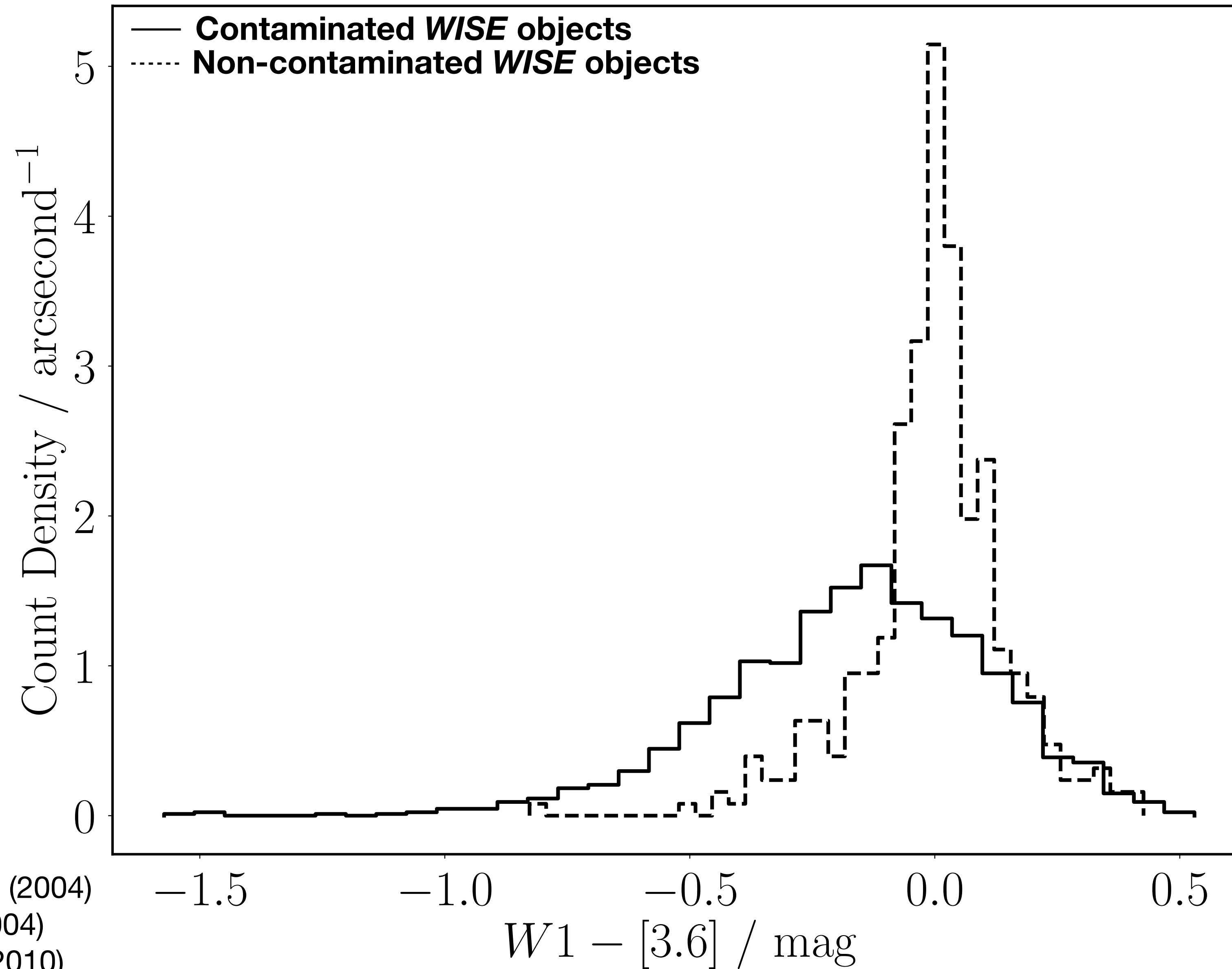
Spitzer - Werner et al. (2004)

IRAC - Fazio et al. (2004)

WISE - Wright et al. (2010)

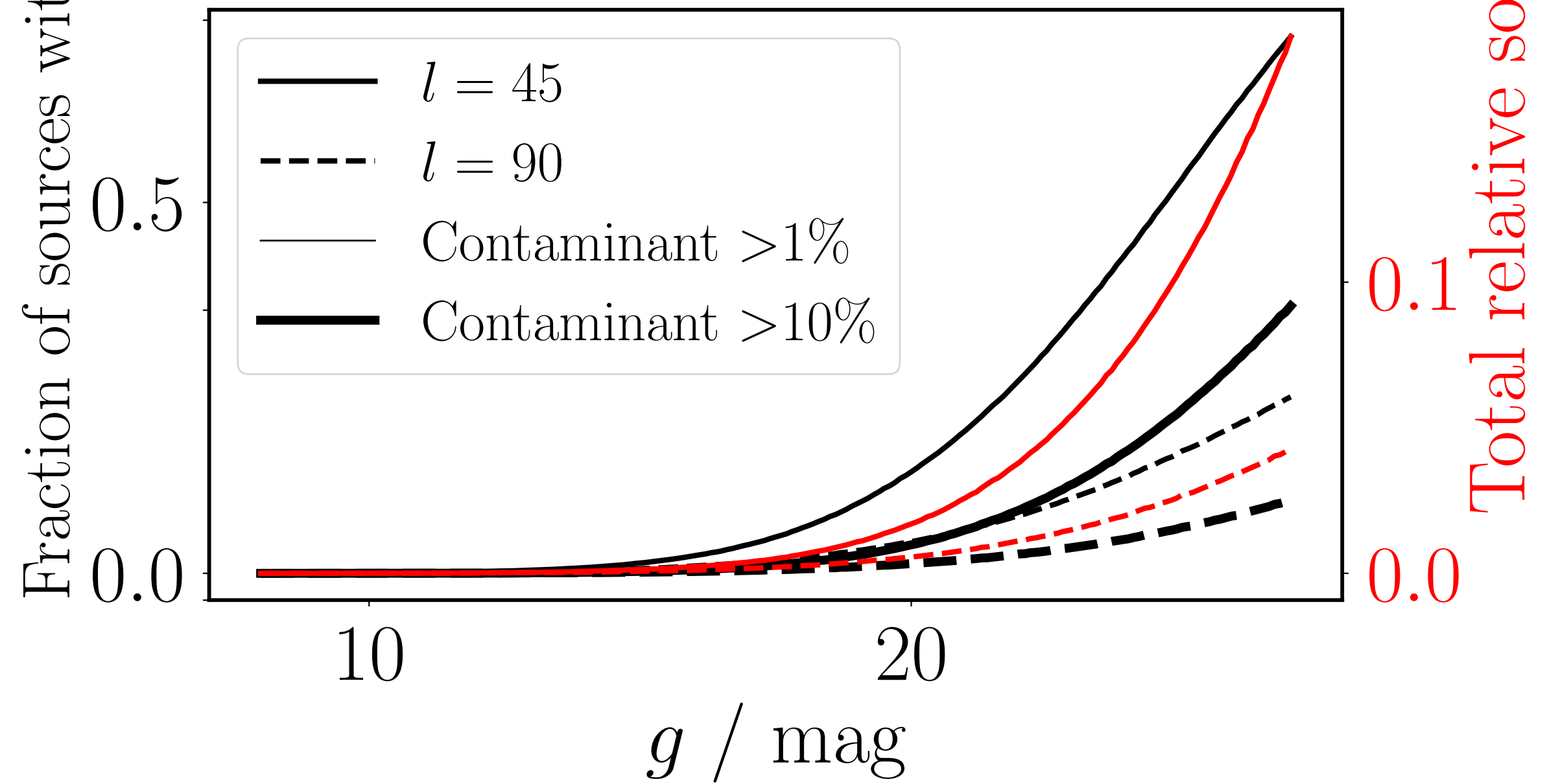
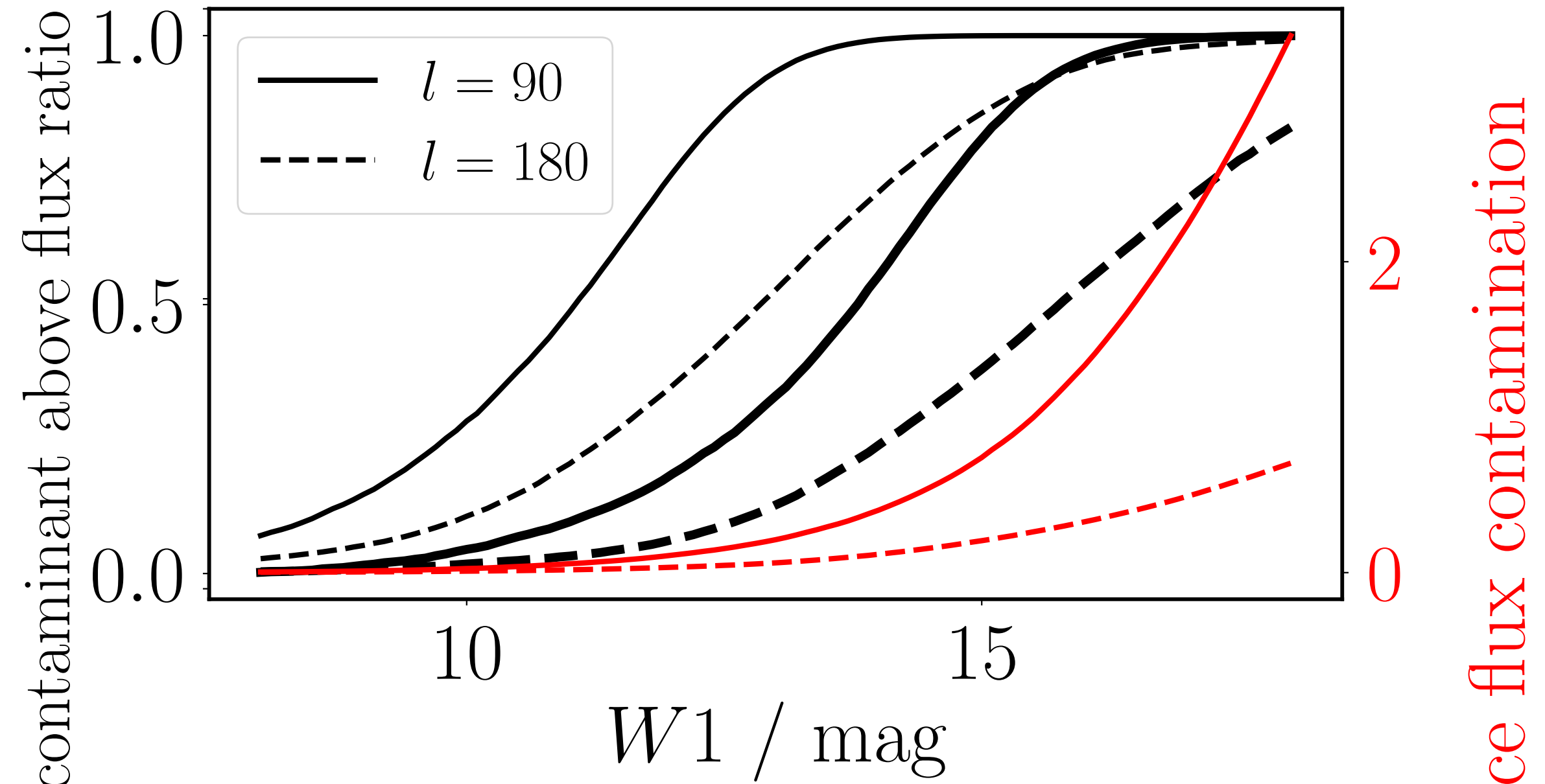
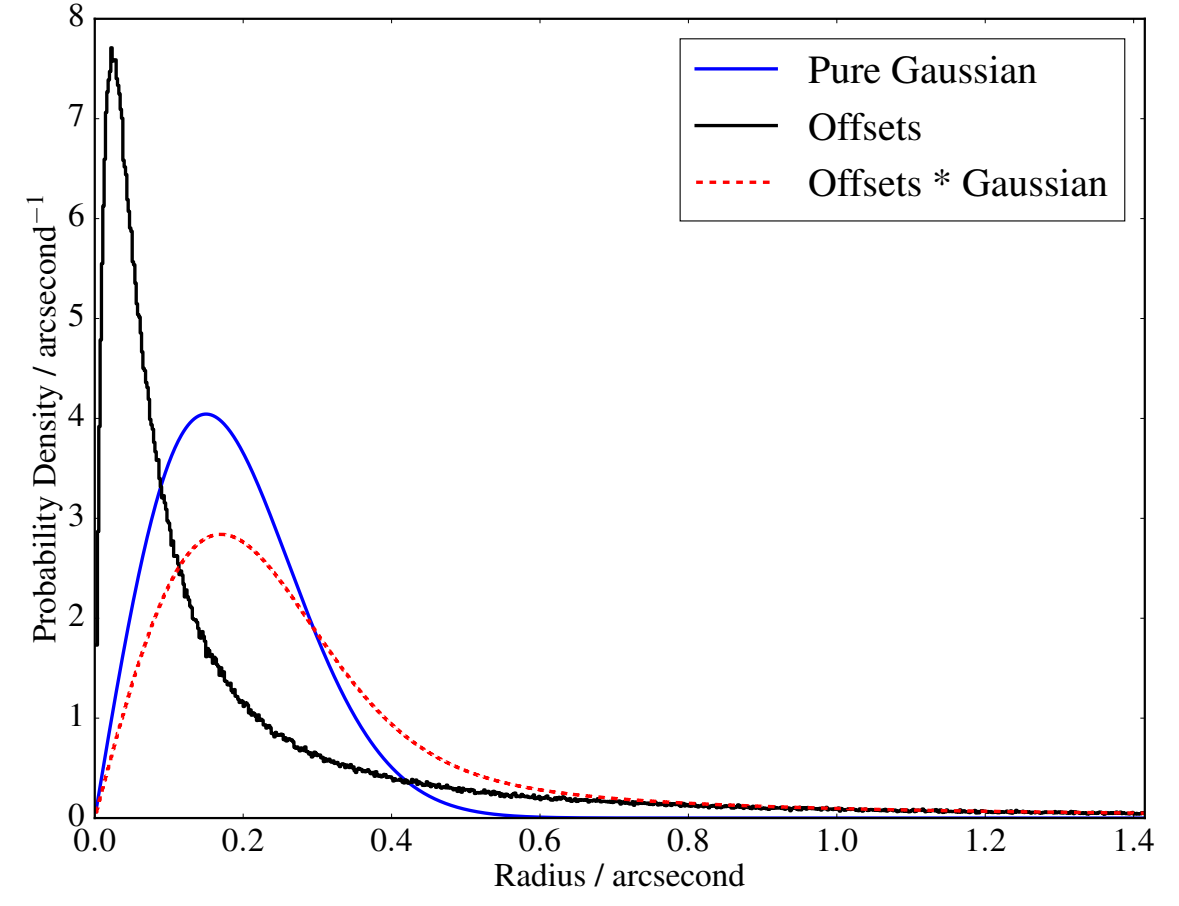
Wilson & Naylor (2018b)

Resolving Contaminant Flux

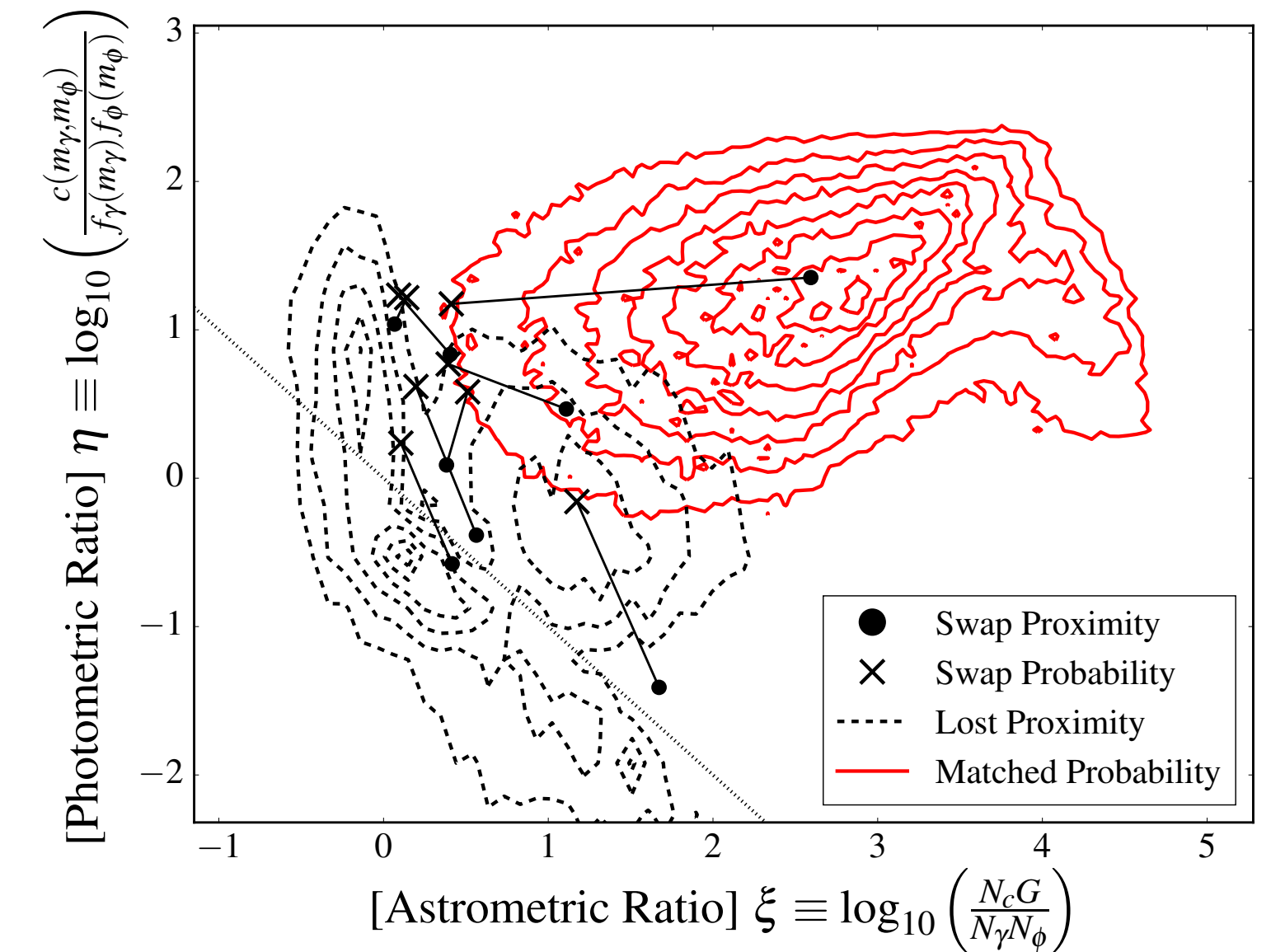
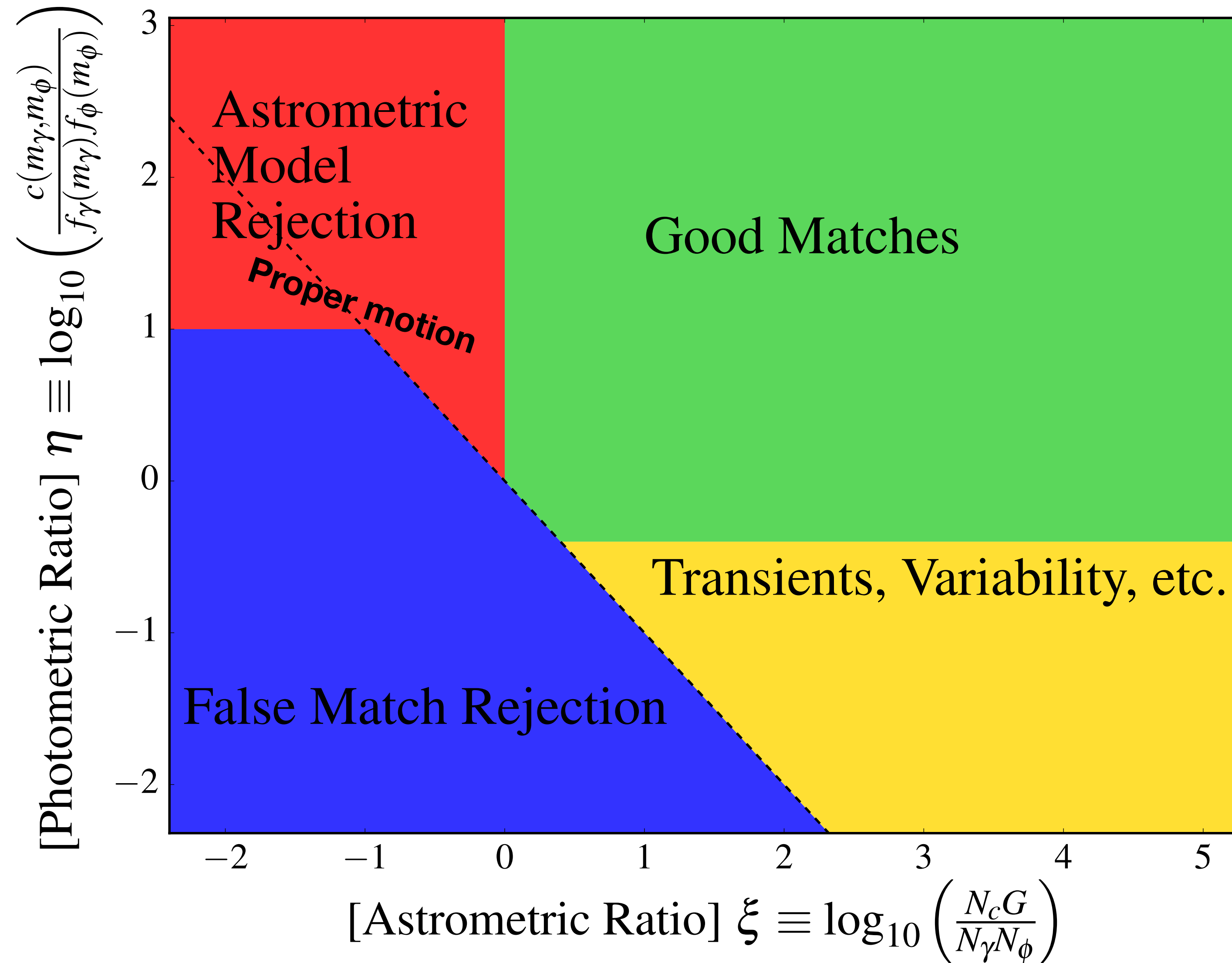


Spitzer - Werner et al. (2004)
IRAC - Fazio et al. (2004)
WISE - Wright et al. (2010)
Wilson & Naylor (2018b)

Contamination Rates and Amounts



The Likelihood Space



Open Source Code: macauff

Matching Across Catalogues using the Astrometric Uncertainty Function and Flux



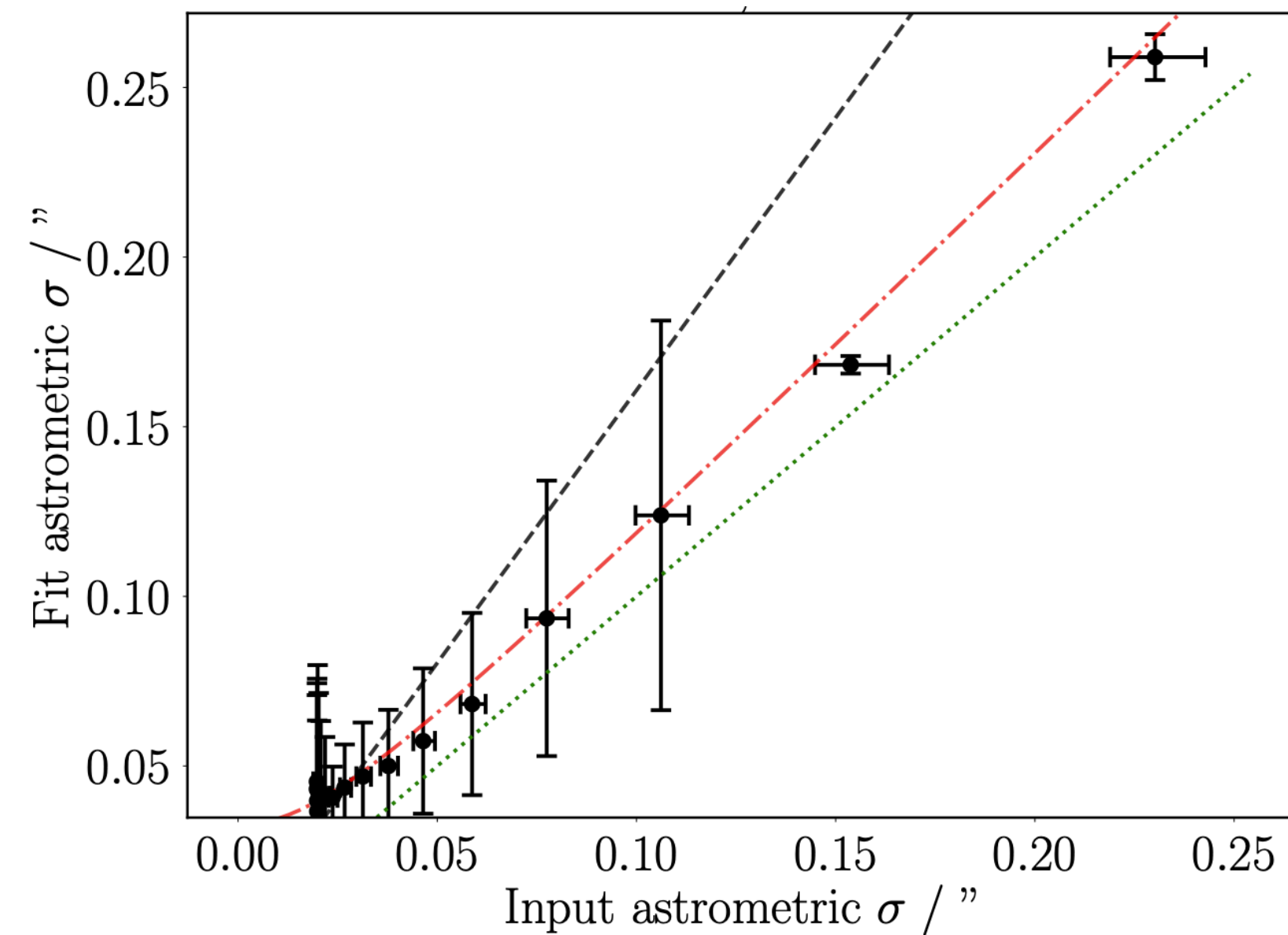
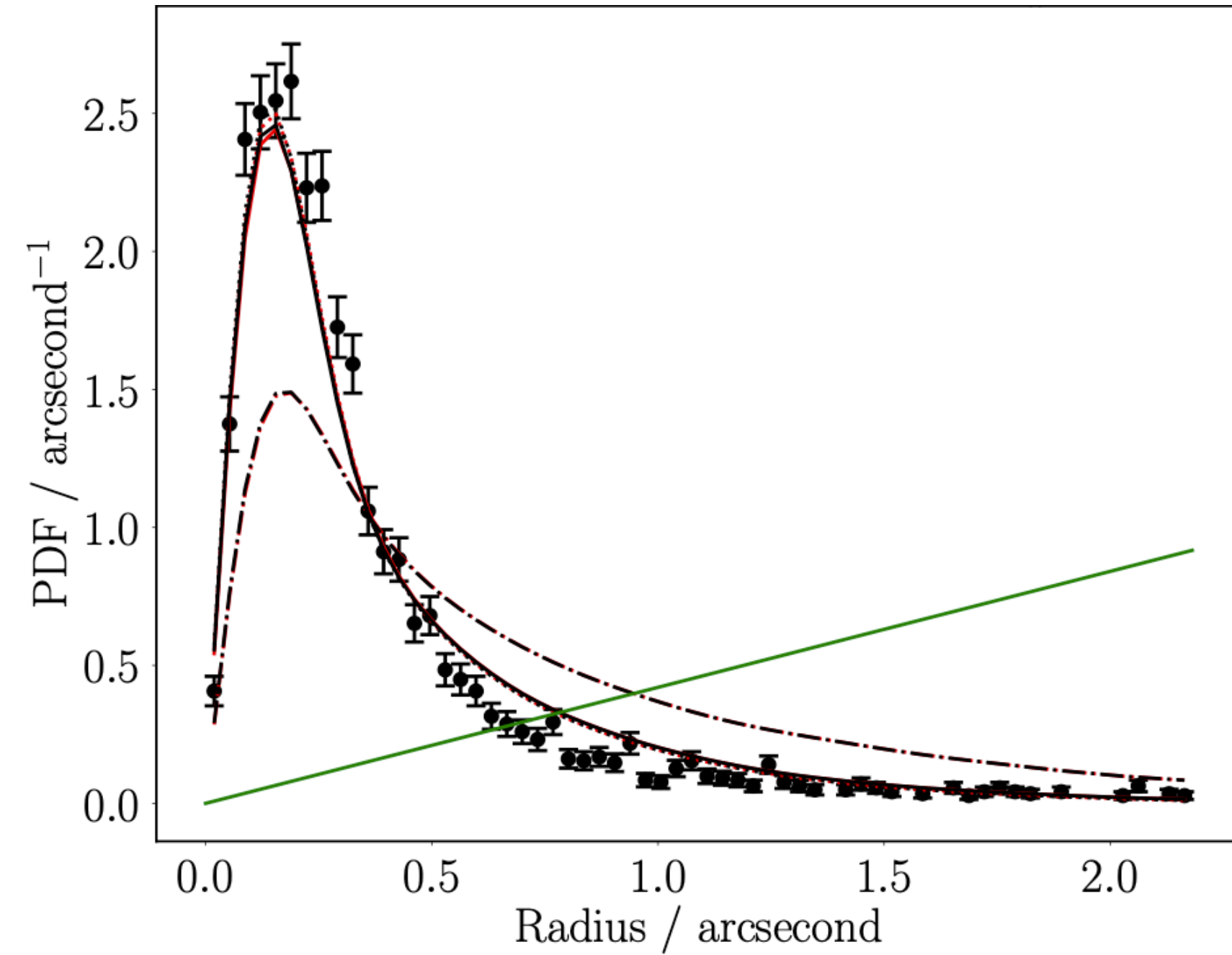
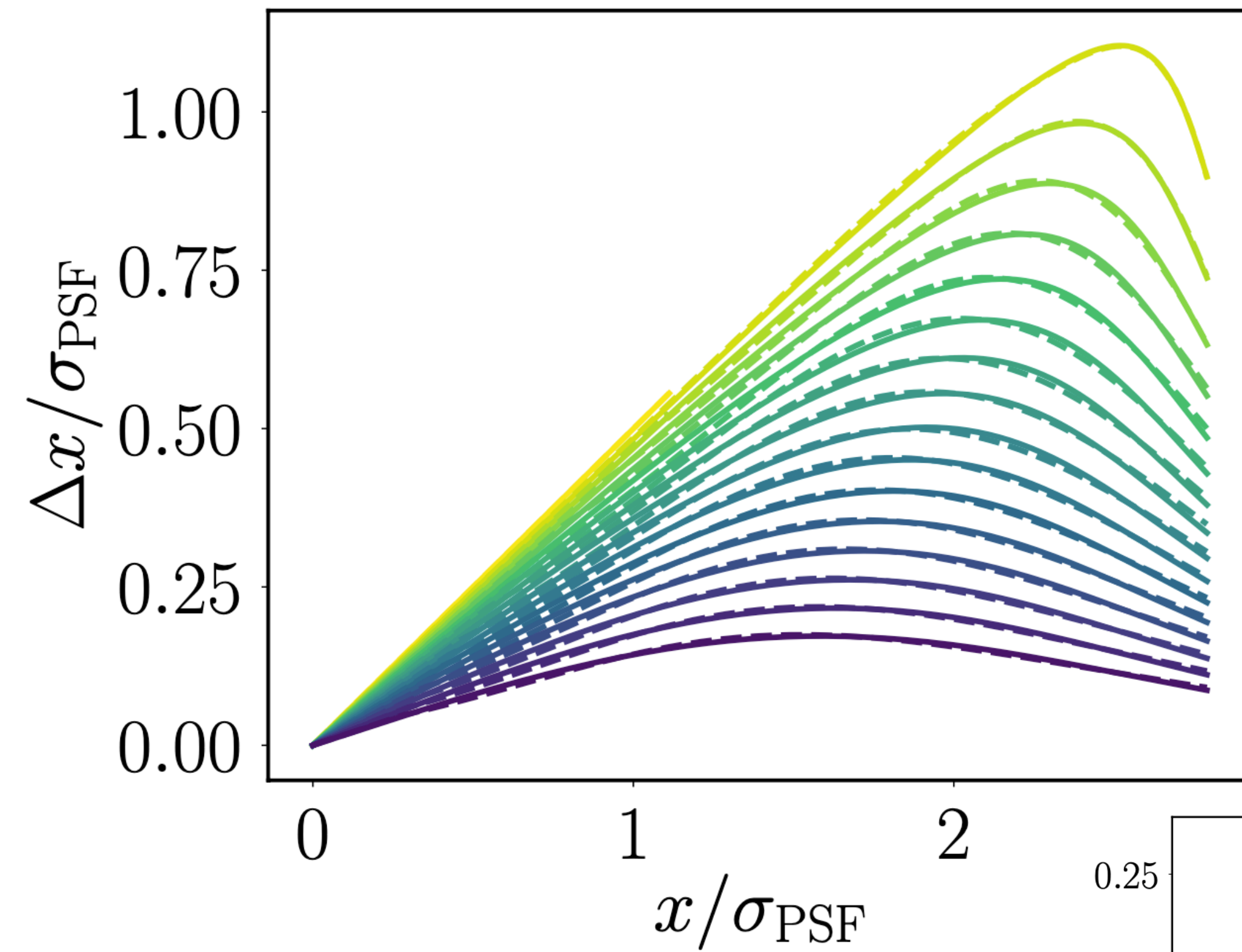
<https://github.com/Onoddil/macauff>



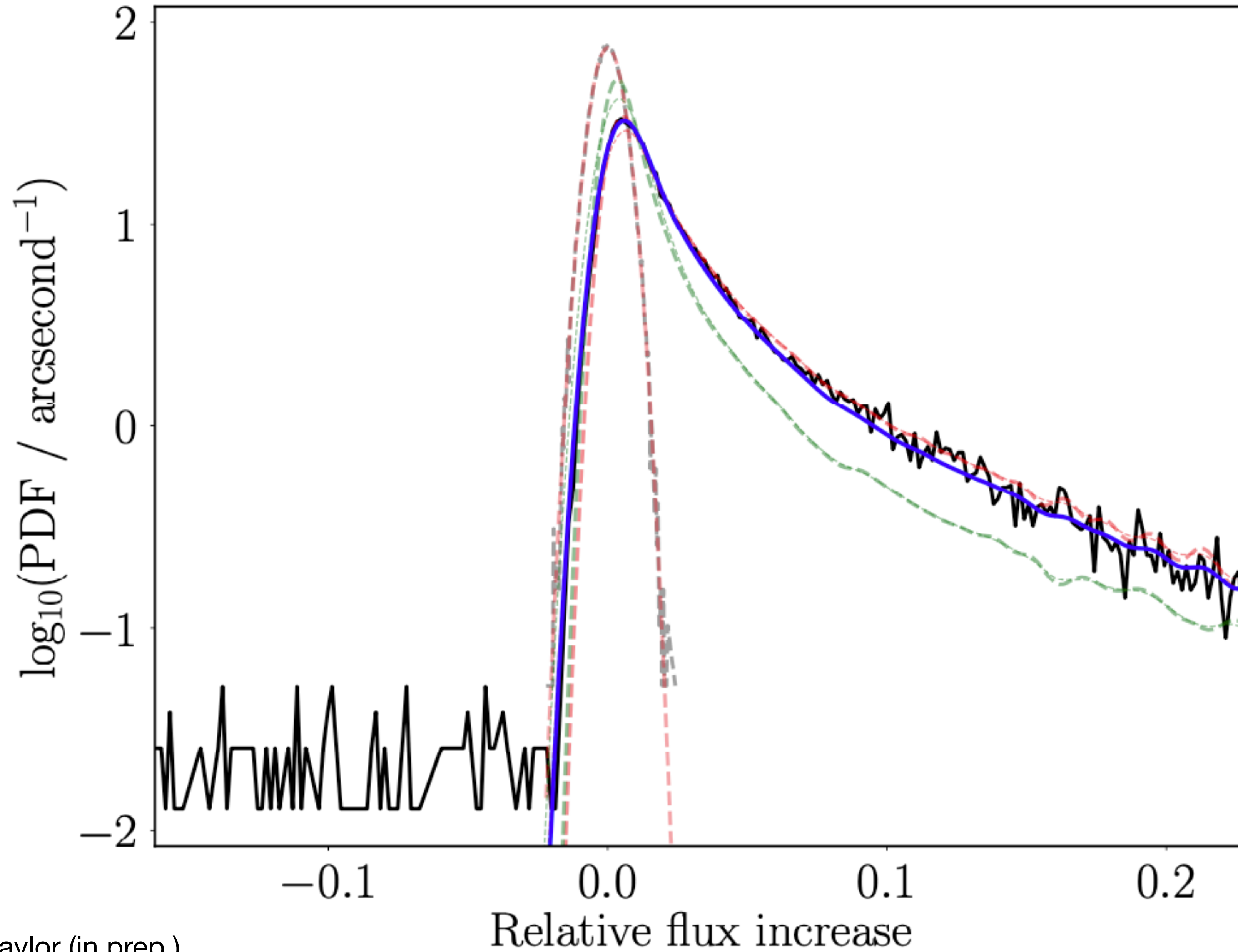
(Points if you know your tartans!)

Tom J Wilson @onoddil

Probing the Faintest Sources

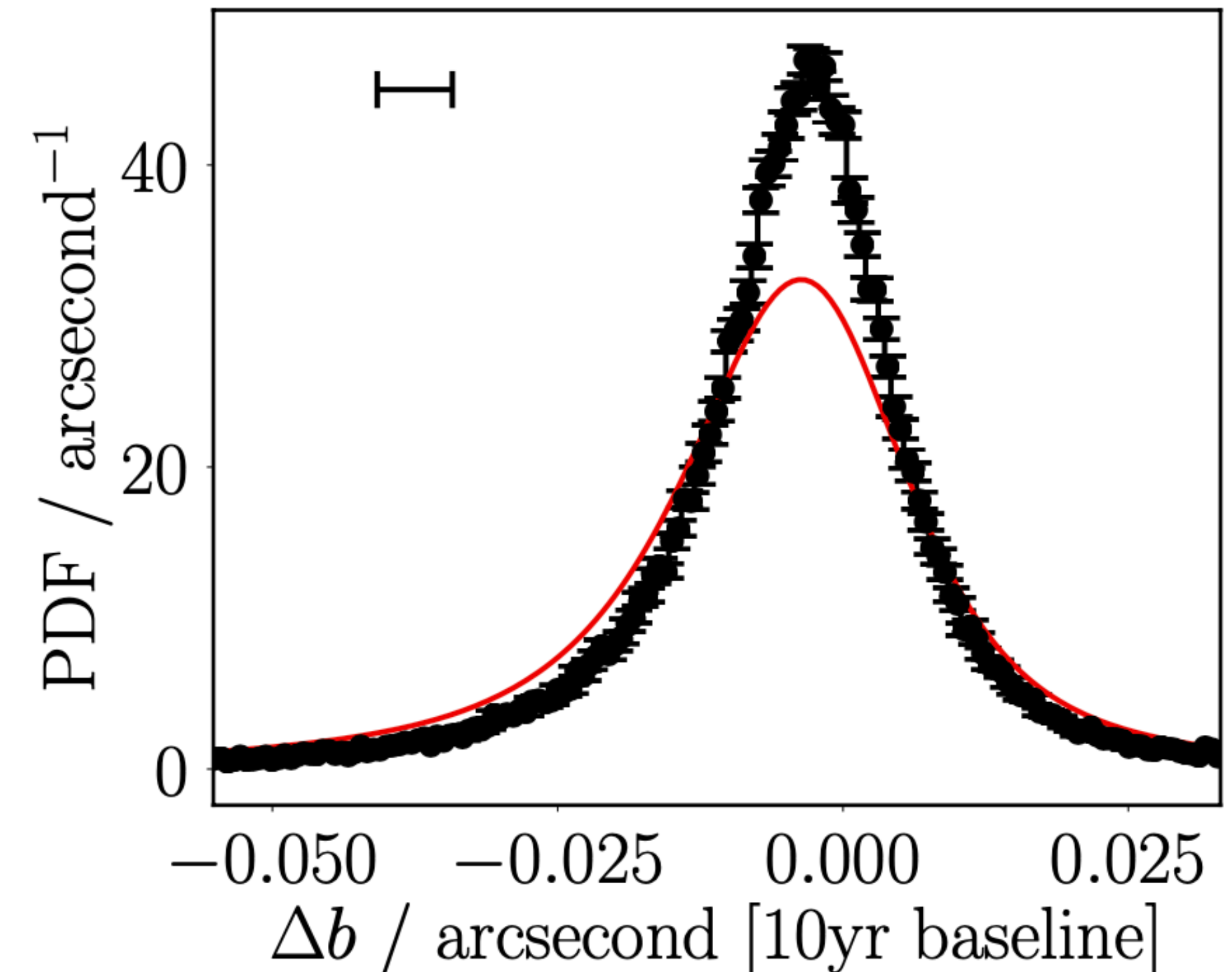
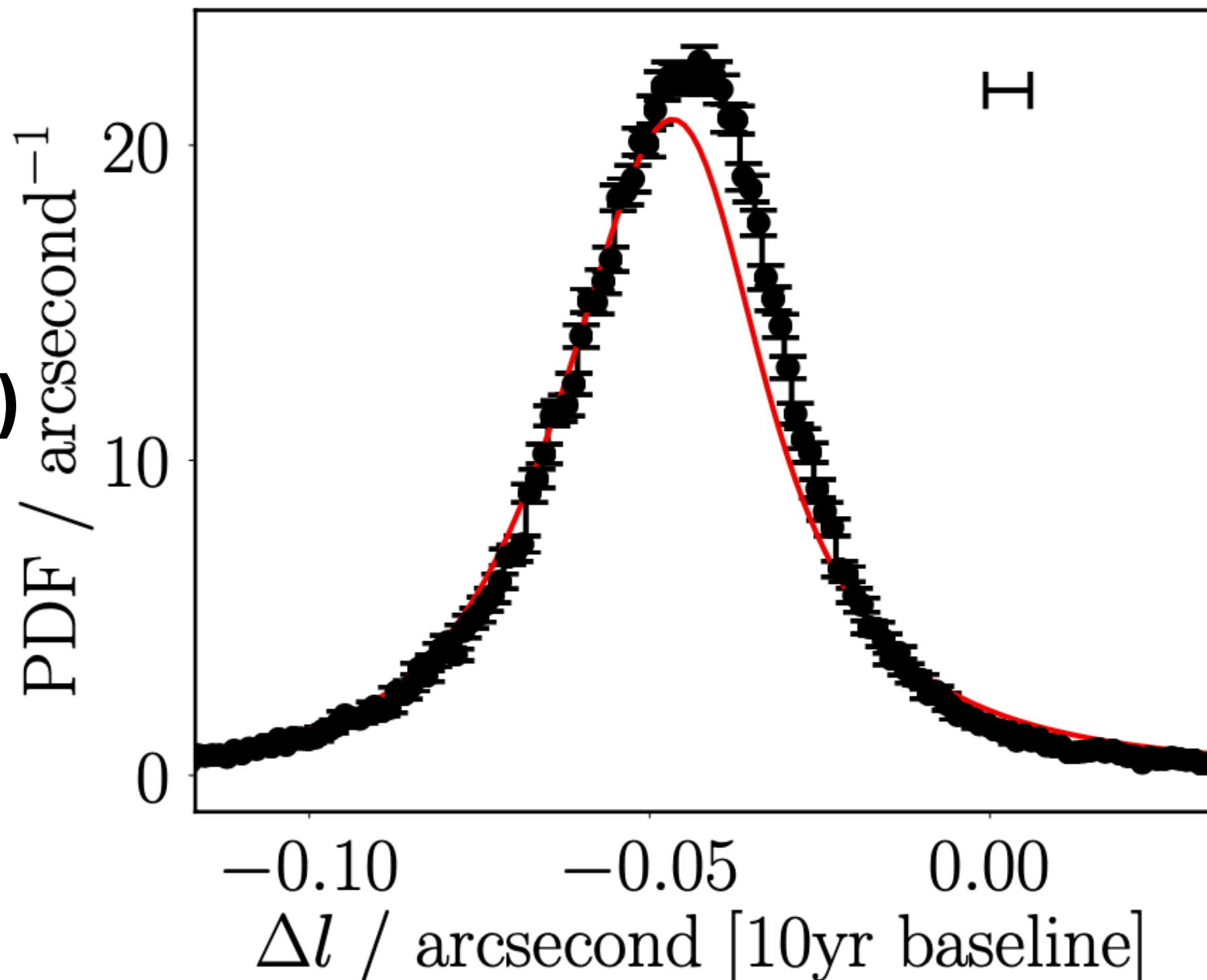
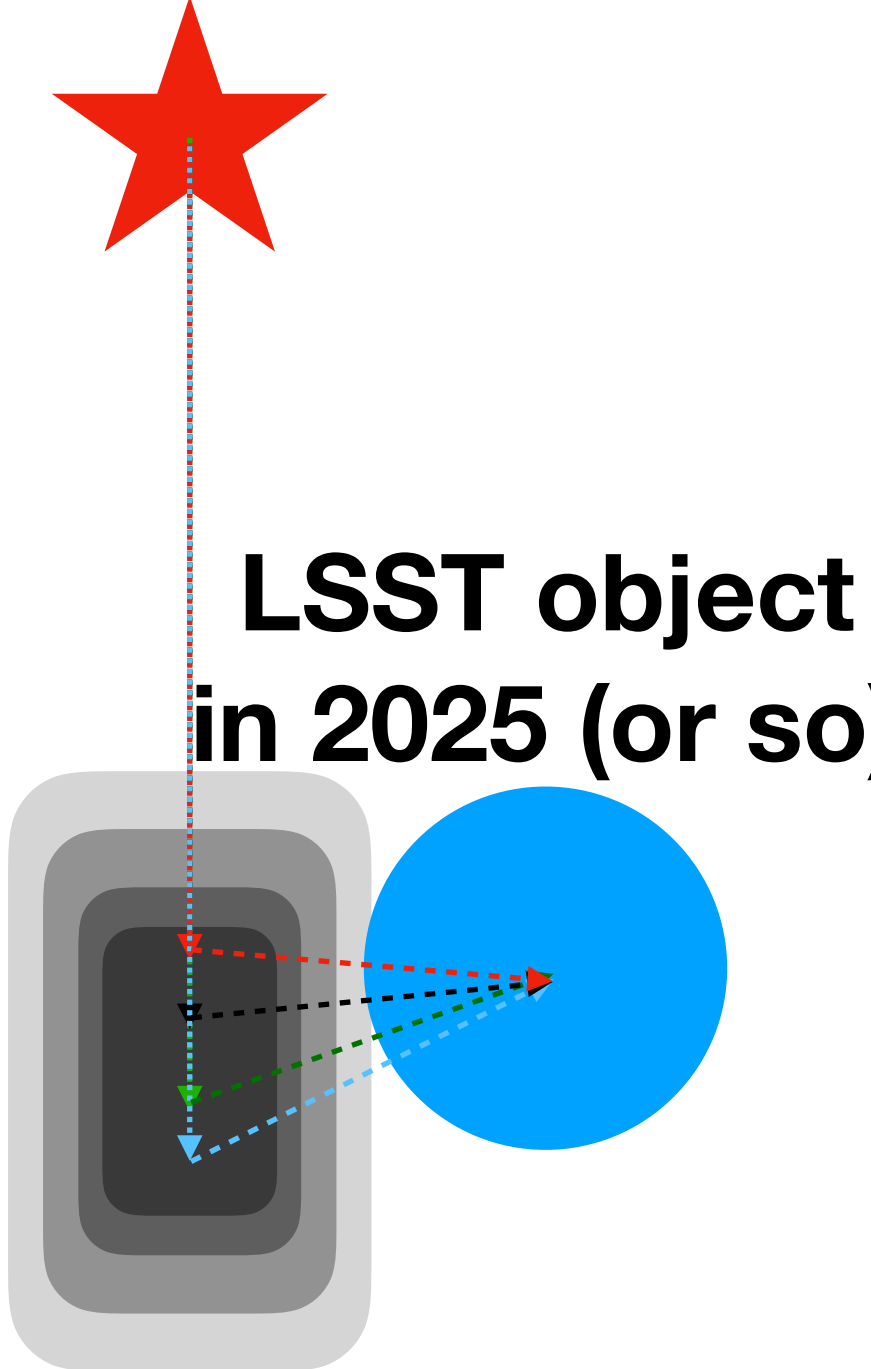


Photometric Contamination Function



Including Unknown Proper Motions

e.g. *WISE* object in 2010



Because this function works in *separation*, rather than pure *position*, space, we apply the distribution after calculating G .

$$G' = G * h'_{pm} \quad G = h_{\gamma} * h_{\phi}$$

$$h_{\gamma} = h_{\gamma,centroiding} * h_{\gamma,perturbation} * \dots$$

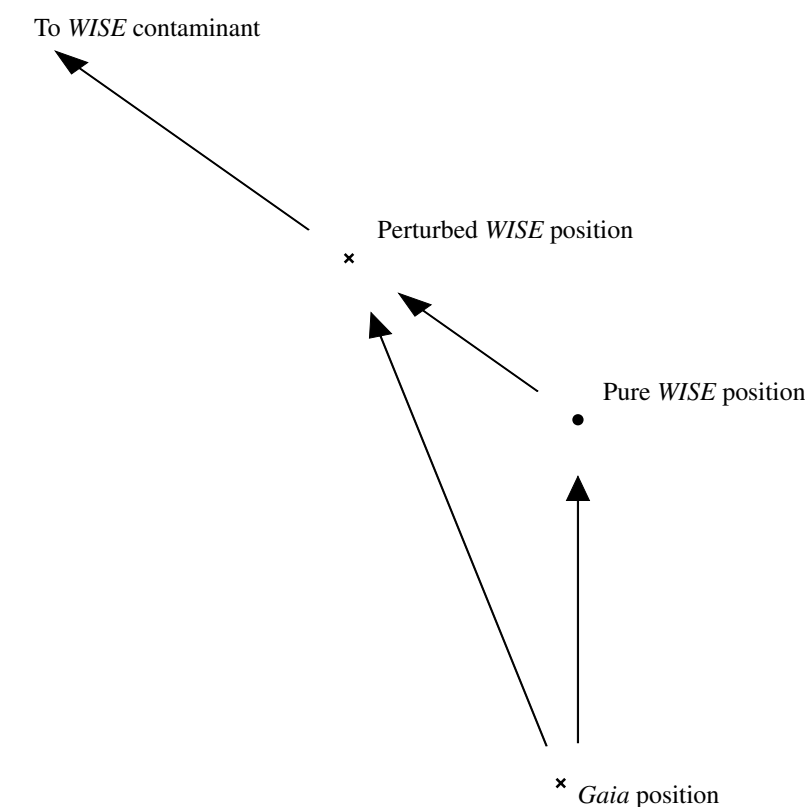
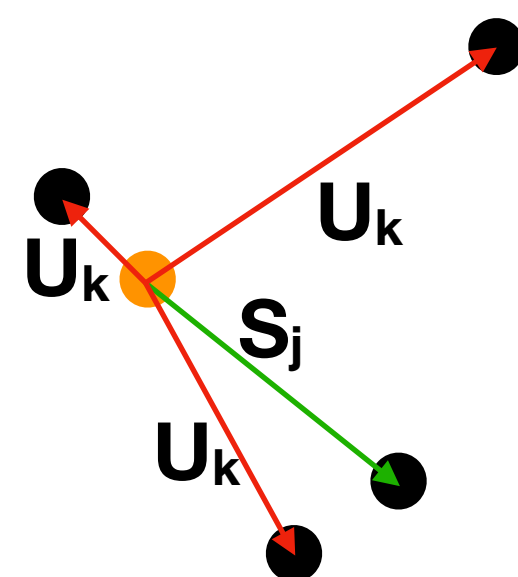
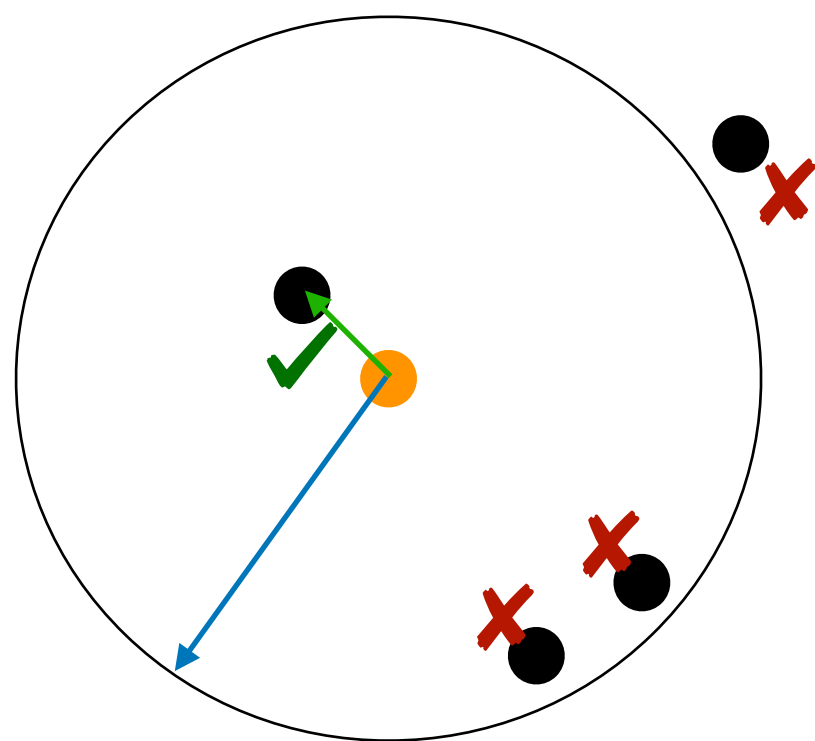
What does this mean for you?

The “busy” astronomer: uses a quick and simple 2” match -> Too many matches

The “Bayesian” astronomer: uses astrometric centroid uncertainty to reduce match radius -> Too few matches

The “careful” astronomer: includes perturbation from blended objects in the AUF -> Correct number of matches

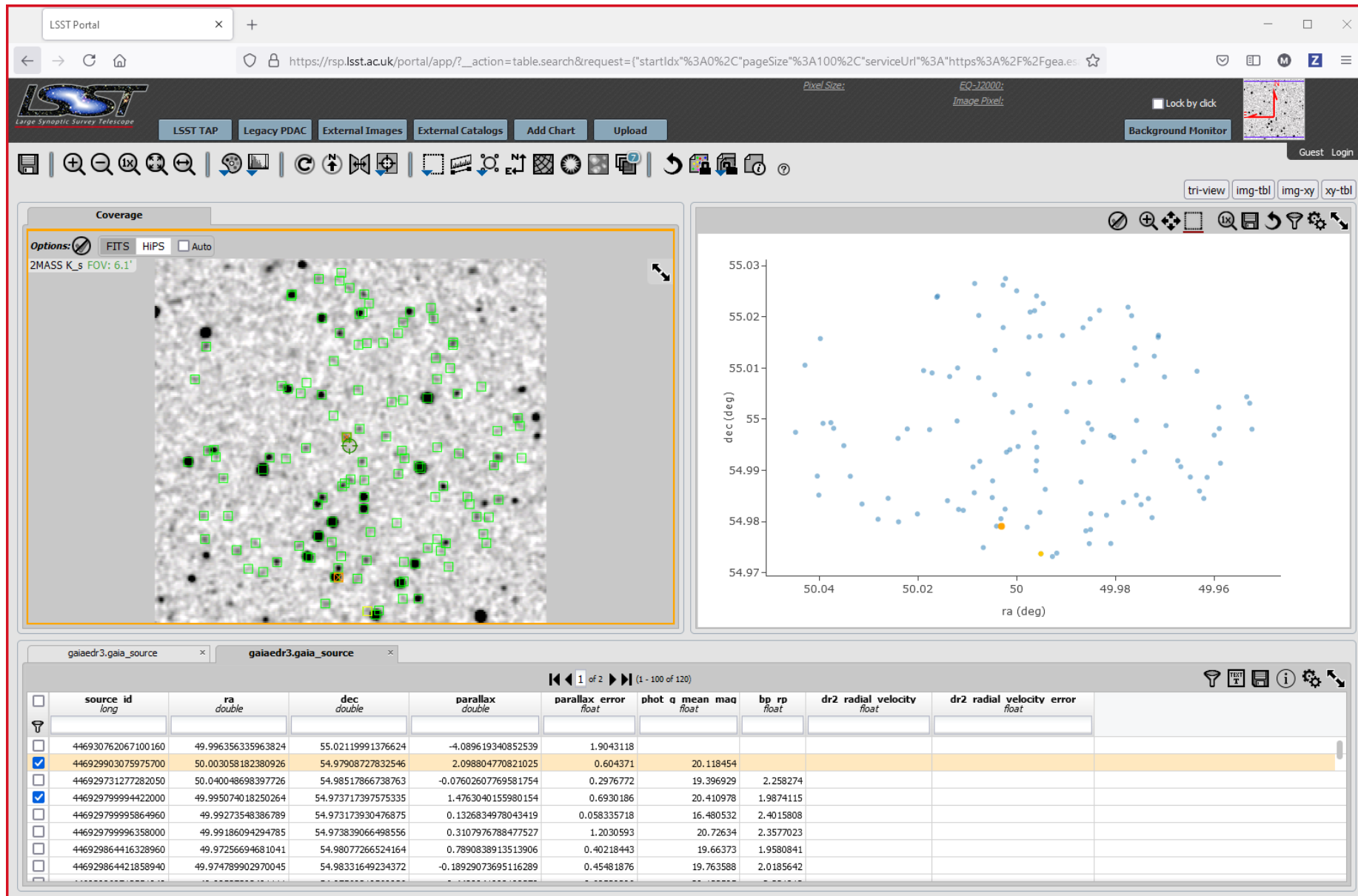
The “smart” astronomer: uses our cross-matches to get the correct number of matches *and* information on how much flux contamination is affecting their object!



you downloading your favourite cross-matches, probably

How To Use Our Cross-Matches

(Or, how this impacts you on a day-to-day basis)



Three tables per cross-match: merged catalogue dataset, and 2x non-match dataset (one per catalogue)

Example columns:

- Designations of the two sources (e.g., WISE J... and *Gaia* EDR3...)
- RA and Dec (or Galactic l/b) of the two sources
- Magnitudes (corrected for necessary effects, such as e.g. *Gaia*) in all bandpasses for both objects
- Match probability — probability of the most likely permutation (see equation 26 of Wilson & Naylor 2018a)
- Eta - Photometric likelihood ratio (counterpart vs non-match probability, just for brightnesses; see eq37 of WN18a)
- Xi - Astrometric likelihood ratio (just position match/non-match comparison; see eq38 of WN18a)
- Average contamination - simulated mean (percentile) brightening of the two sources, based on number density of catalogue
- Probability of sources having blended contaminant above e.g. 1% relative flux

We will provide a two match runs per catalogue pair match: one with, and one without, the photometry considered, to allow for the recovery of sources with “weird” colours but otherwise agreeable astrometry

Conclusions

- **Blended star contamination causes positional shifts, now modelled robustly for the first time in the AUF**
- **Symmetric data-driven photometric likelihood now possible**
- **WISE objects are up to 30% flux contaminated**
- **LSST will suffer of order 10% flux contamination in the future**
 - **Important for extinction/distance; “1% photometry”?**
 - **Modelling of statistical flux contamination allows for the recovery of “true” fluxes**
- **LSST will suffer at least one extra source (possibly up to 10!) in each 2” matching circle**
 - **Can use photometry in catalogues to break these false match degeneracies**
- **Can include unknown proper motions easily within AUF match framework**
- **High dynamic range matches must account for differential crowding matching to ancillary or historic data**
- **Upcoming LSST:UK cross-match service macauff – let me know your thoughts/needs/hopes/dreams**

Wilson & Naylor, 2017, MNRAS, 468, 2517

Wilson & Naylor, 2018a, MNRAS, 473, 5570

Wilson & Naylor, 2018b, MNRAS, 481, 2148

Wilson (2022, RNAAS)

Wilson (2022, RASTI, in review)

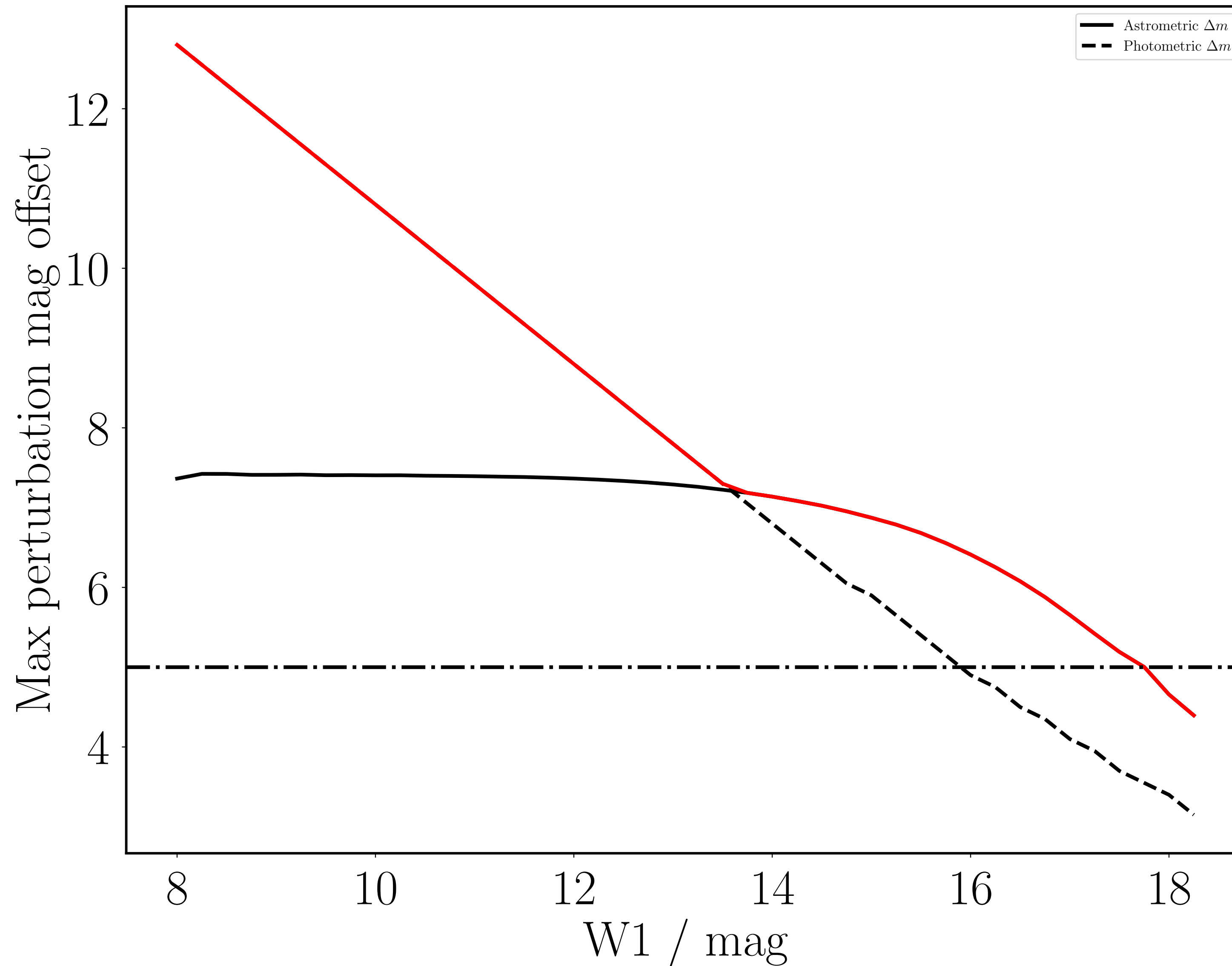
<https://github.com/Onoddil/macauff>



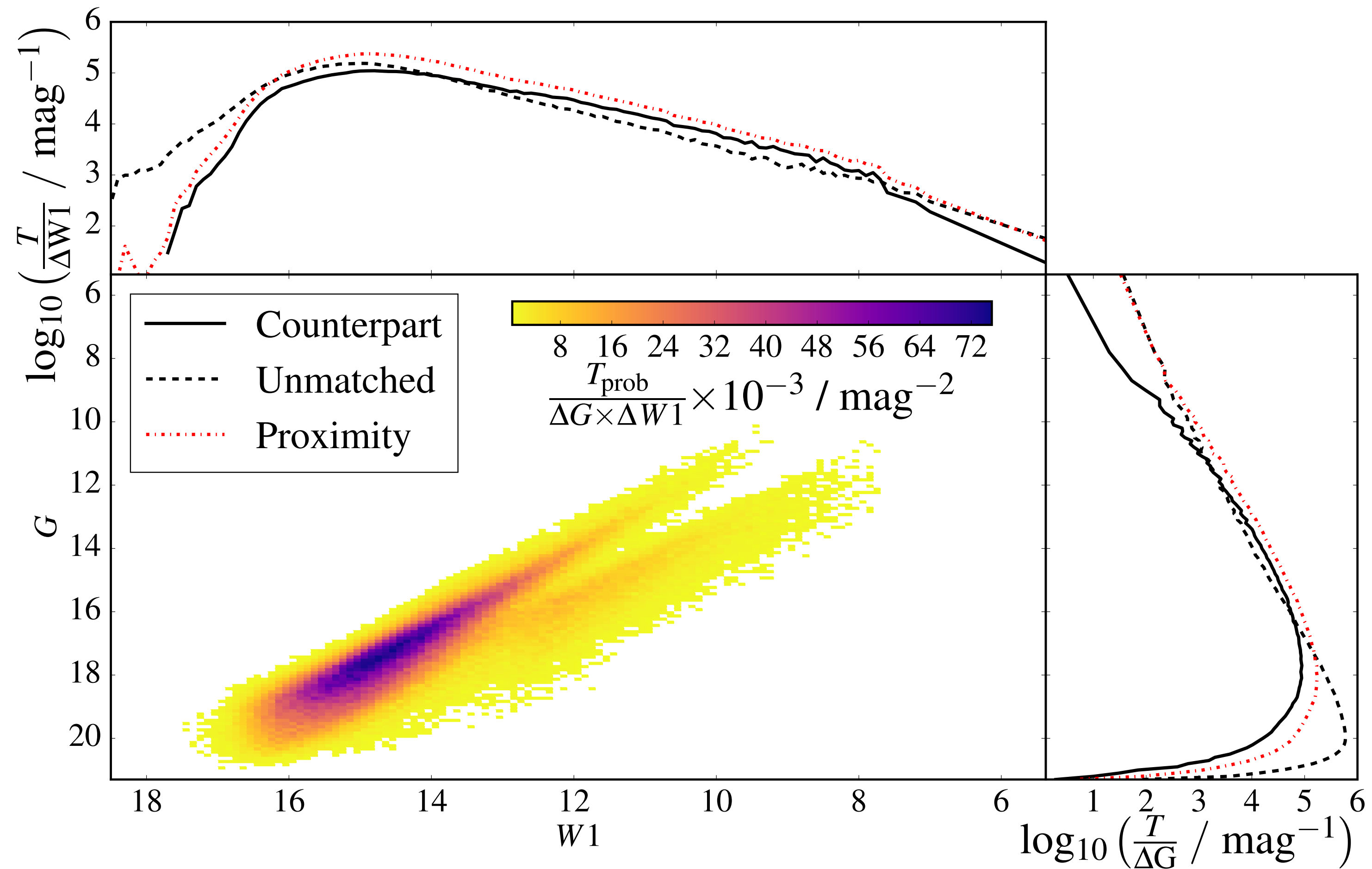
Carnegie EPL, 27/May/22

Tom J Wilson @onoddil

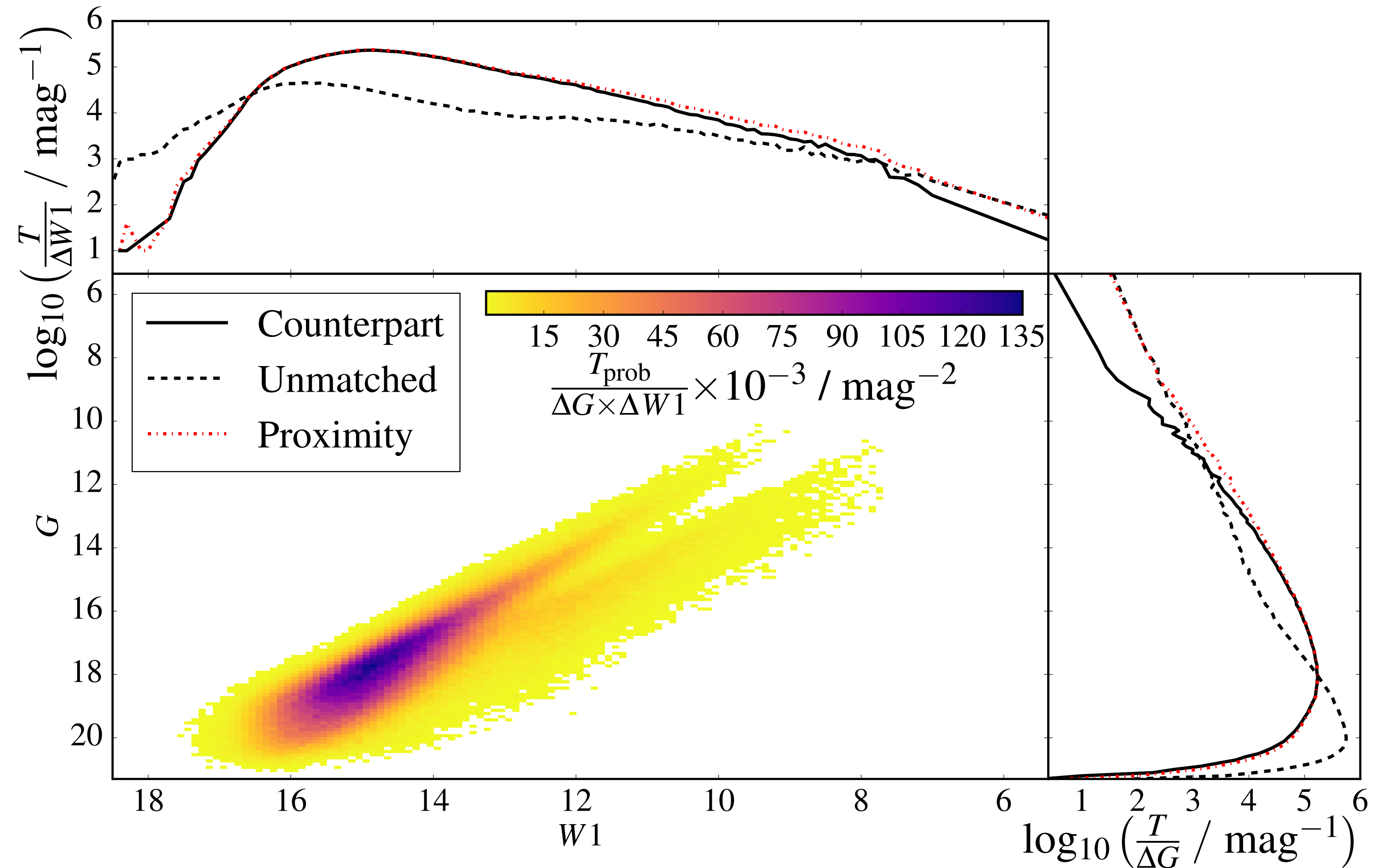
The Astrometric Uncertainty Function and LSST: A Crisis of Completeness Limit



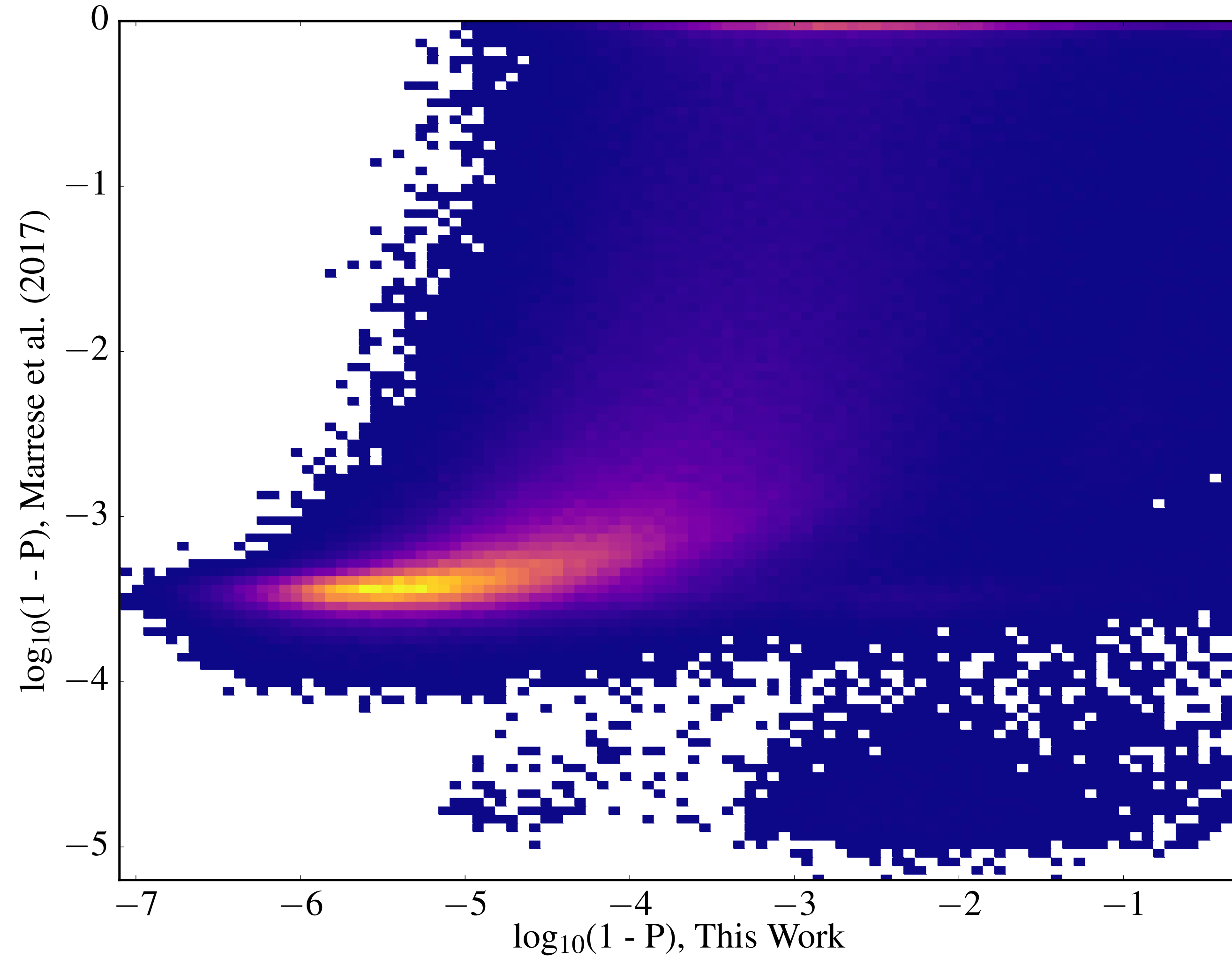
Contamination Effects: Gaia-WISE Gaussian Matches



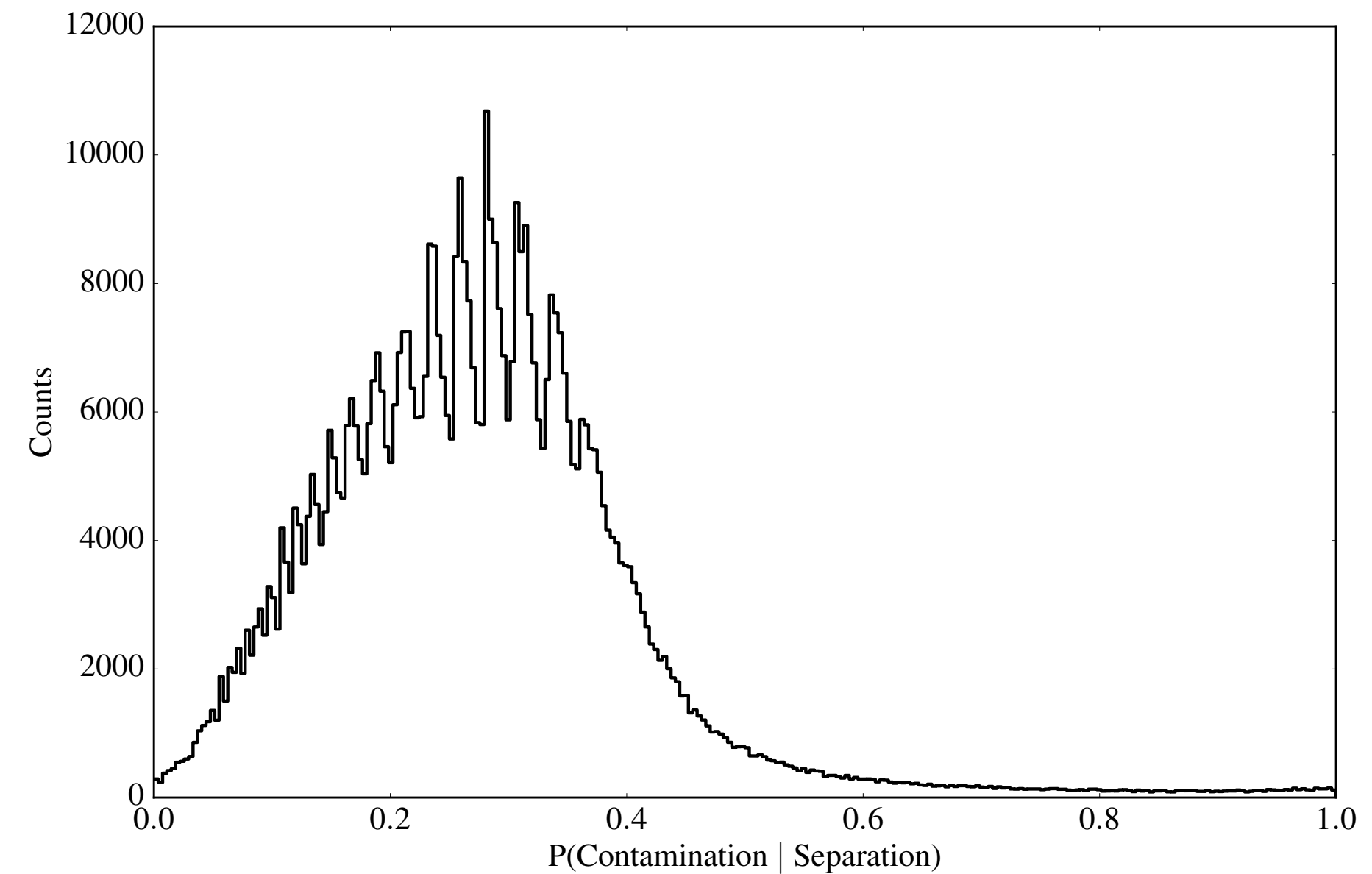
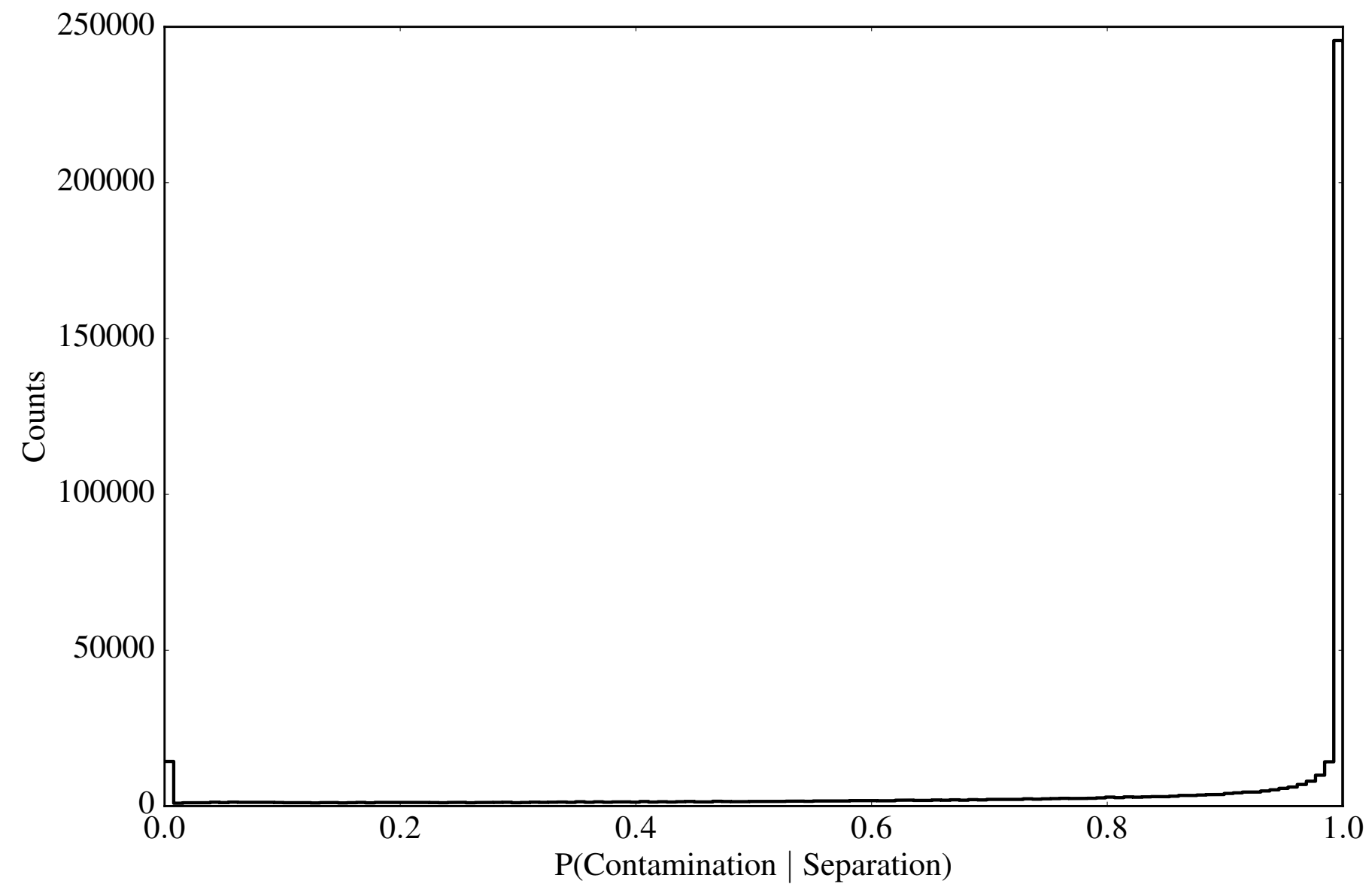
Contamination Effects: Gaia-WISE Empirical Matches



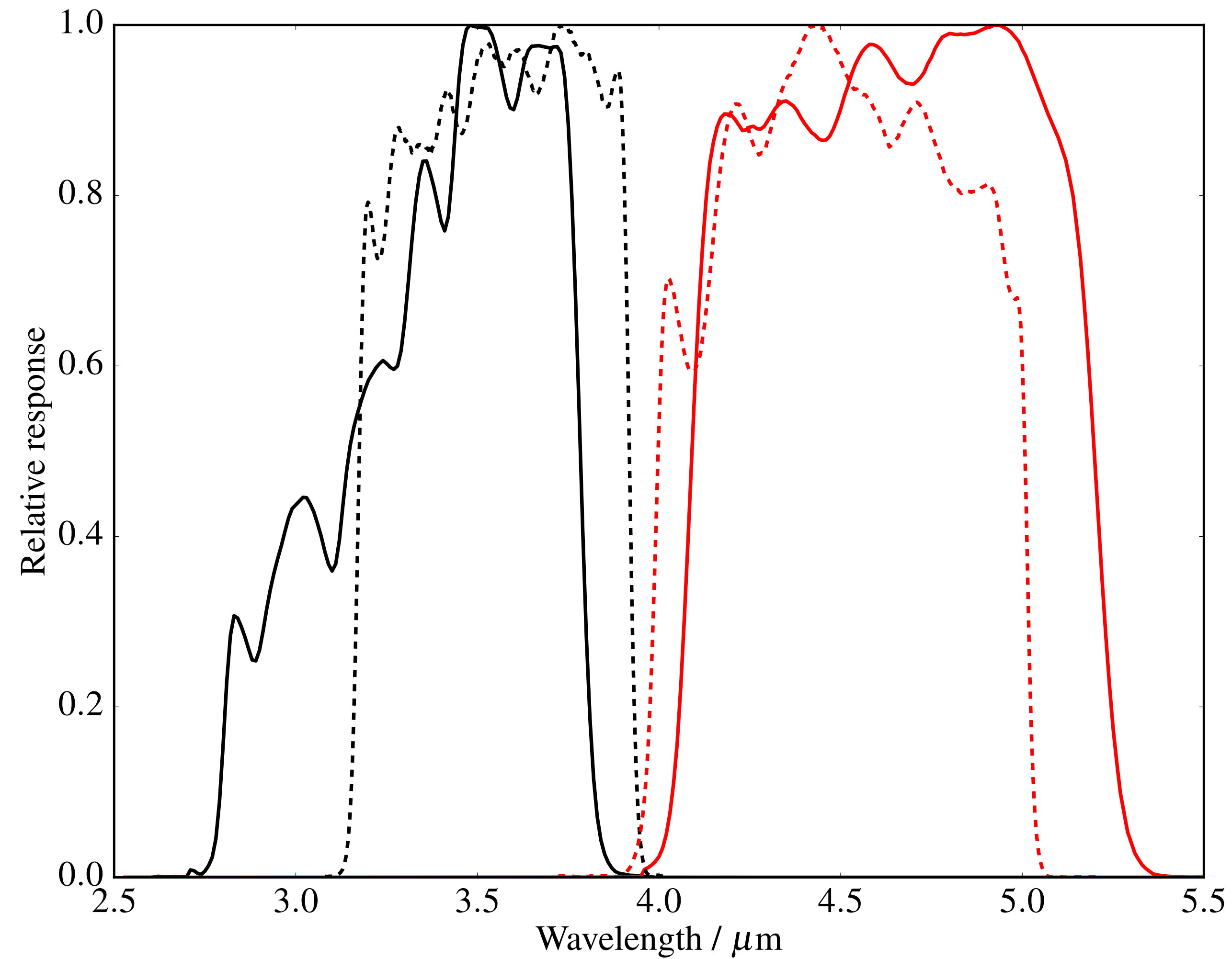
Contamination Effects: Gaia Lost Matches



Photometric Contamination: WISE/Spitzer Contamination %



Contamination Effects: Wavelength Coverage



The Astrometric Uncertainty Function: Analytical perturbations

