Solving the Catalogue Cross-Match Problem in the Era of LSST

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Science and Technology Facilities Council

University of Exeter

LSST:UK Consortium

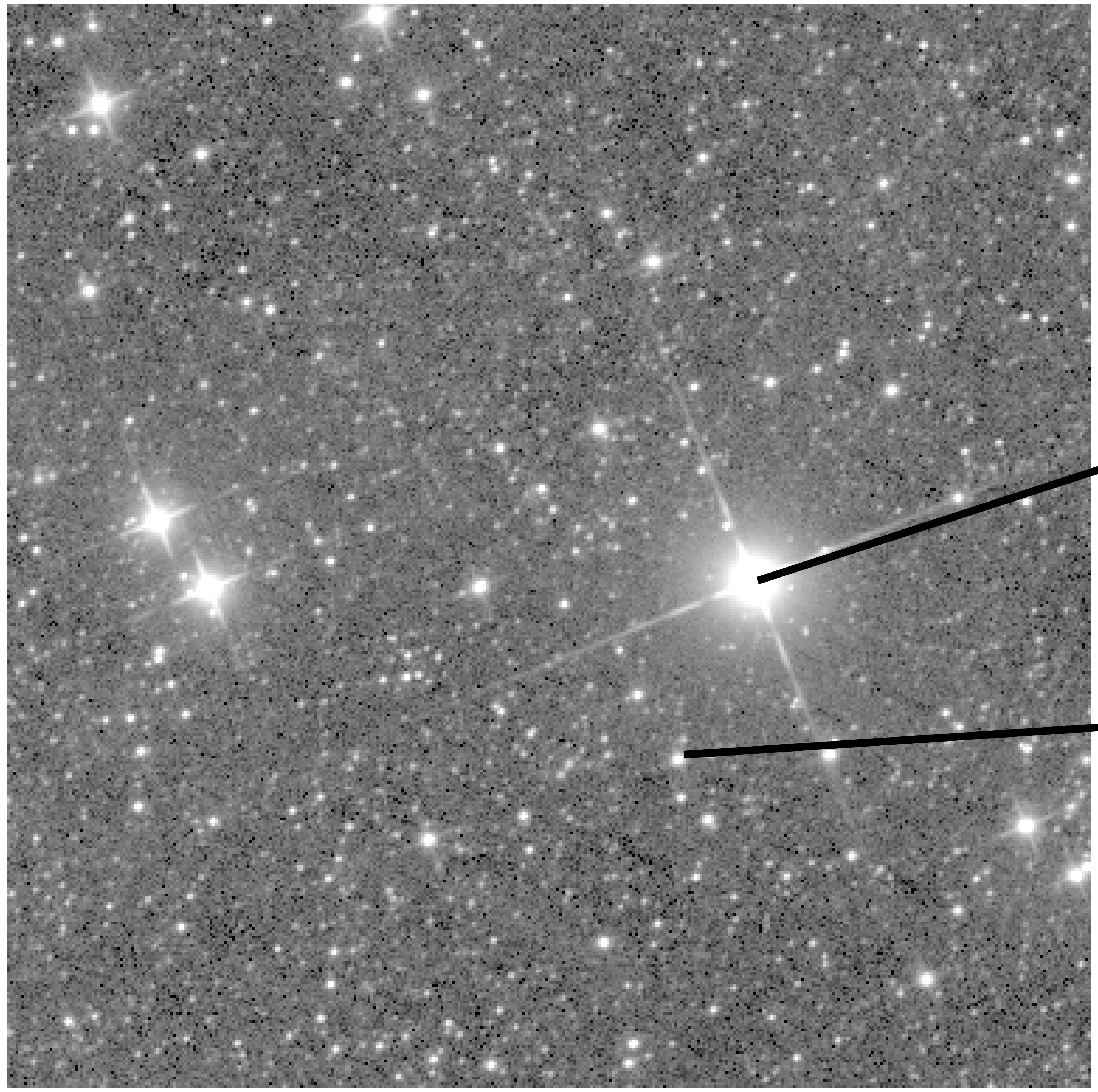
VERA C. RUBIN OBSERVATORY

Macauff

@Onoddil @pm.me .github.io

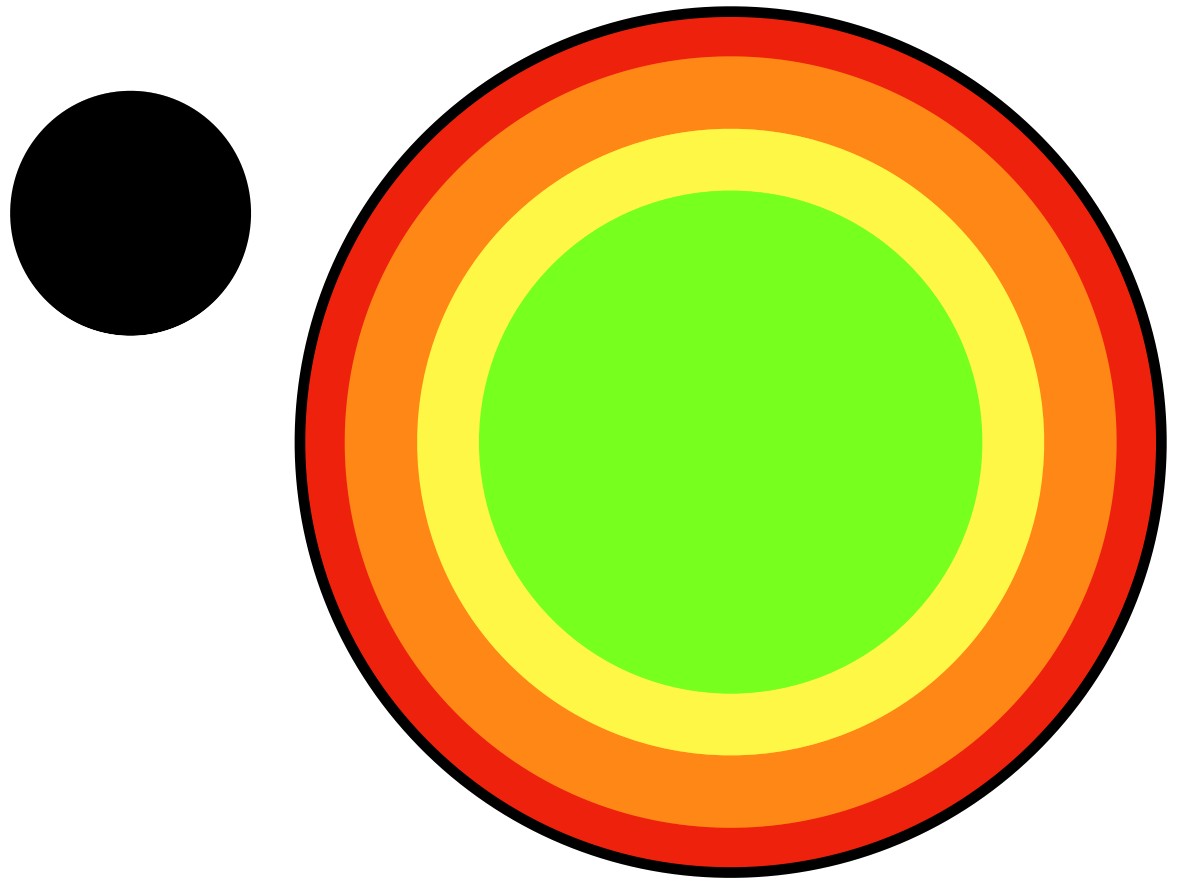
What's In A Photometric Catalogue?

(Ironically, it's half astrometry!)



WISE - Wright et al. (2010)

Source ID	Position (deg)	Uncertainty (arcsecond)	Brightness (mag)	Uncertainty (mag)
1	218.4763	0.073	14.94	0.04
2	218.3951	0.217	20.32	0.15



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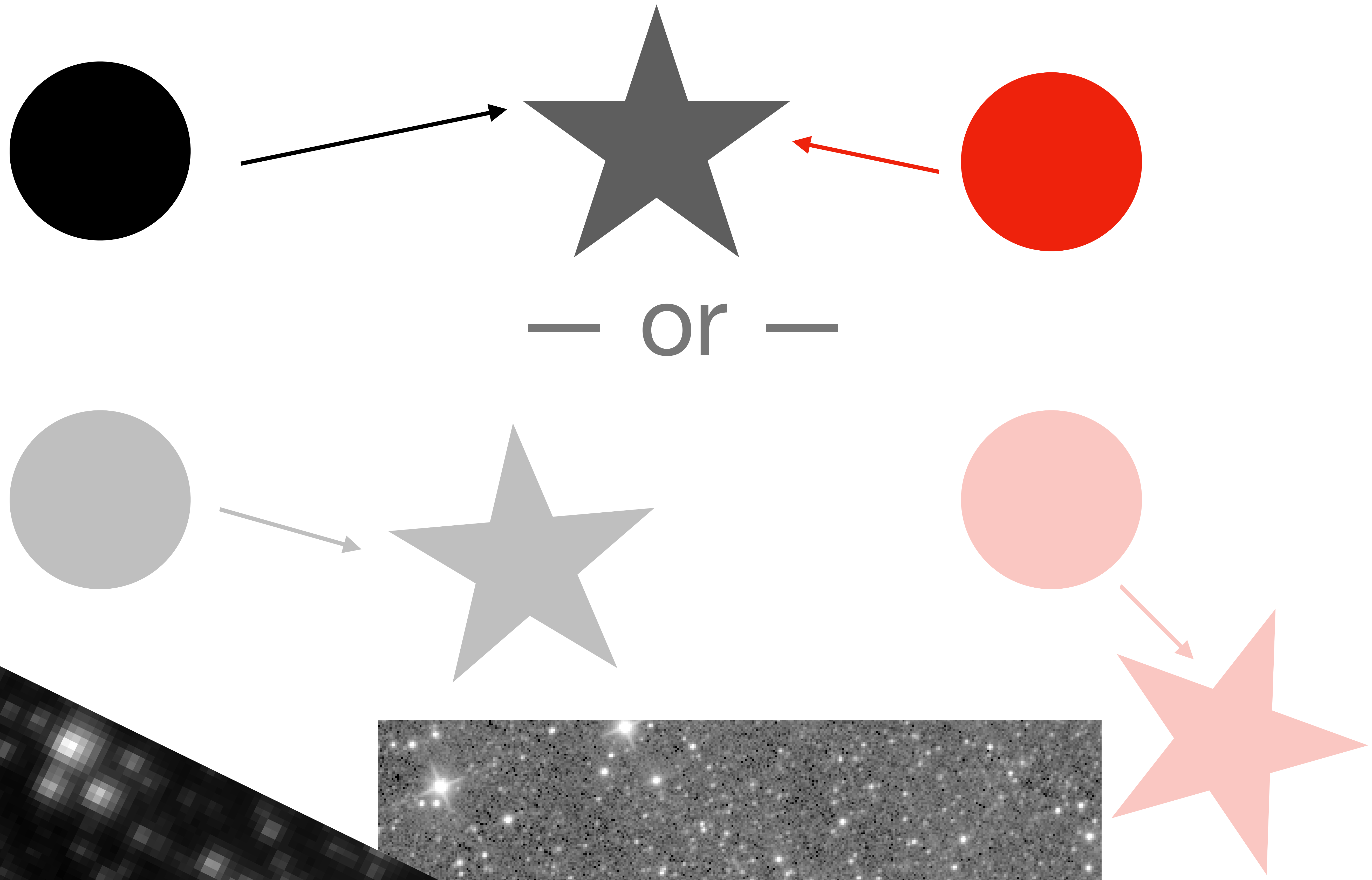
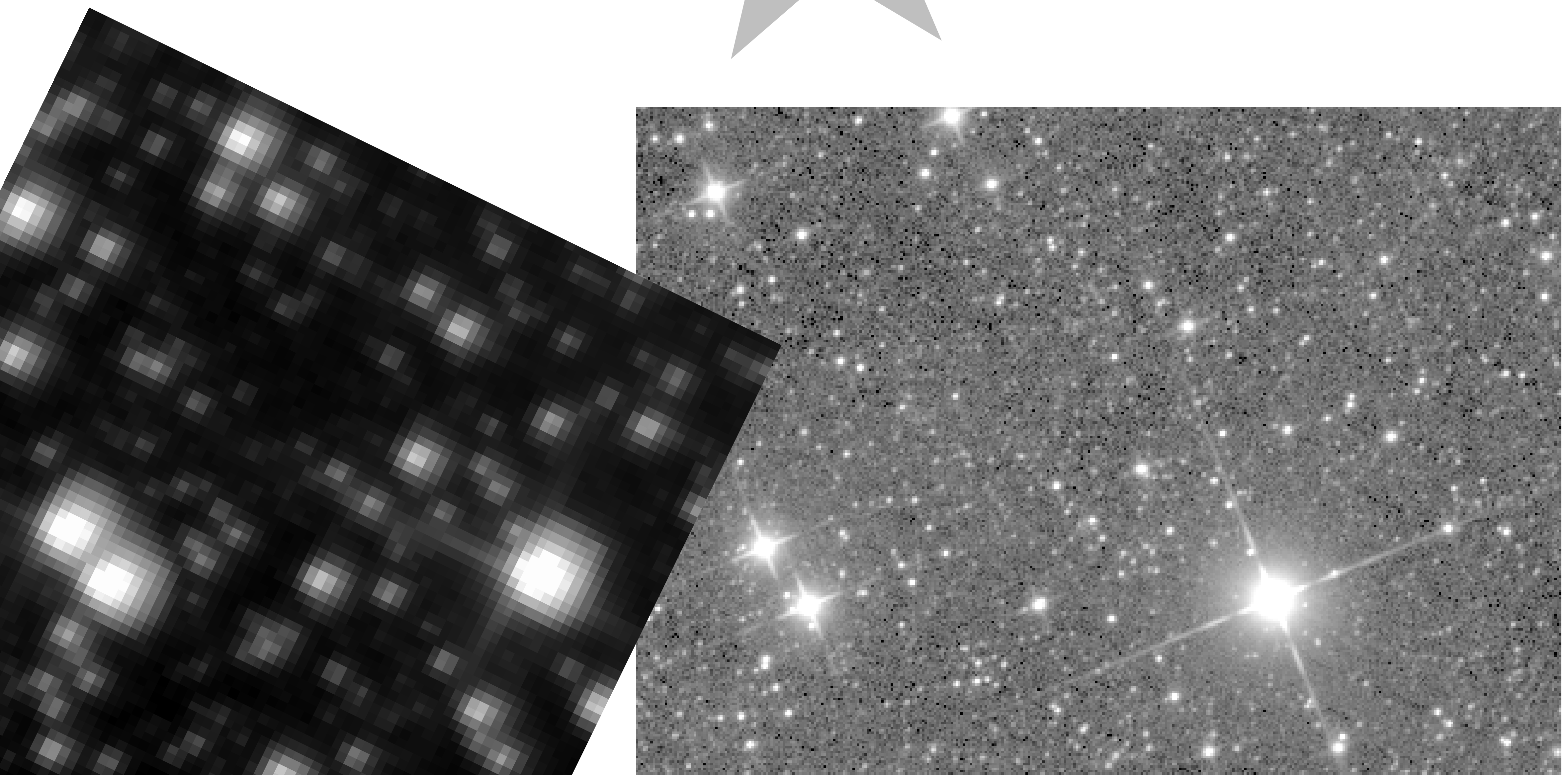
Cross-Matching and Counterpart Assignment



ID A	ID 1	A magnitude	Magnitude 1
A J…	CAT1 1	14.94	17.53
…	…	…	…

ID A	ID 1	A magnitude	Magnitude 1
A J…	NULL	14.94	NULL
NULL	CAT1 1	NULL	17.53
…	…	…	…



WISE - Wright et al. (2010)
TESS - Ricker et al. (2015)

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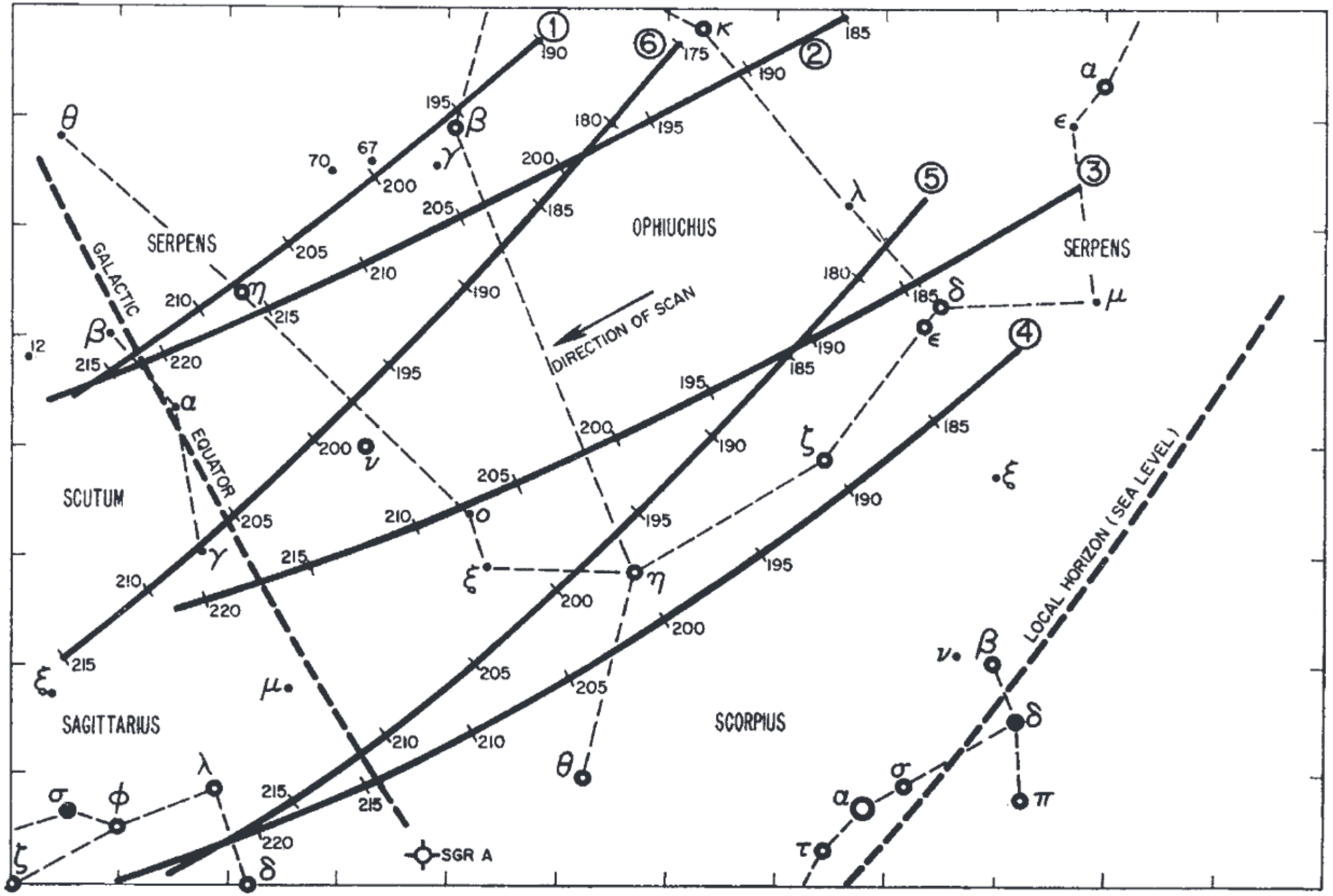
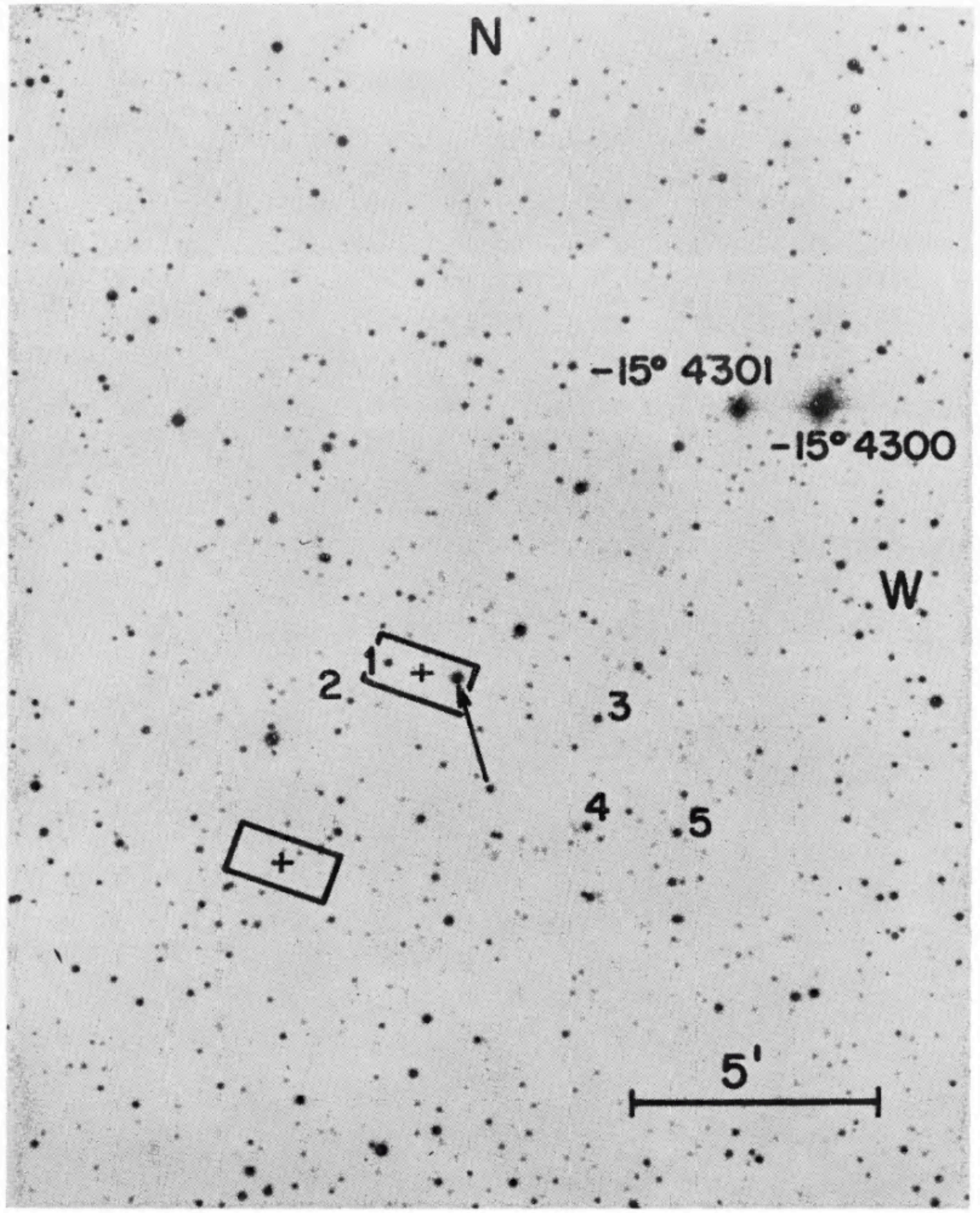
Technology Abounds

- **Ancient lists of stars (Ptolemy, 150; Brahe, 1598)**
- **Galileo invents the telescope (1610)**
- **Greenwich Observatory catalogues (e.g. Bradley, 1798)**
- **Astrophotography invented (Bond & Whipple, 1850)**
- **Harvard Observatory surveys (8th magnitude, 1882-1886)**
- **Astrographic Chart (11th magnitude; 1887-1962)**
- **Carte Du Ciel (14th magnitude; 1880s-never finished)**
- **Invention of the CCD (Boyle & Smith, 1970)**
- **InfraRed detector invented (Forrest et al. 1985)**
- **4- and 5-m class telescopes (1970s-1980s; e.g. LAT, MMT, UKIRT, CFHT, WHT)**
- **Space Telescopes (1980s-2010s; e.g. IRAS, ISO, AKARI, *WISE*, *Spitzer*)**
- **All-sky ground-based surveys (e.g. 2MASS, 1997-2001; SDSS, 2000-; Pan-STARRS, 2010-).**

X-ray Detections: Hunting for Sco X-1



Giacconi, Gursky, & Waters (1964)



Sandage et al. (1966)

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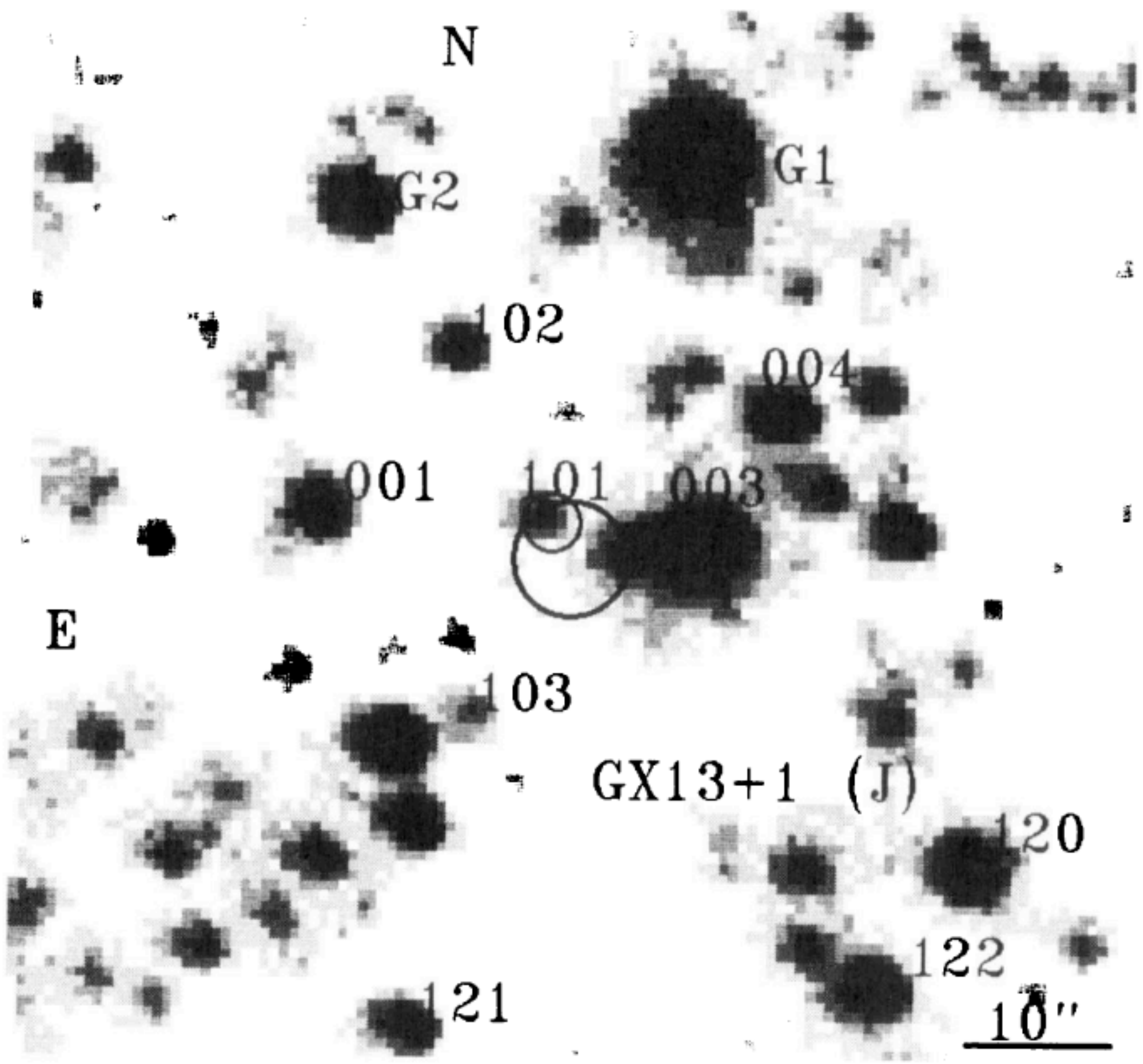
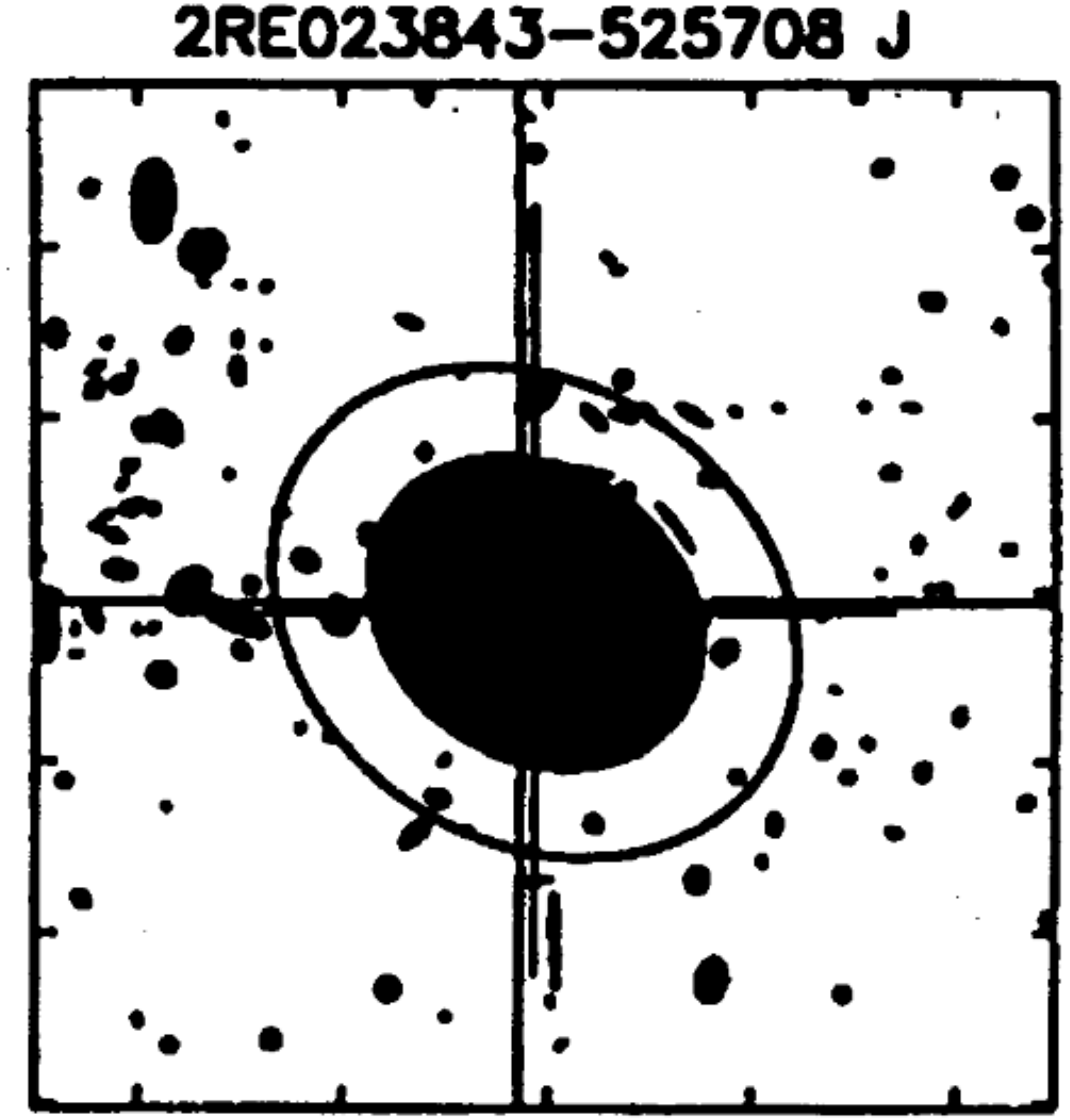
The Brightest Star in the Sky



Naylor, Charles, & Longmore (1991)



Mason et al. (1995)

"...X-ray sources are rare events; bright optical sources are also rare events, so the observation of an X-ray source and a bright optical source in the same region of the sky is considered a non-random event"

Fotopoulou et al. (2016)

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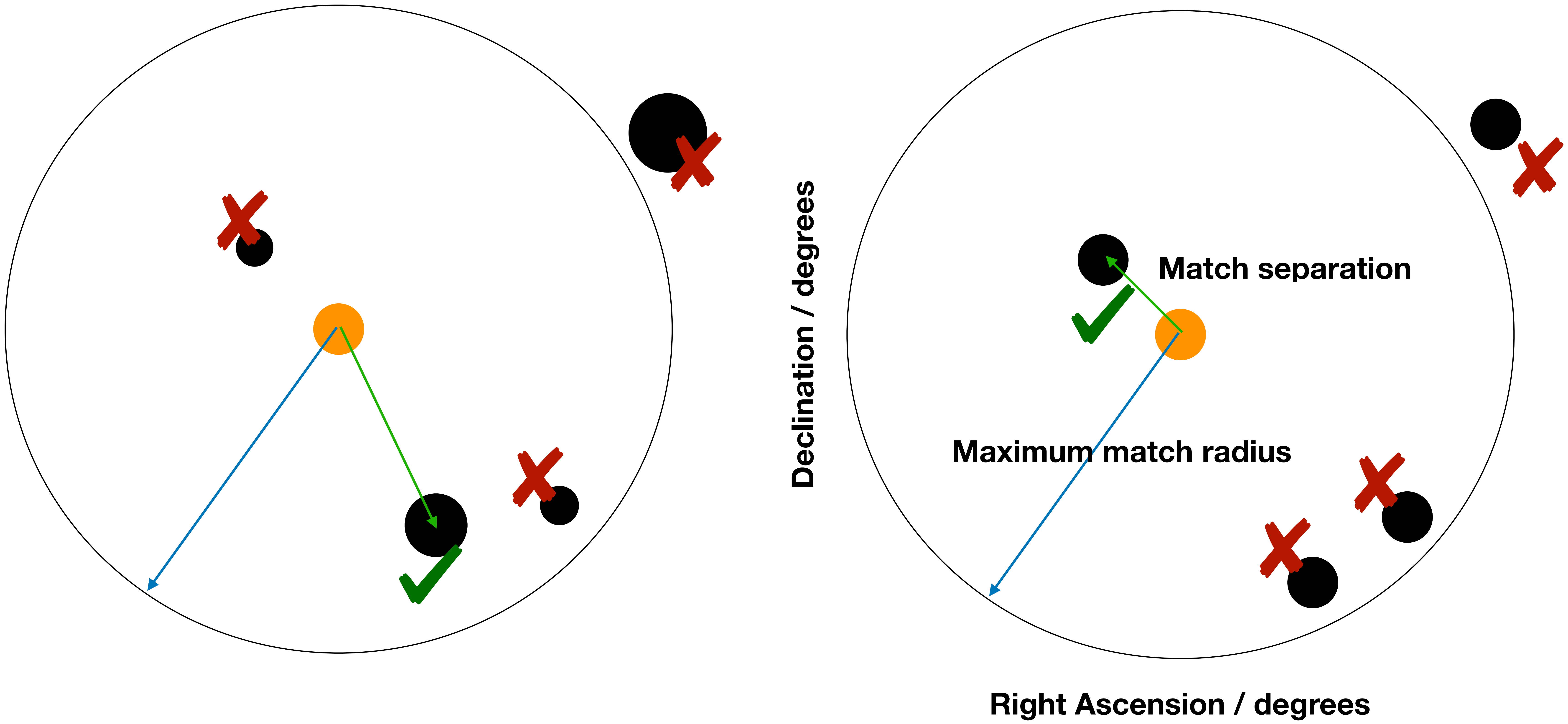
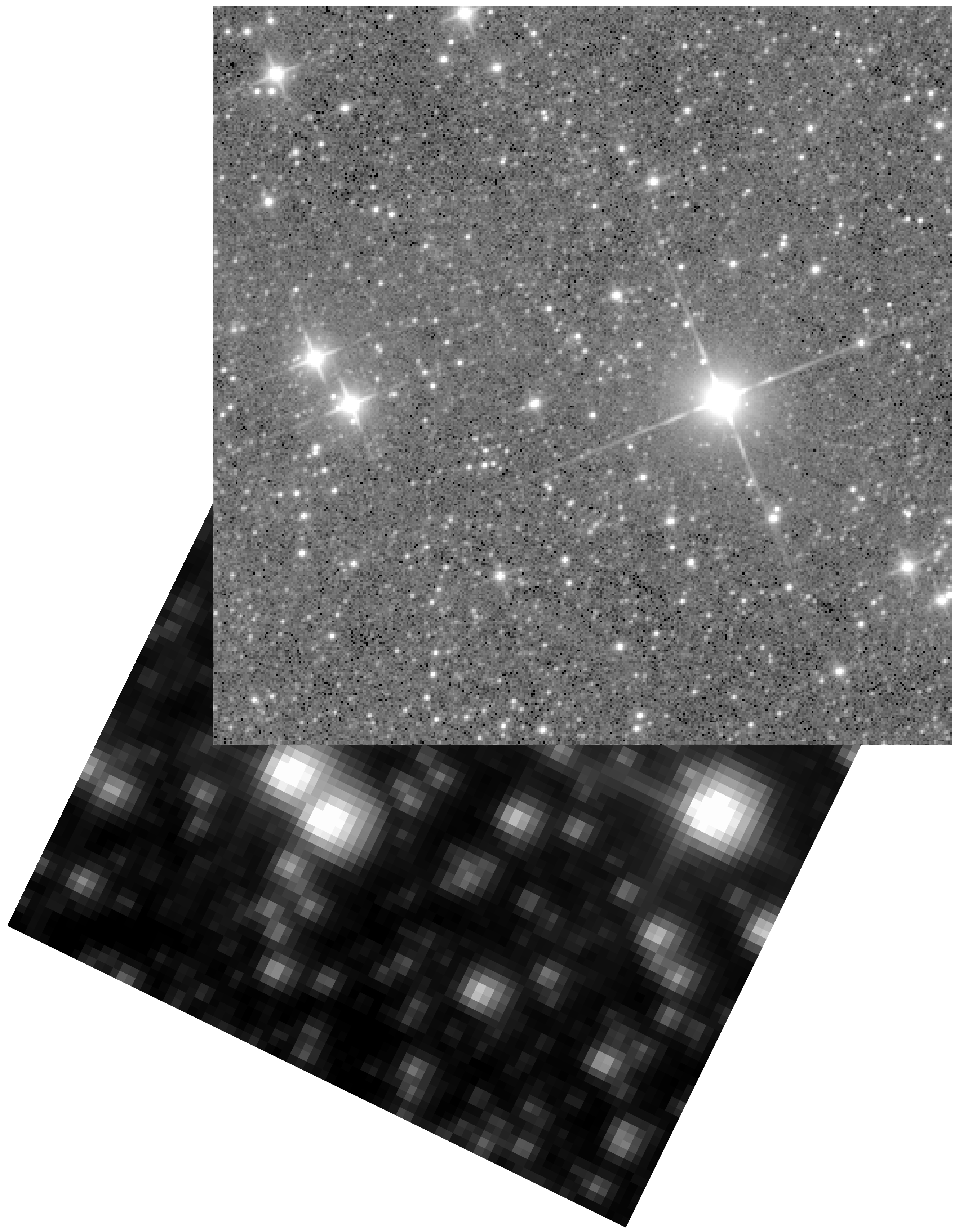
“Traditional” Cross-Matching





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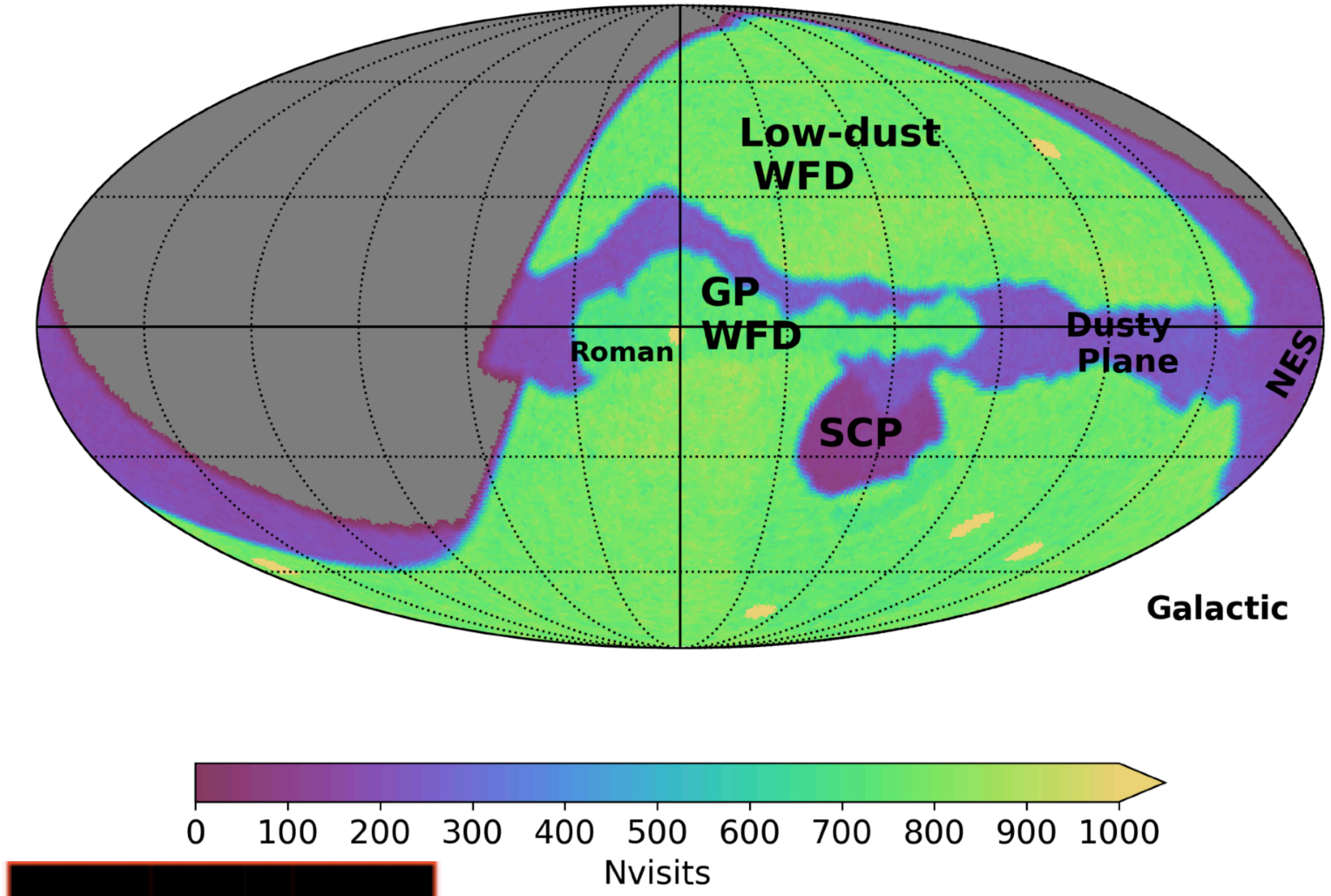
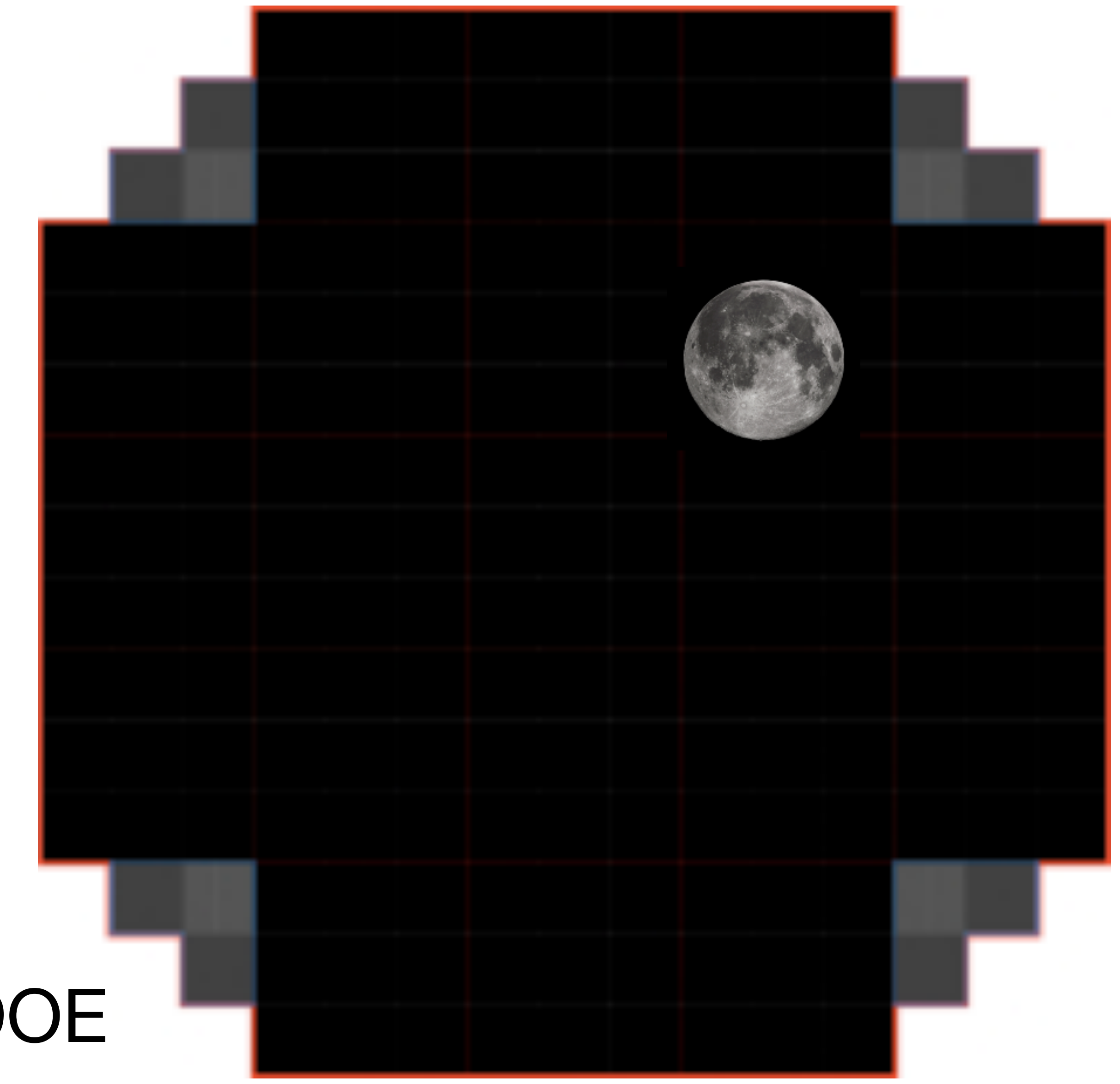
The Vera C. Rubin Observatory's LSST

NOIRLab/NSF/AURA/F. Bruno





LSST/DOE

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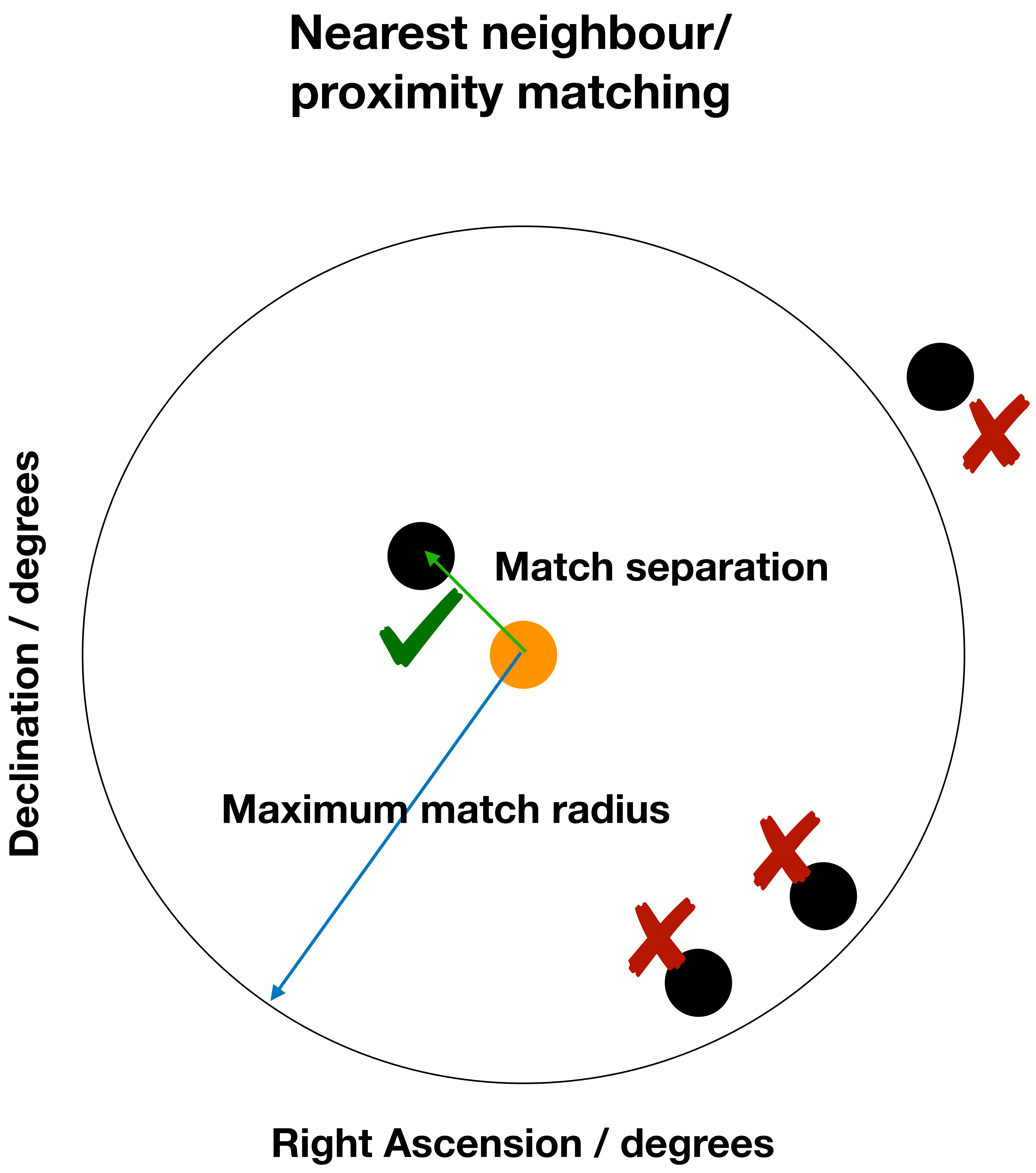
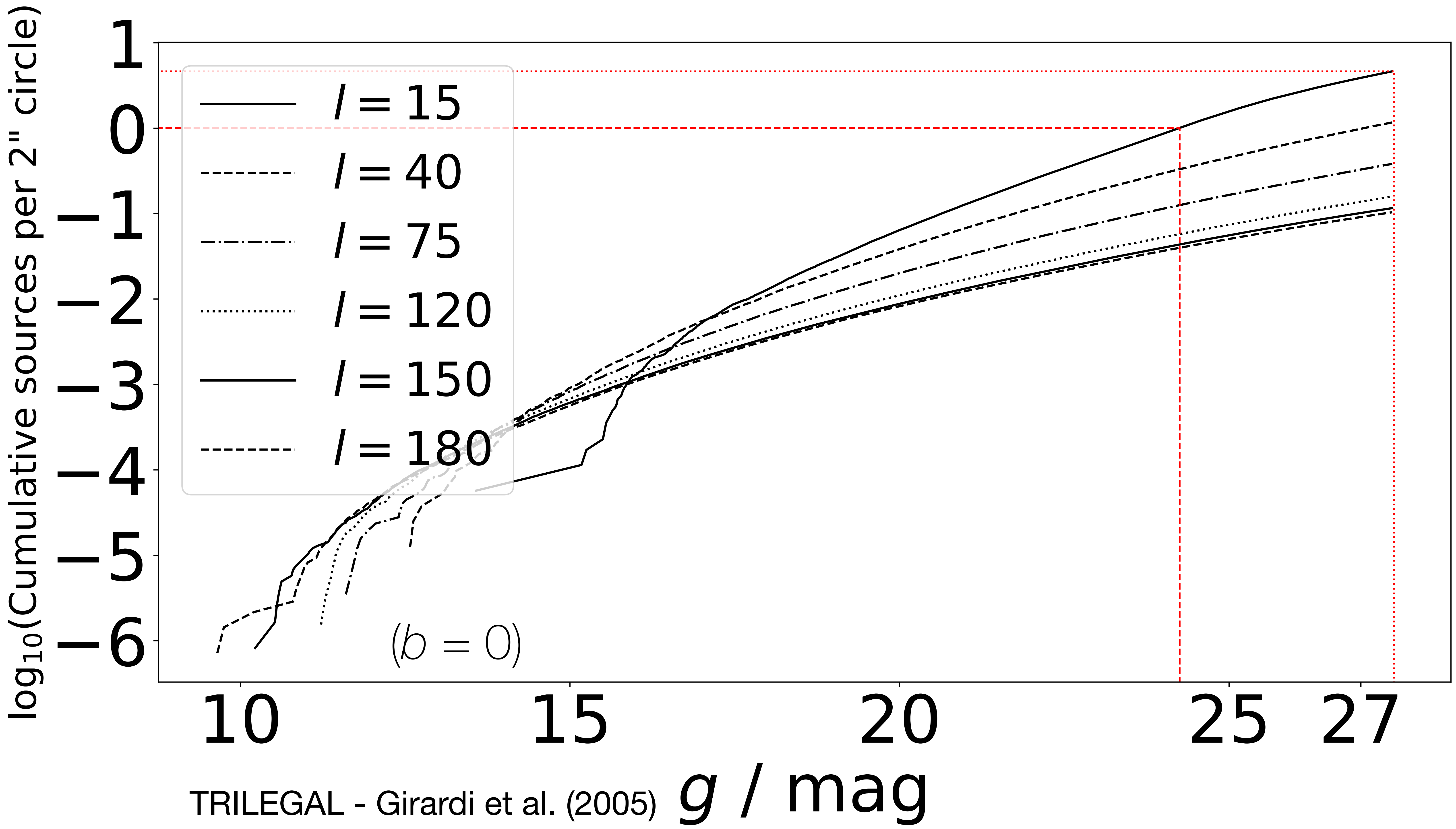
The Looming Problem With LSST





TRILEGAL - Girardi et al. (2005)

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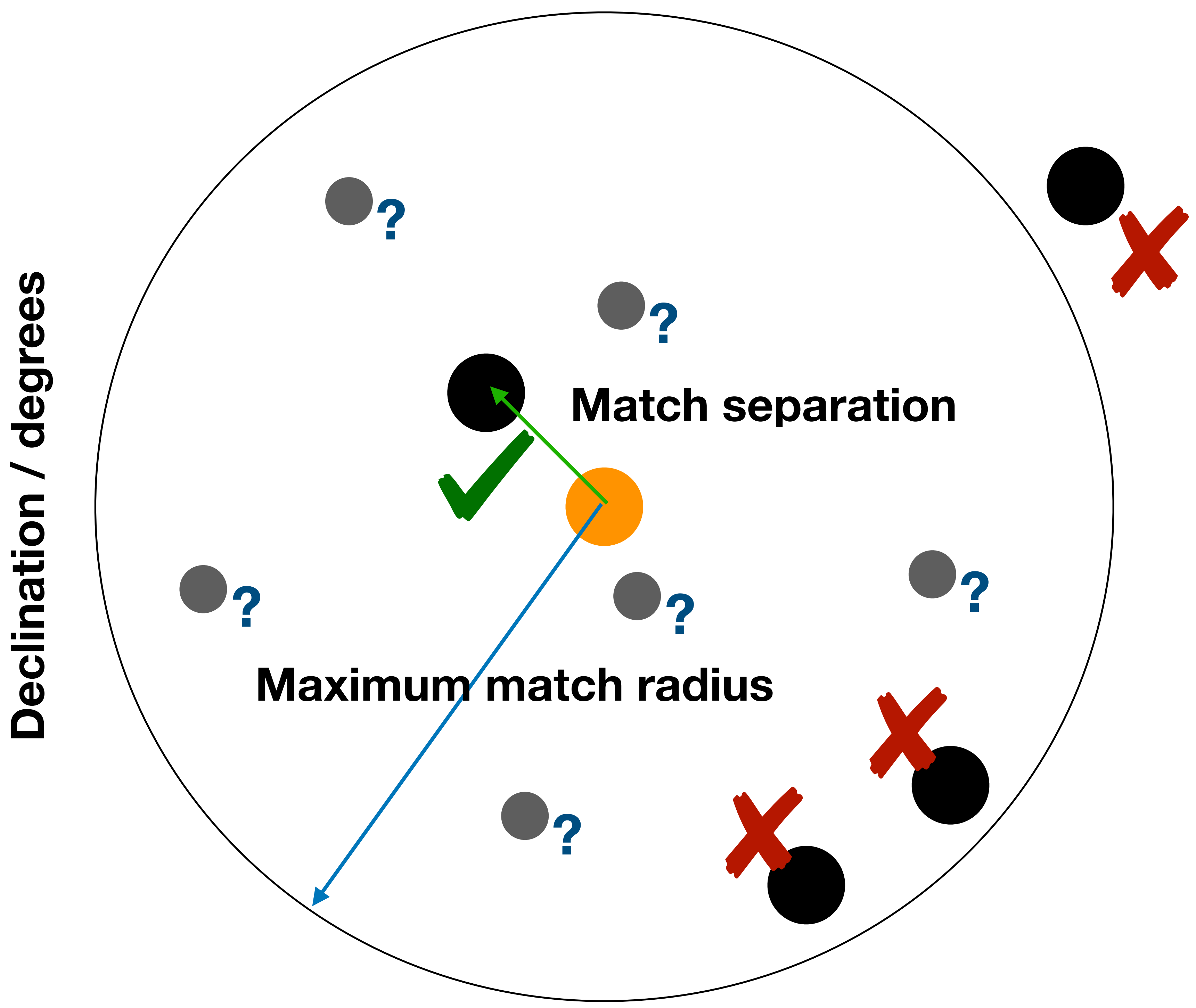
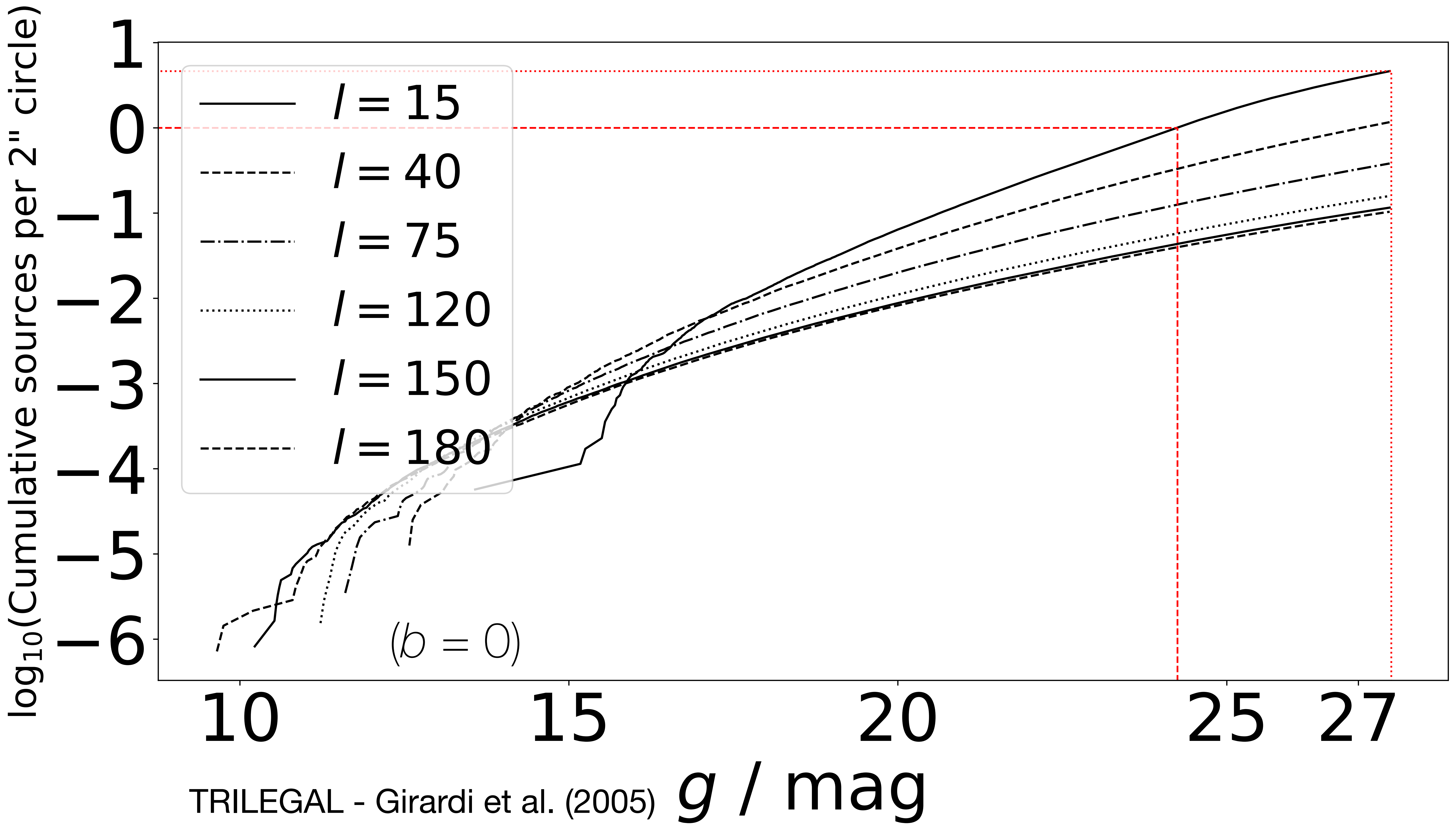
The Looming Problem With LSST

Nearest neighbour/ proximity matching



(It's 2-4 randomly placed objects in every match radius at high Galactic latitudes)



TRILEGAL - Girardi et al. (2005)

Nearest-neighbour matching <u>*will not*</u> work in the era of Rubin!

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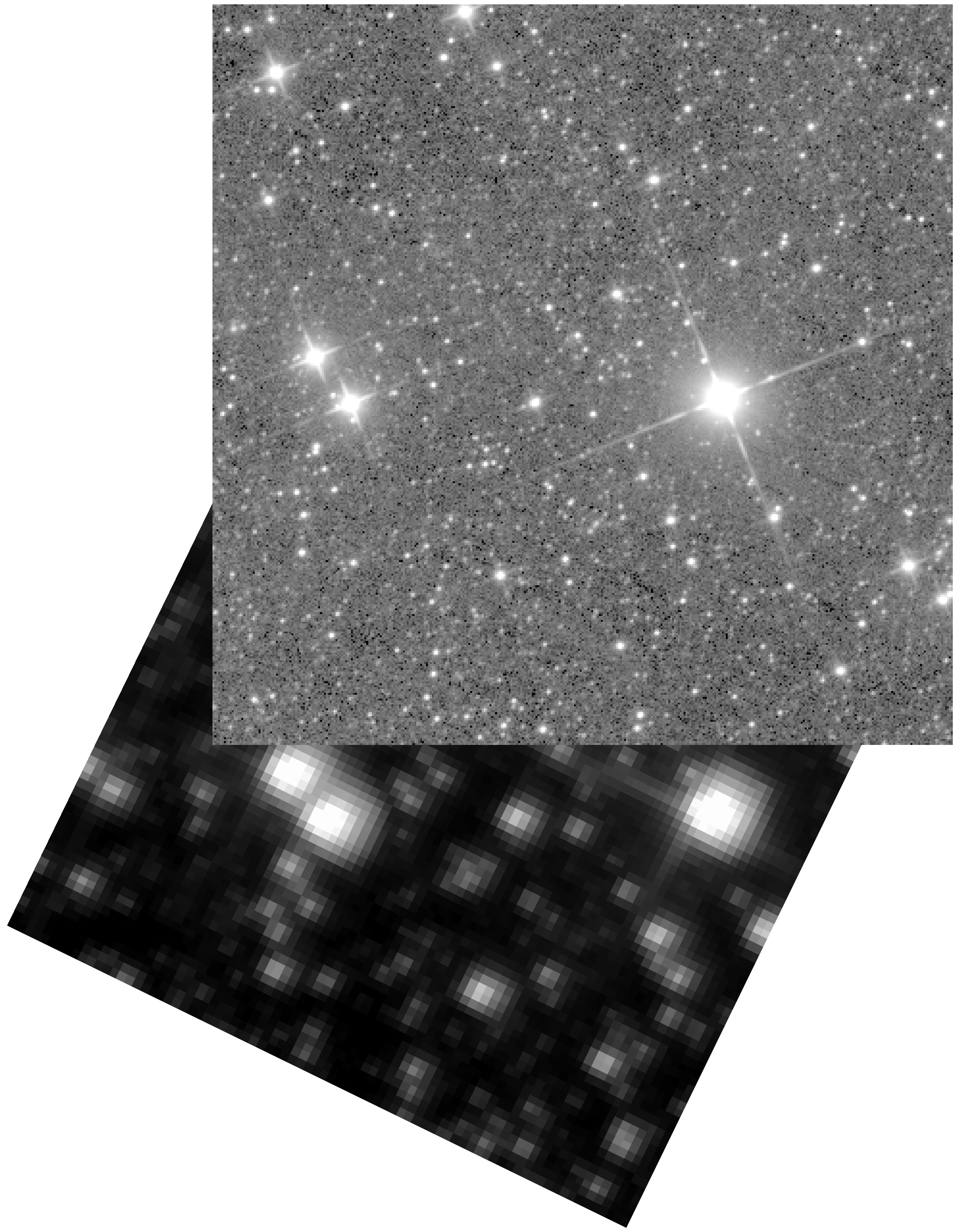
The Astronomy Error Function



1) $p(x)$ decreases as x increases
2) $p(x \text{ and } y) = p(x)p(y)$
3) $p(x) = p(-x) \Rightarrow p(x^2)$

J. F. W. Herschel, "Quetelet on Probabilities", 1850

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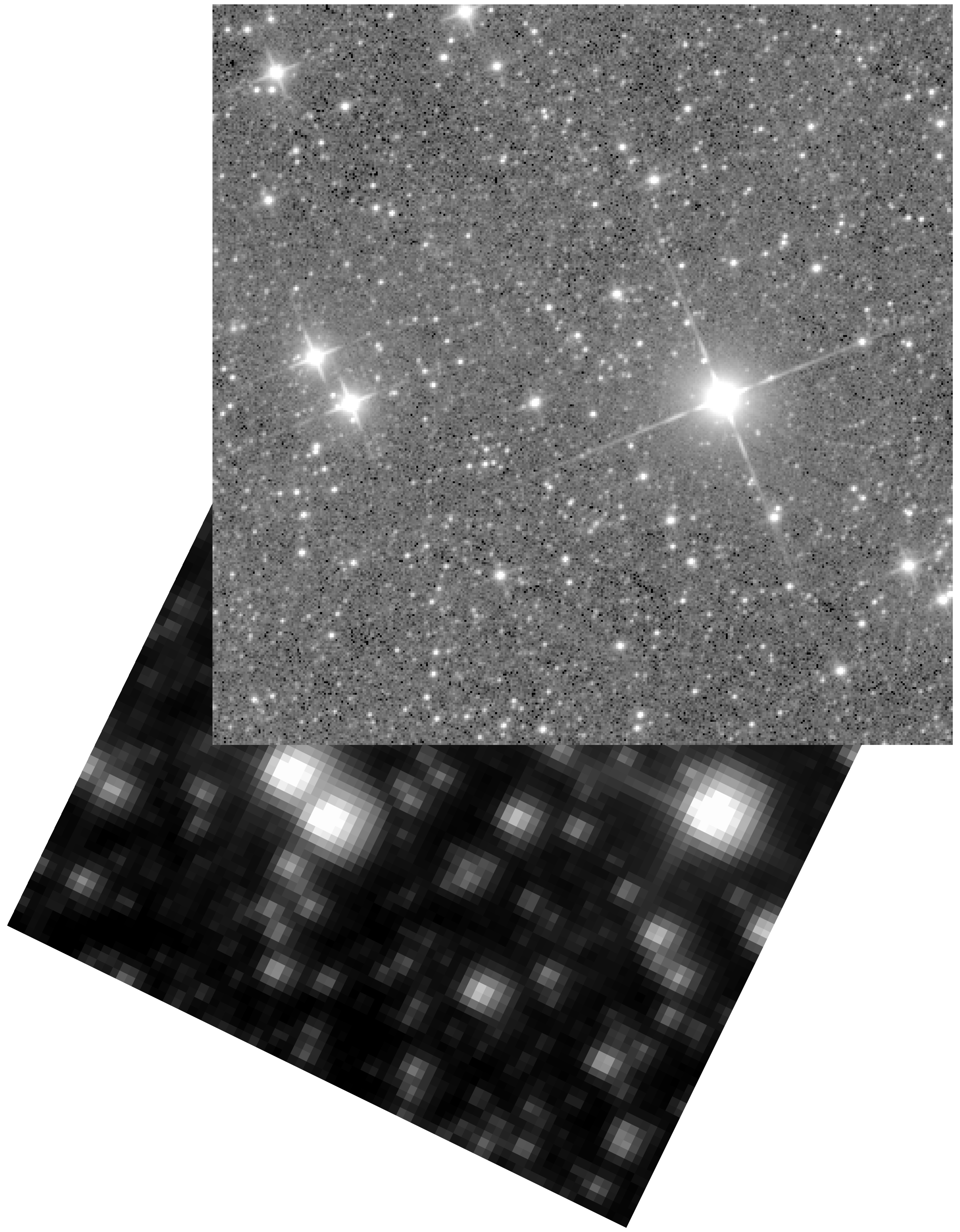

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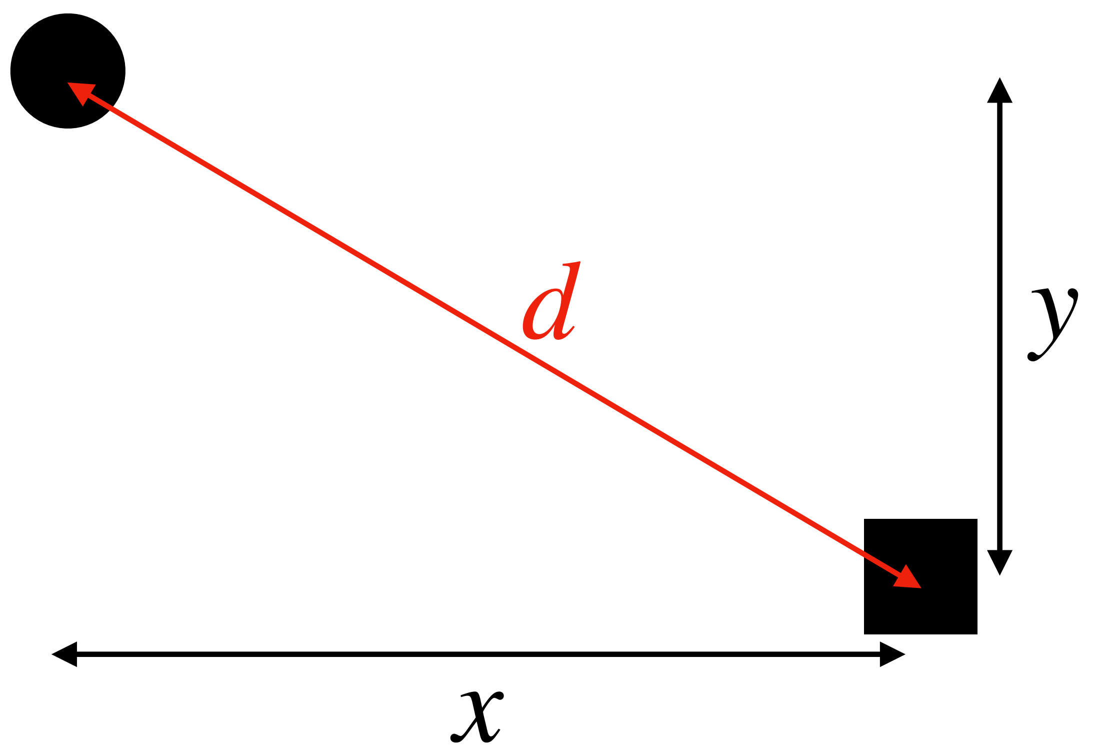

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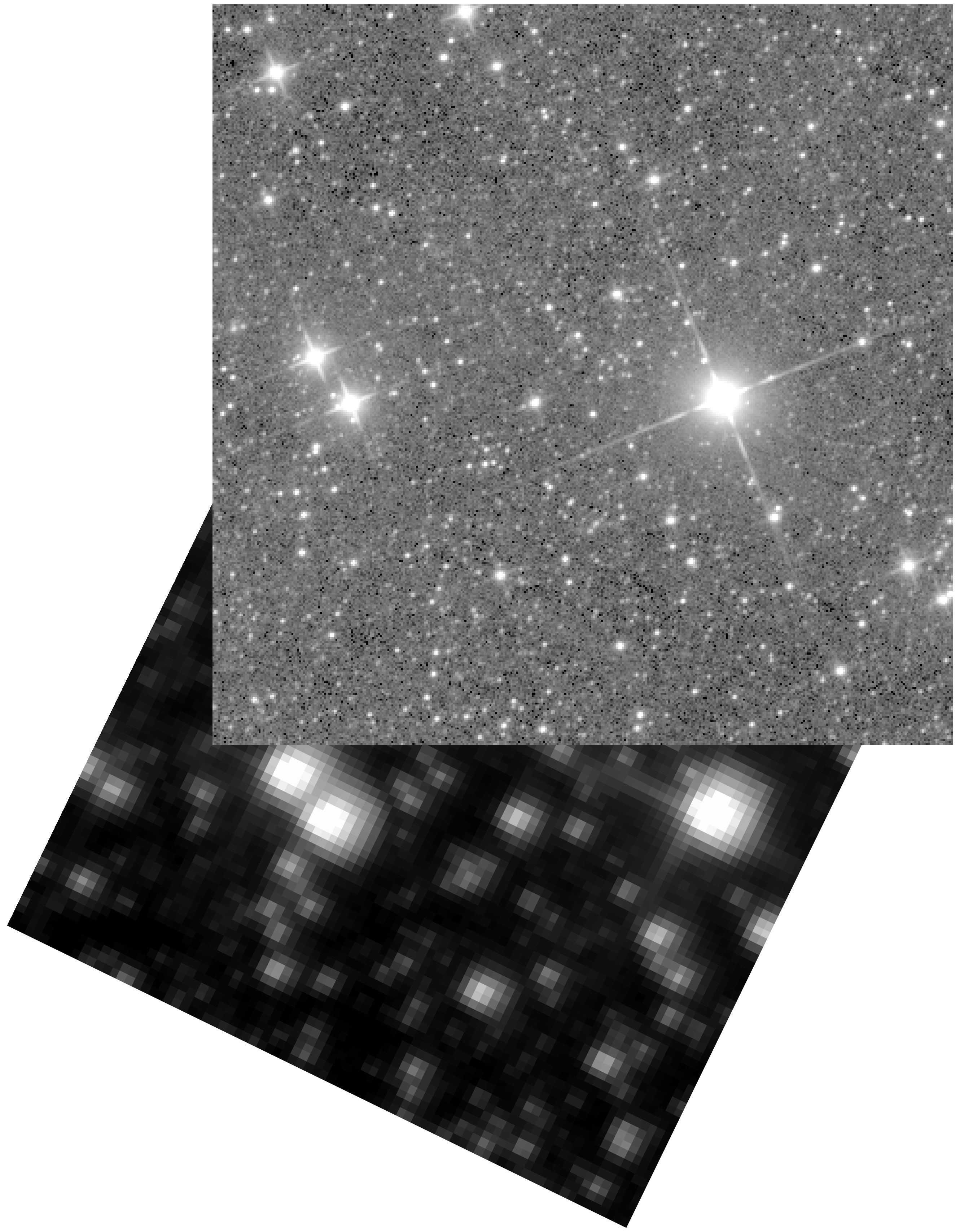

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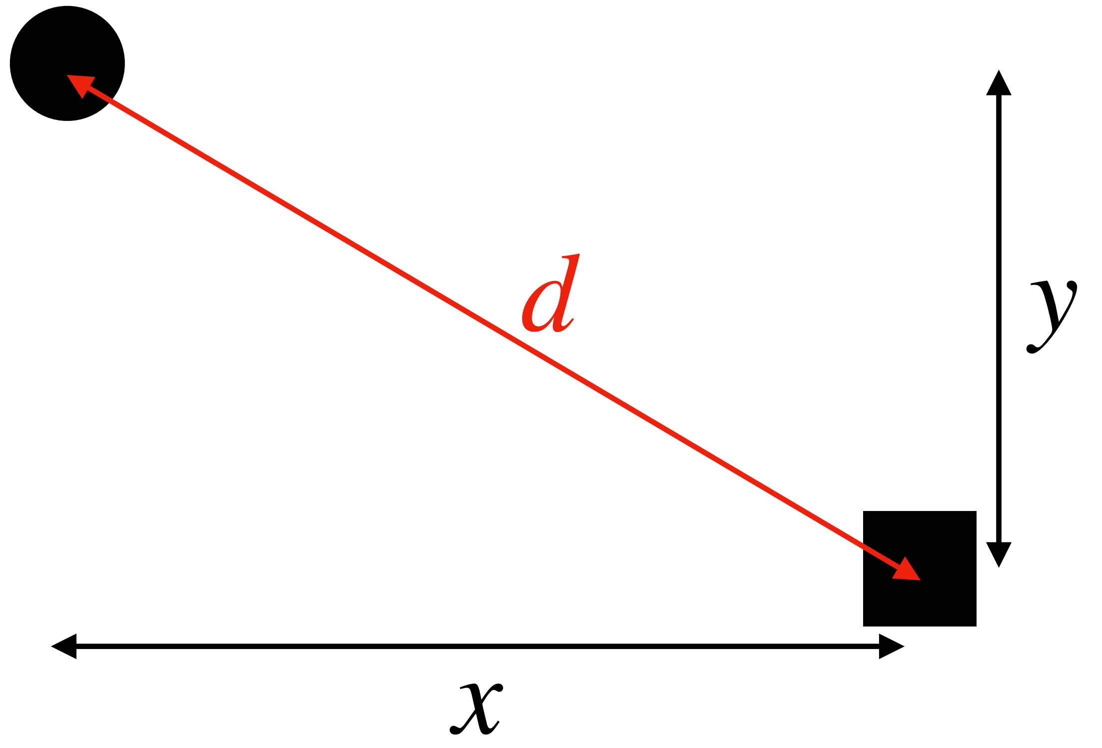

$$p(d^2) = p(x^2 + y^2) = p(x^2)\,p(y^2)$$

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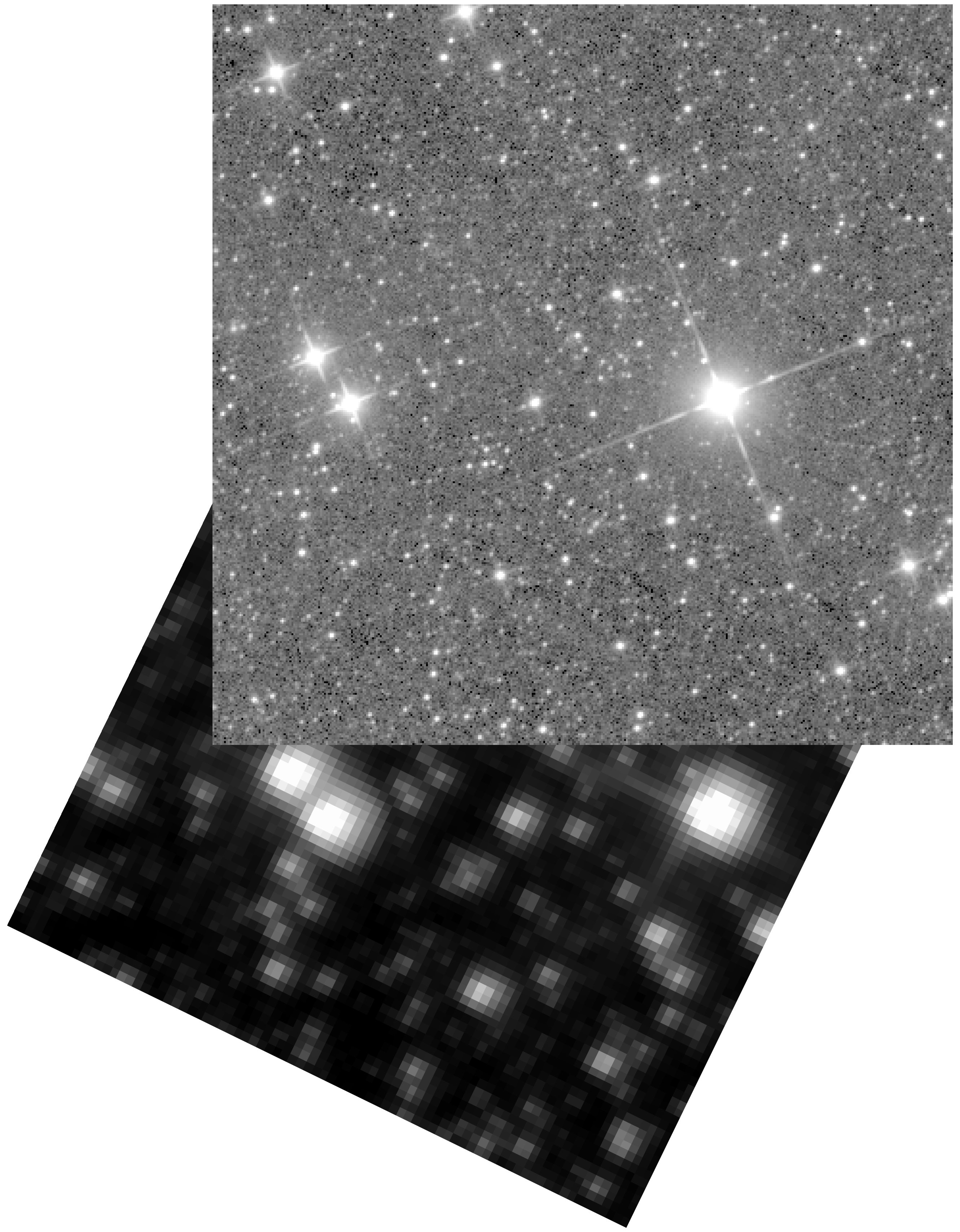

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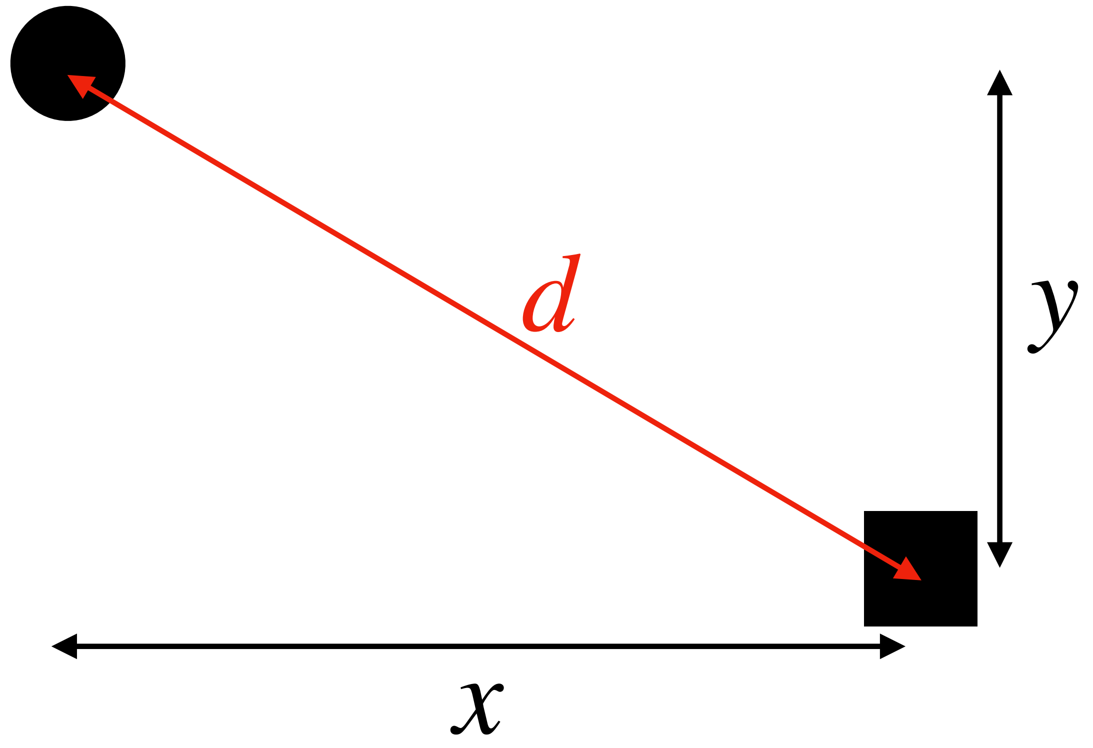

$$p(d^2) = p(x^2 + y^2) = p(x^2)\,p(y^2)$$

$$g(x, y, \sigma) = (2\pi\sigma^2)^{-1}\exp\left(-\frac{1}{2}\frac{x^2 + y^2}{\sigma^2}\right)$$

Nearest-neighbour match radius: ~2”; typical precision σ on source position: $\lesssim$0.2”

J. F. W. Herschel, “Quetelet on Probabilities”, 1850

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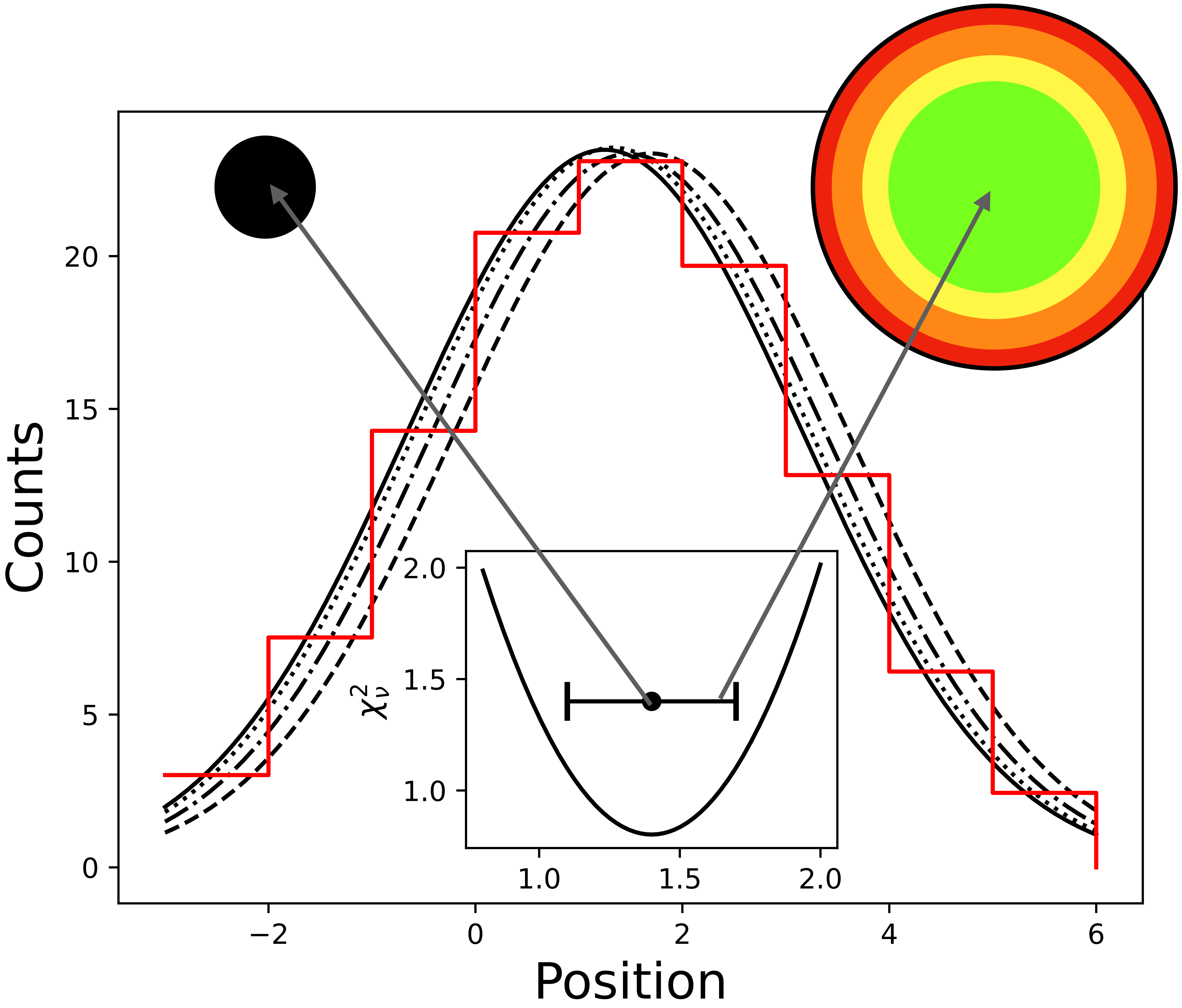
Centroid Positions and Uncertainties



$$p(D\,|\,M) \propto \frac{\exp\left(-\frac{1}{2}\left(\mathbf{x}-\boldsymbol{\mu}\right)^T \boldsymbol{\Sigma}^{-1}\left(\mathbf{x}-\boldsymbol{\mu}\right)\right)}{\sqrt{(2\pi)^k\,|\,\boldsymbol{\Sigma}\,|}}$$

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$$

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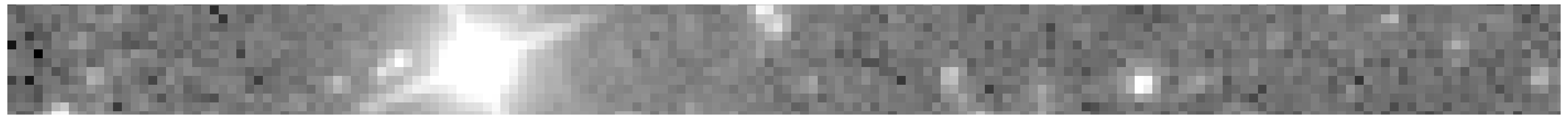

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Probabilistic Cross-Matching

The Likelihood Ratio

$$dp(r|id) = r \times e^{-r^2/2}\, dr.$$

$$dp(r|c) = 2\lambda r \times e^{-\lambda r^2}\, dr$$

$$LR(r) = dp(r|id)/dp(r|c) = \frac{1}{2\lambda}\exp\left\{\frac{r^2}{2}(2\lambda - 1)\right\}$$

de Ruiter, Willis, & Arp (1977)

$$dp_{\rm id} = Qr\exp\left(\frac{-r^2}{2}\right) dr. \quad dp_{\rm uo} = 2\lambda r\, dr$$

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Wolstencroft et al. (1986)

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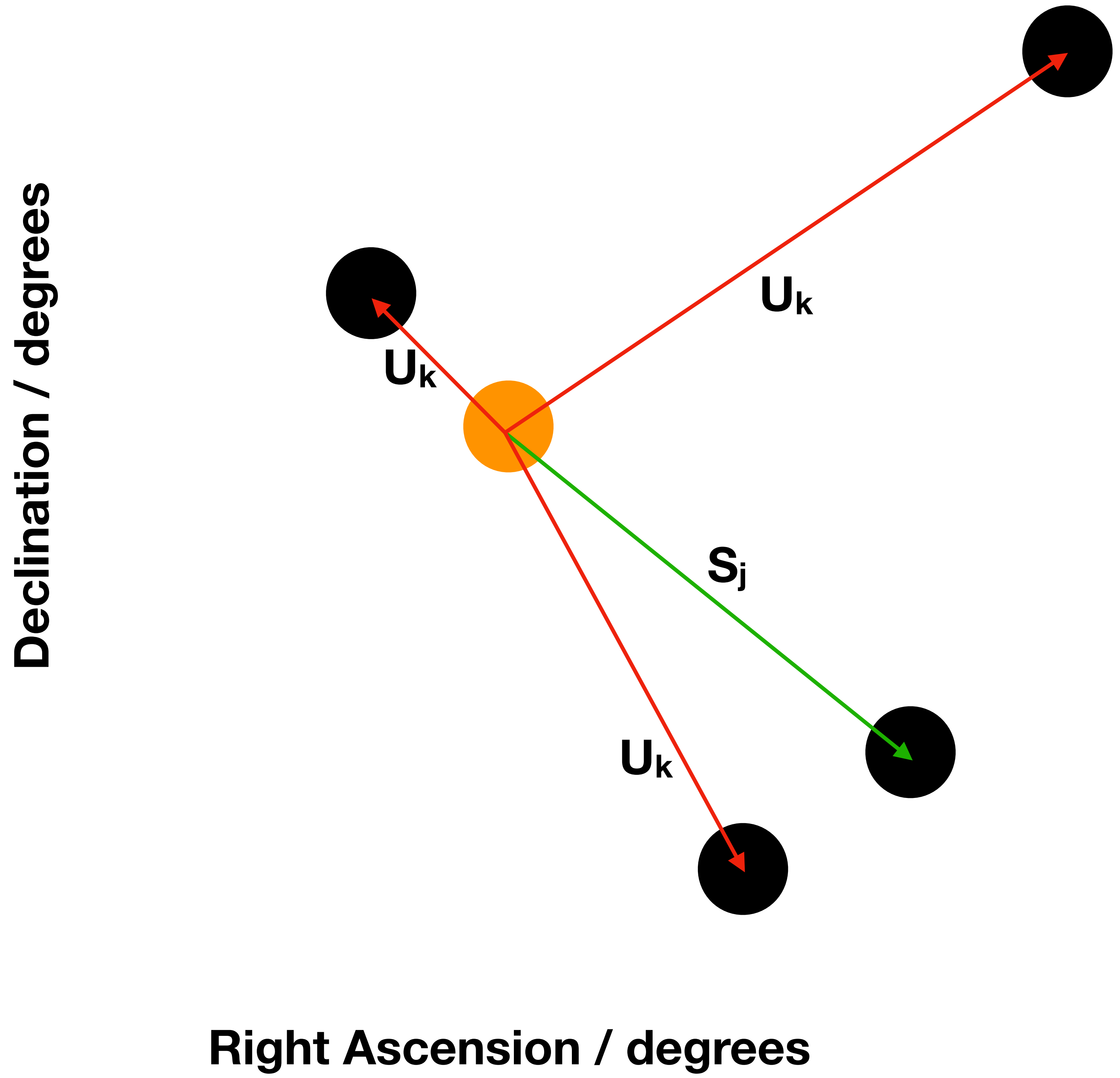
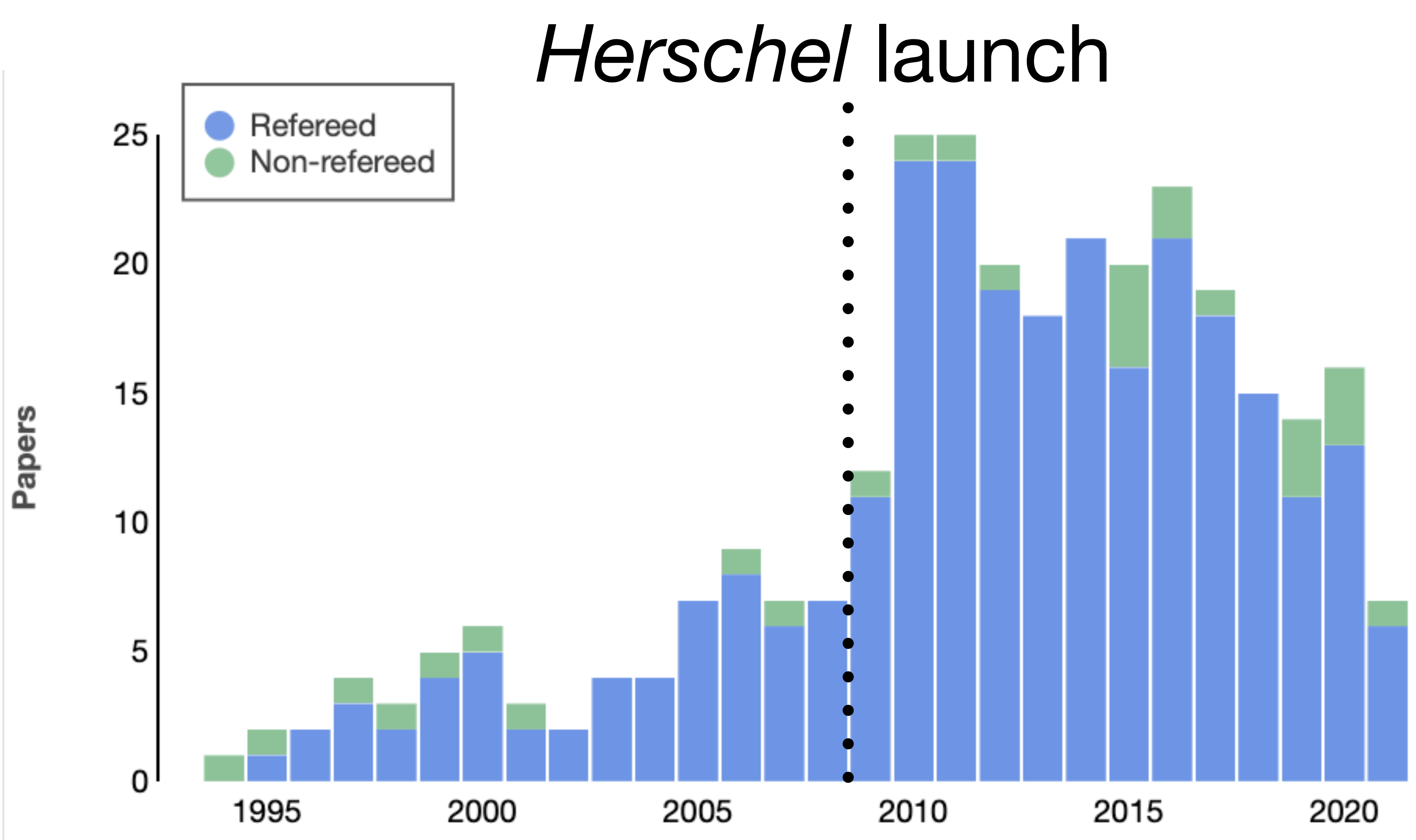
The “Reliability” — Sutherland & Saunders (1992)



$$R_j = \frac{\Pr\left[S_j \cap \left(\bigcap_{k\neq j} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]}{\sum_i \Pr\left[S_i \cap \left(\bigcap_{k\neq i} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right] + \Pr\left[(m_S > m_{\mathrm{lim}}) \cap \left(\bigcap_{k} U_k\right) \cap \left(\bigcap_{k'} E_{k'}\right)\right]} = \frac{L_j}{\sum_i L_i + (1-Q)}$$



$$L = \frac{q(m,c)\,f(x,y)}{n(m,c)}$$

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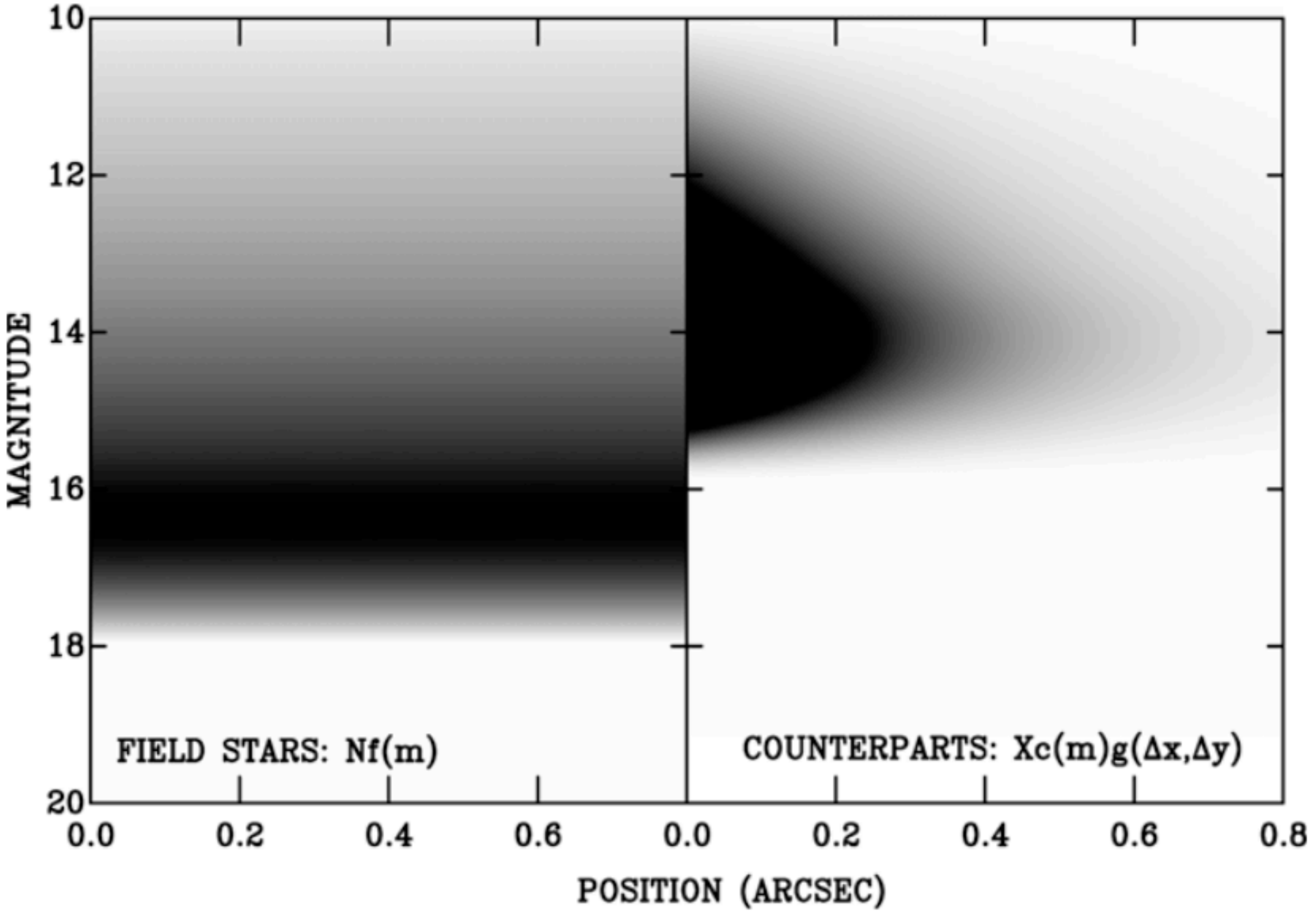
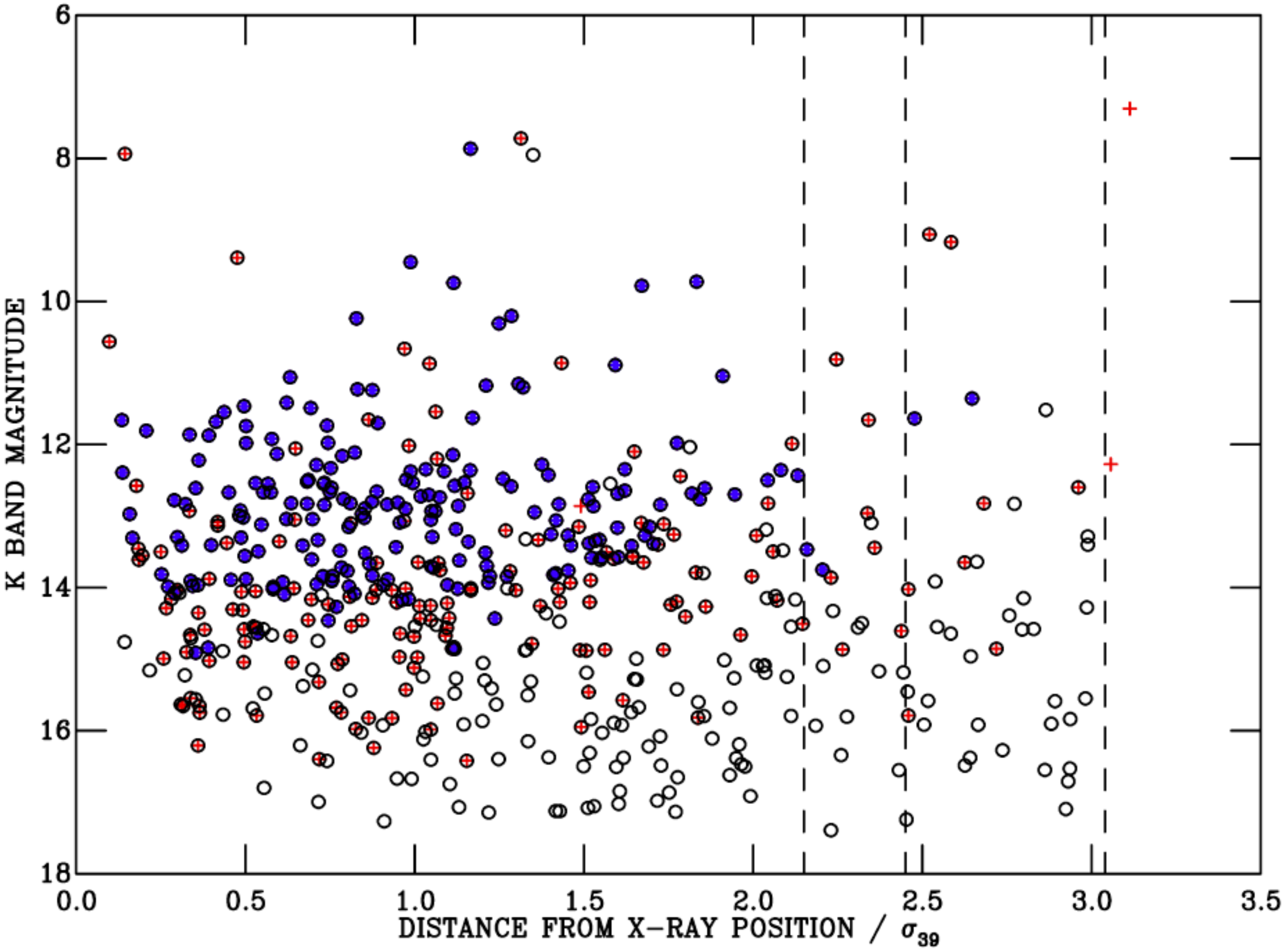
Probabilistic Cross-Matching

$$P(0) = \frac{1-X}{1-X+\sum_j \frac{Xc(m_j)\,g(\Delta x_j,\Delta y_j)}{Nf(m_j)}} \qquad P(i) = \frac{\frac{Xc(m_i)\,g(\Delta x_i,\Delta y_i)}{Nf(m_i)}}{1-X+\sum_j \frac{Xc(m_j)\,g(\Delta x_j,\Delta y_j)}{Nf(m_j)}}$$





Naylor, Broos, & Feigelson (2013)

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Probabilistic Cross-Matching

$$p(D|H) = \int p(\boldsymbol{m}|H) \prod_{i=1}^{n} p_i(\boldsymbol{x}_i|\boldsymbol{m}, H)\, d^3 m$$

$$p(D|K) = \prod_{i=1}^{n} \left[\int p(\boldsymbol{m}_i|K) p_i(\boldsymbol{x}_i|\boldsymbol{m}_i, K)\, d^3 m_i \right]$$

$$B(H, K|D') = \frac{\int p(\boldsymbol{\eta}|H) \prod_{i=1}^{n} p_i(\boldsymbol{g}_i|\boldsymbol{\eta}, H)\, d^r \eta}{\prod_{i=1}^{n} \left[\int p(\boldsymbol{\eta}_i|K) p_i(\boldsymbol{g}_i|\boldsymbol{\eta}_i, K)\, d^r \eta_i \right]}$$

Budavári & Szalay (2008)

Includes SED model fitting to all sources

(Apologies for the lack of nice figures in this paper!)

Probabilistic Cross-Matching

Nearest neighbour or brightest neighbour: one-to-one, either astrometry OR photometry

Likelihood ratio: one-to-one matches, mostly just astrometry (e.g., Wolstencroft et al. 1986)

Reliability: One-to-many matches, uses photometry from one dataset (e.g. Naylor et al. 2013)

Budavári & Szalay (2008): one-to-one-to-one-to… matches, include SED fitting

e.g. Pineau et al. (2017): many-to-many-to-many-to… matches, no photometry implemented

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One assumption made in all of these works: source positions uncertainties are Gaussian!

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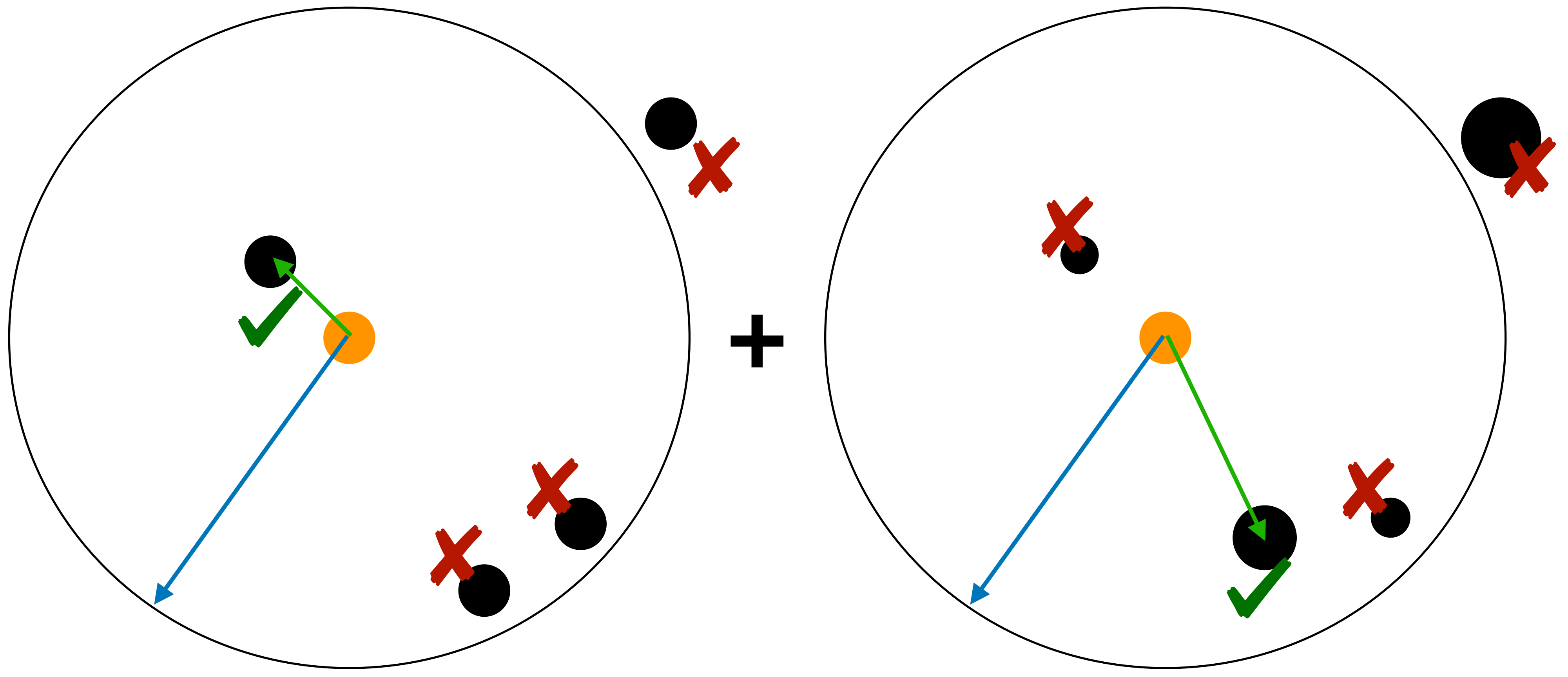
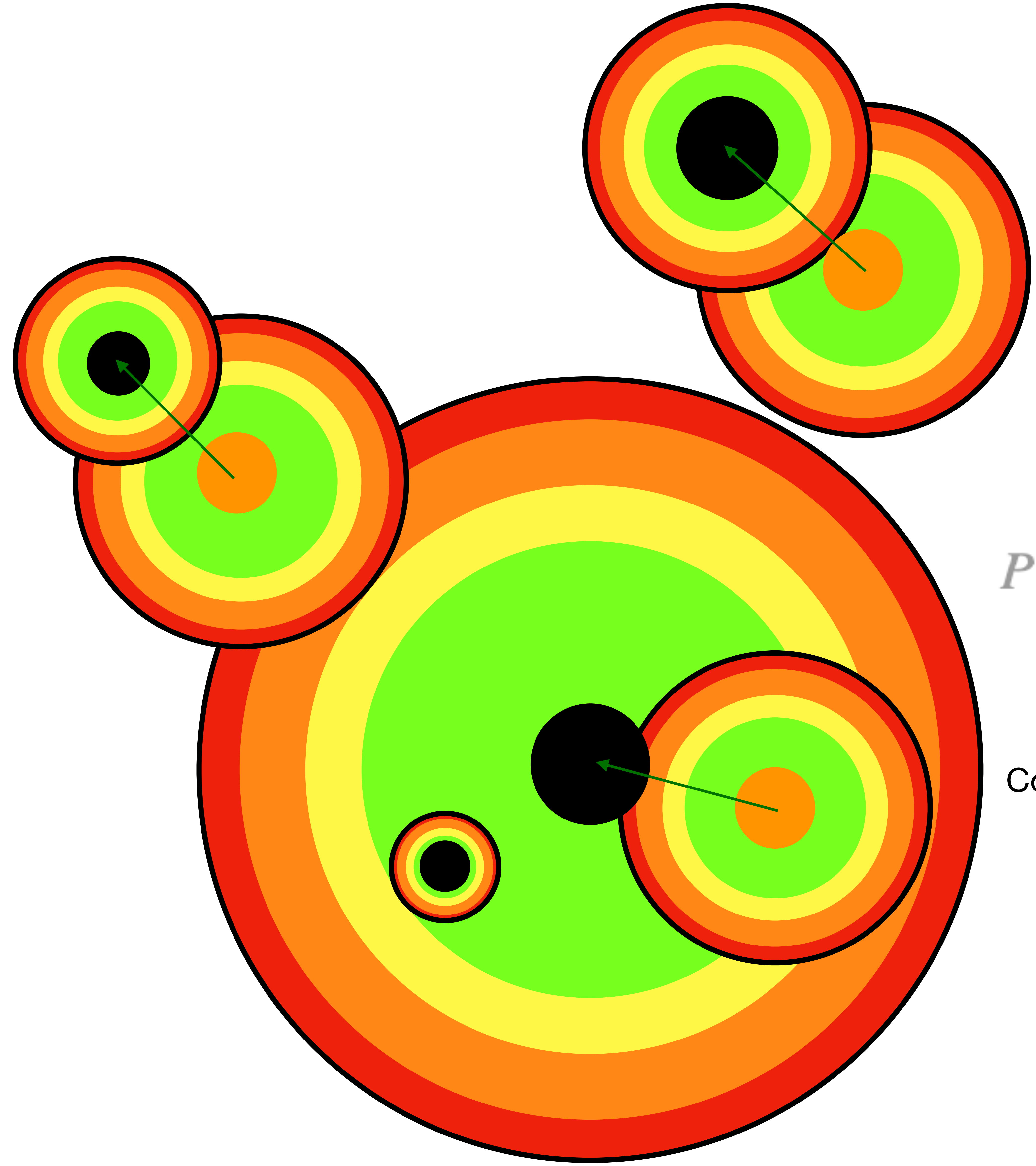
Wilson & Naylor (2017, 2018a, 2018b), Wilson (2022, 2023), and so on:

a many-to-many match with data-driven photometric probabilities

also describes a flexible approach to positional "errors," vital for future surveys

Probabilistic Cross-Matching





Probability of two sources having their <u>on-sky separation</u> given the hypothesis they are counterparts

G includes information on position (un)certainty

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i \lambda_i} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

Consider multiple objects in both catalogues simultaneously

Probability of sources having their <u>brightnesses</u> given they are unrelated to one another ("field stars")

Probability of sources having their <u>brightnesses</u> given they are counterparts

Wilson & Naylor (2018a)

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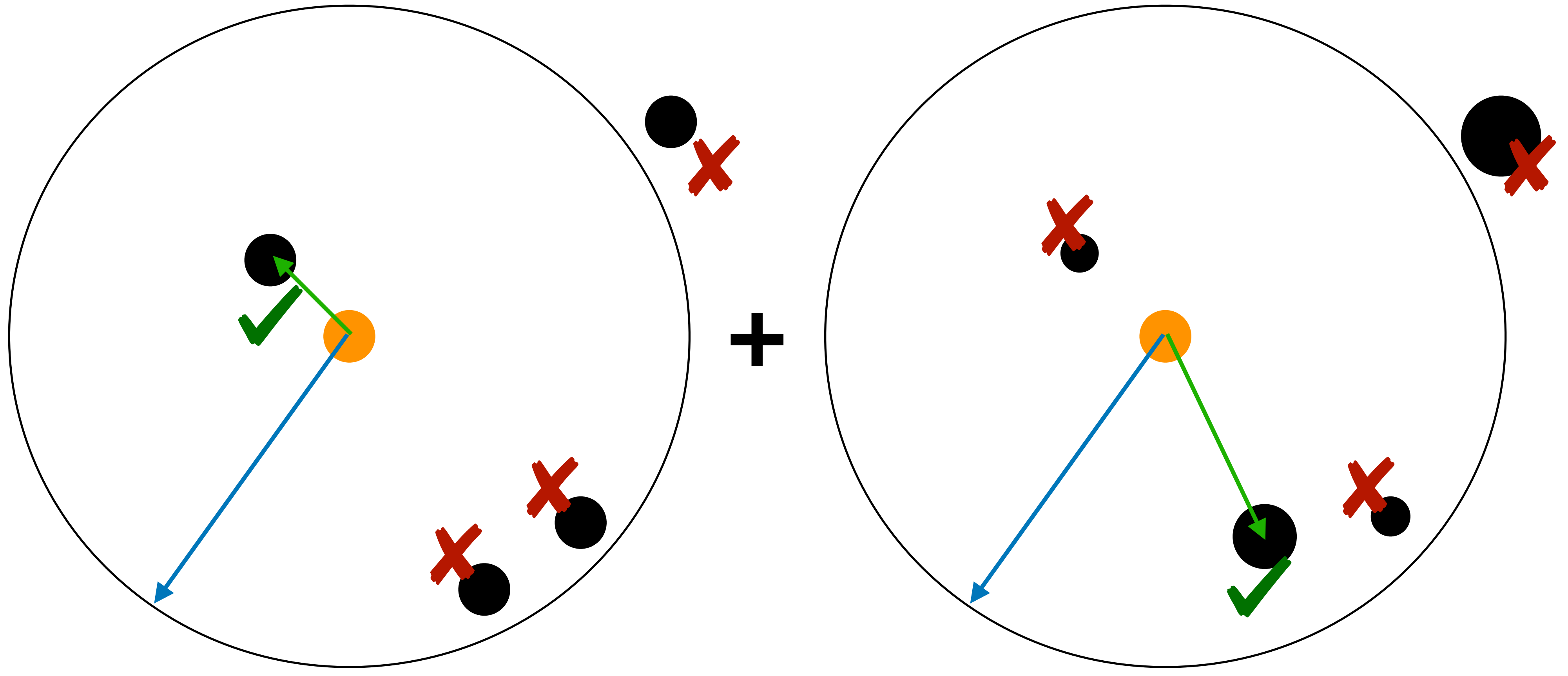
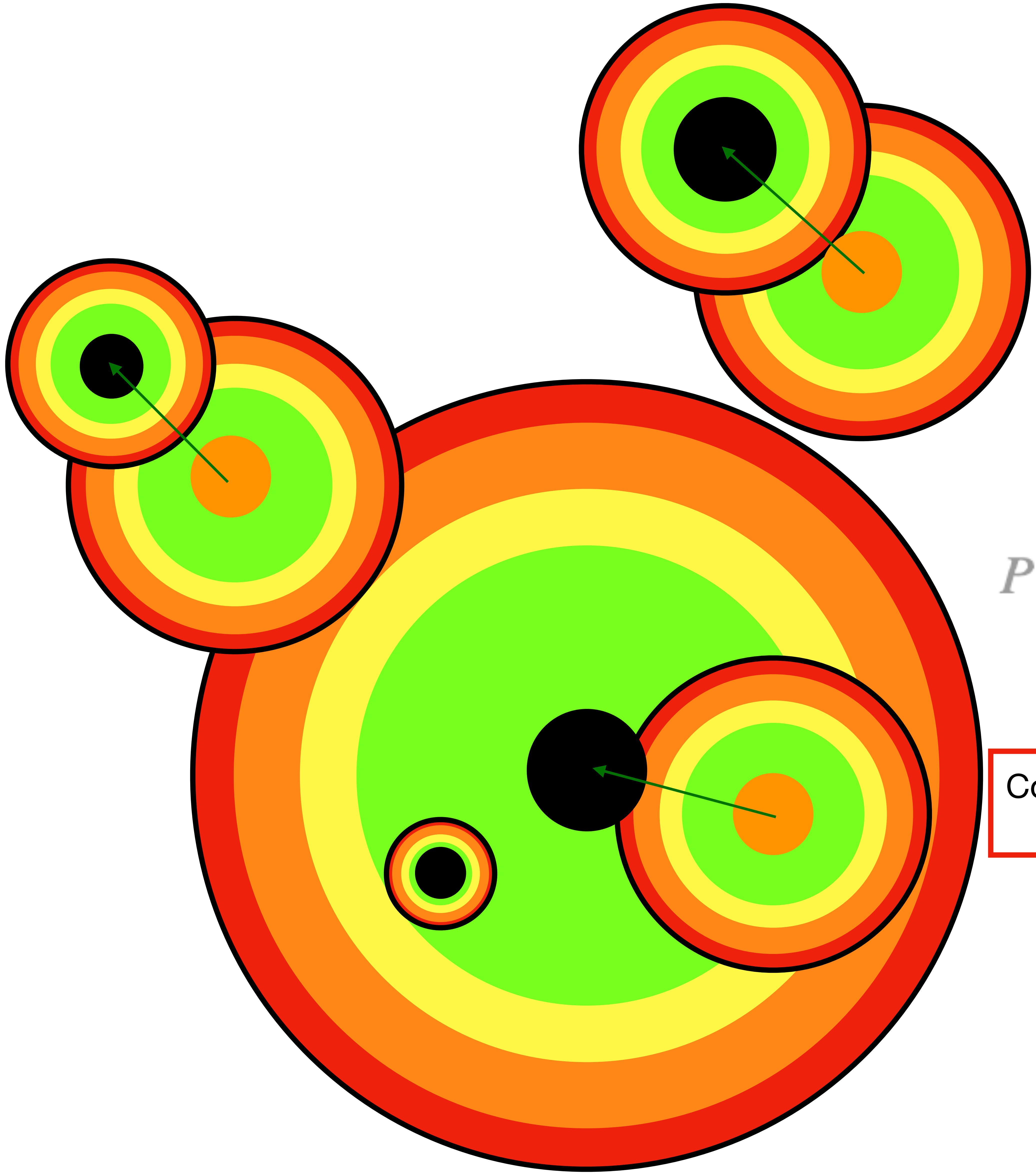
Probabilistic Cross-Matching





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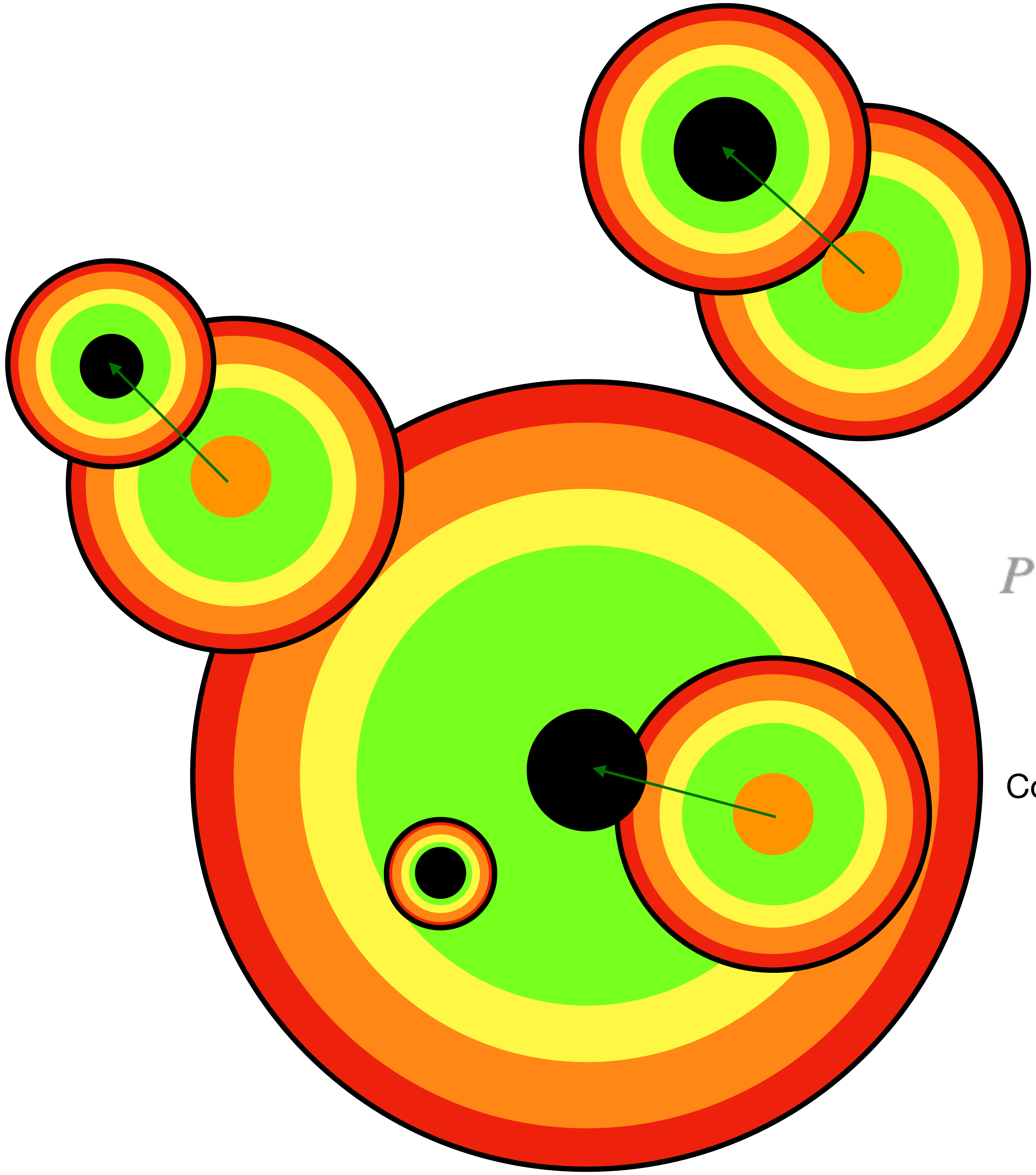

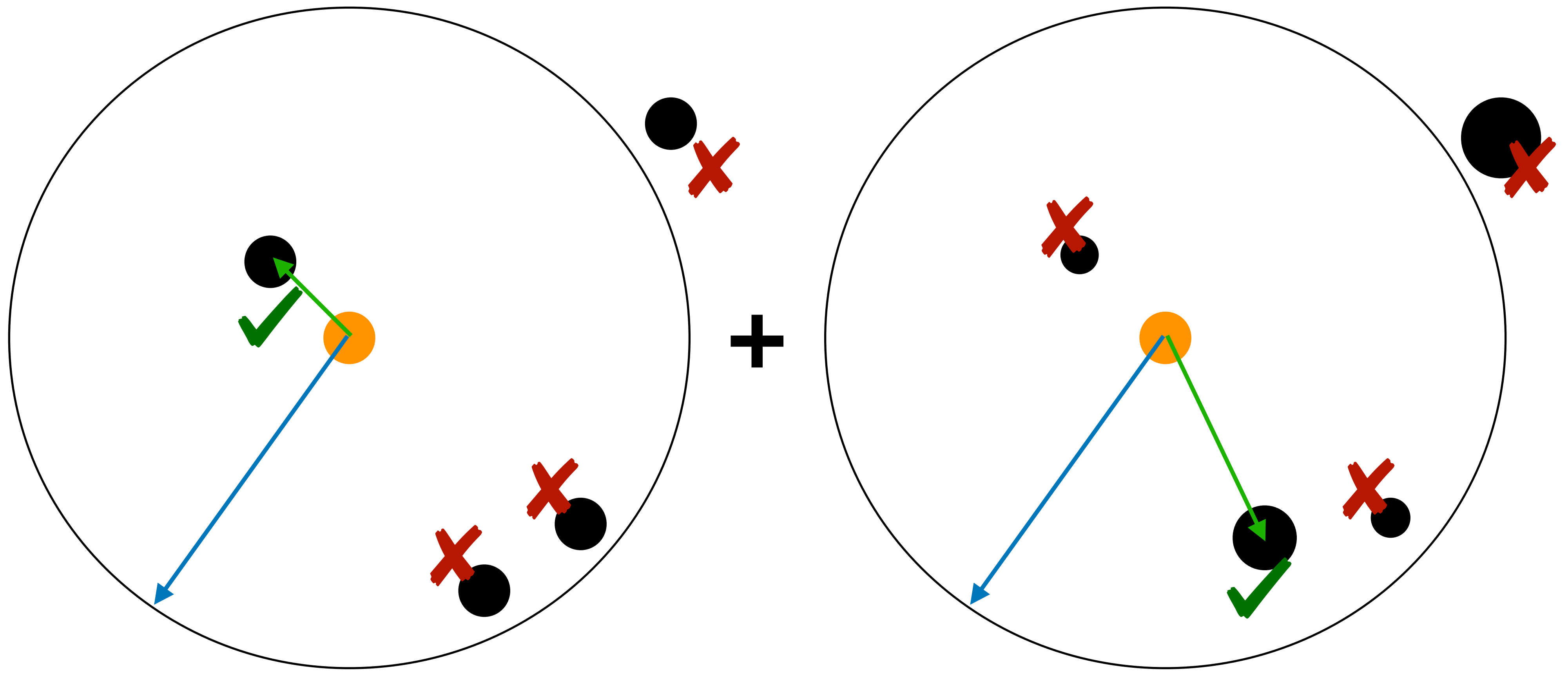

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Wilson & Naylor (2018a)

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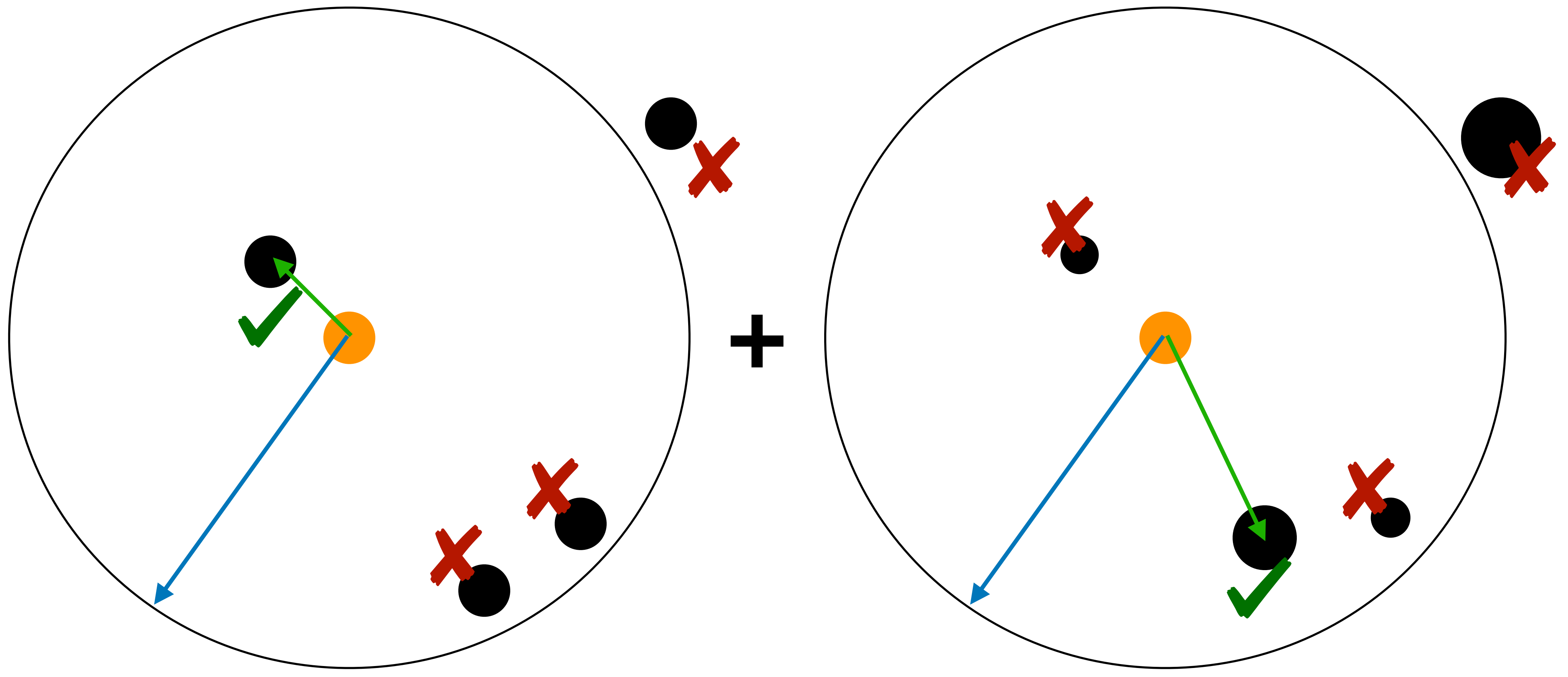
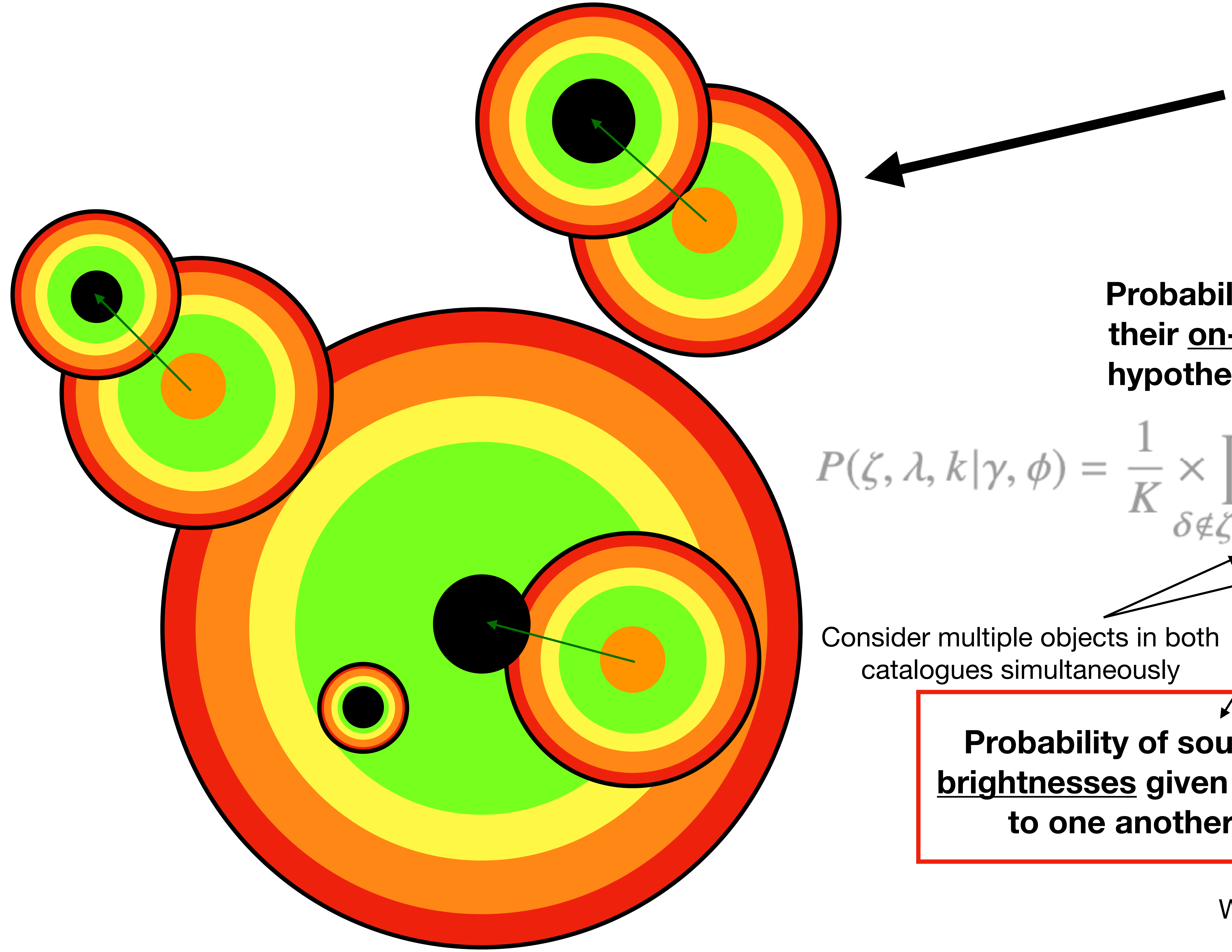
Probabilistic Cross-Matching





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$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i \lambda_i} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

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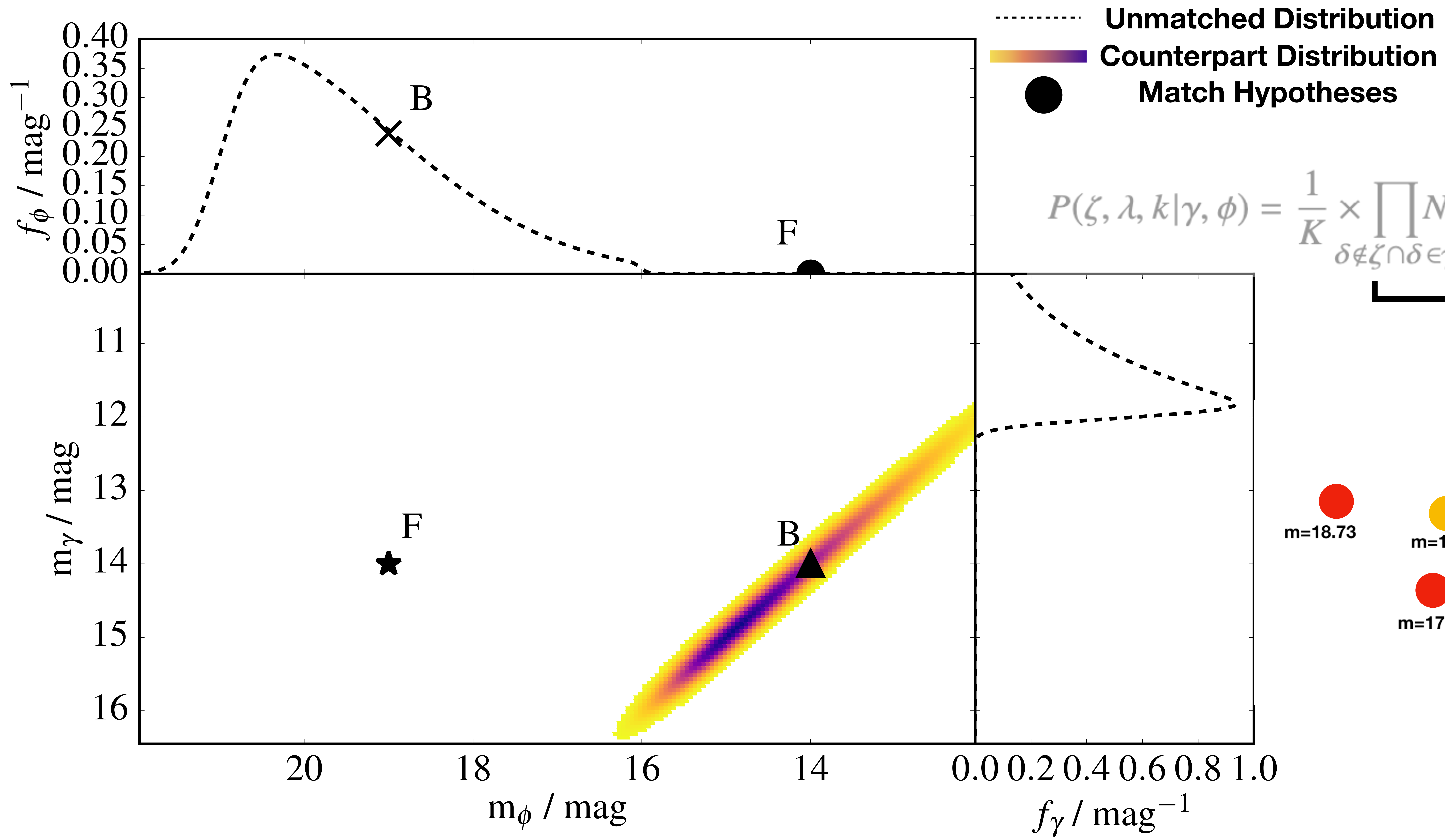
Probability of sources having their <u>brightnesses</u> given they are unrelated to one another (“field stars”)

Probability of sources having their <u>brightnesses</u> given they are counterparts

Wilson & Naylor (2018a)

Tom J Wilson @onoddil

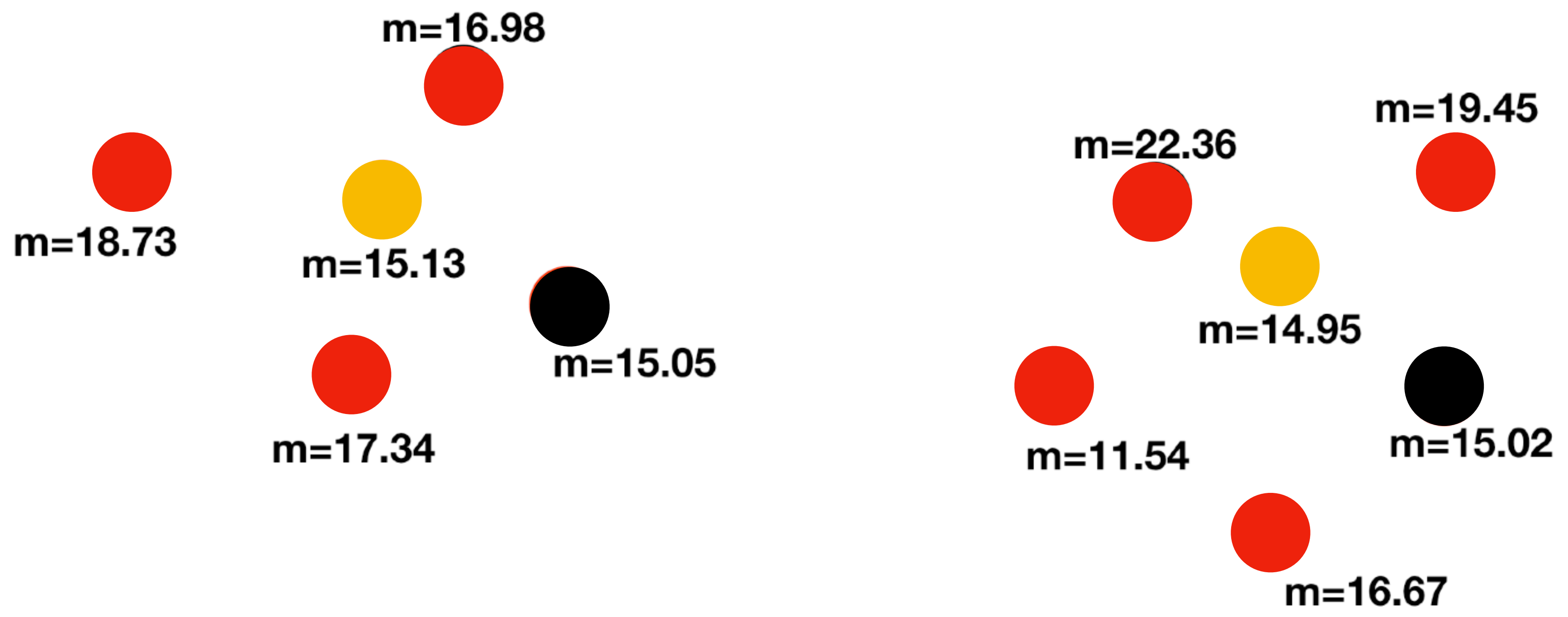
Photometry: Rejecting False Positives



$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \underbrace{\prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma \boldsymbol{f_\gamma^\delta} \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi \boldsymbol{f_\phi^\omega}}_{\text{Unmatched}} \underbrace{\prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i\lambda_i} \boldsymbol{c_{\gamma\phi}^{\zeta_i\lambda_i}}}_{\text{Matched}}$$

Unmatched

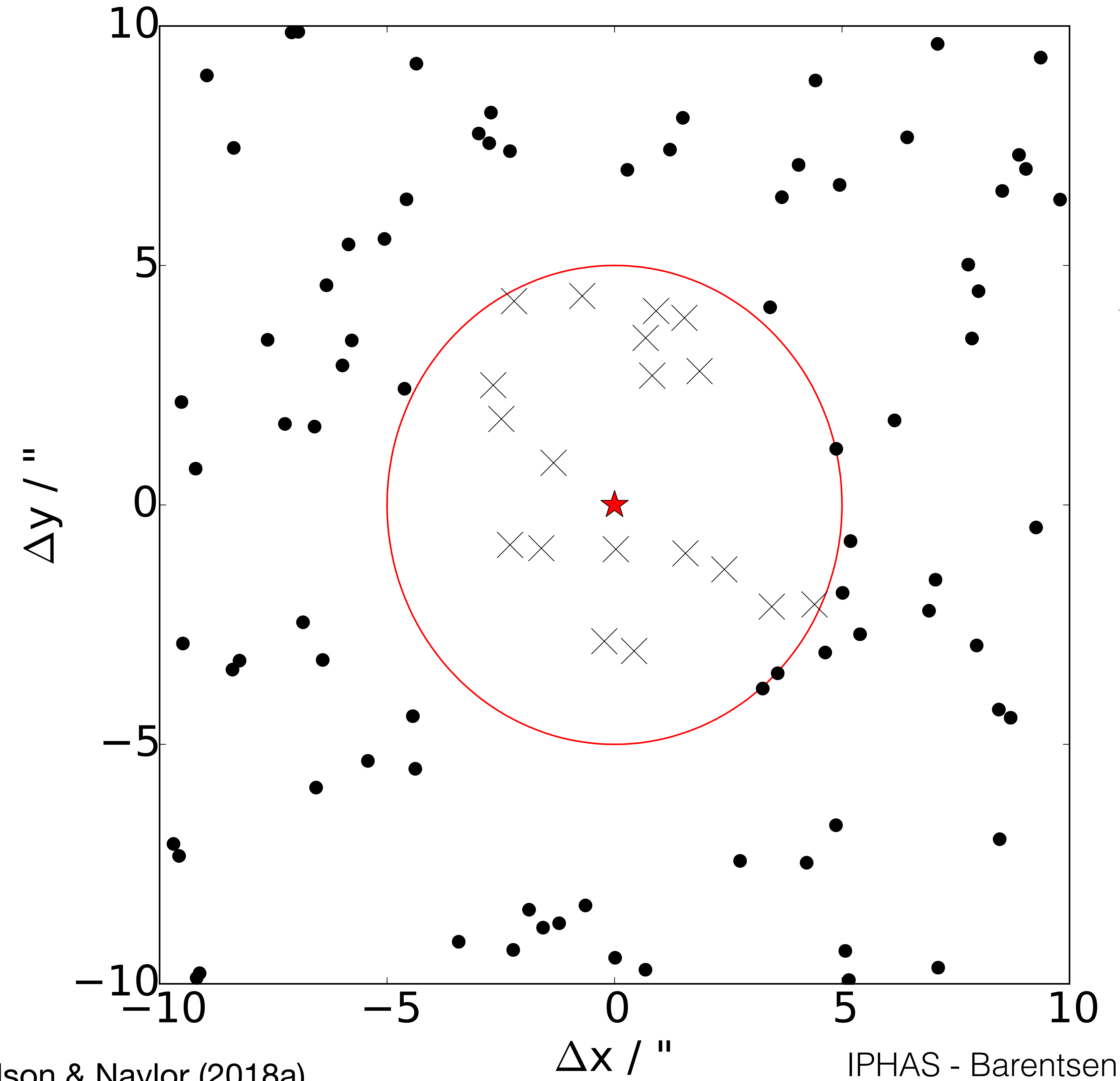
Matched



Wilson & Naylor (2018a)

Tom J Wilson @onoddil

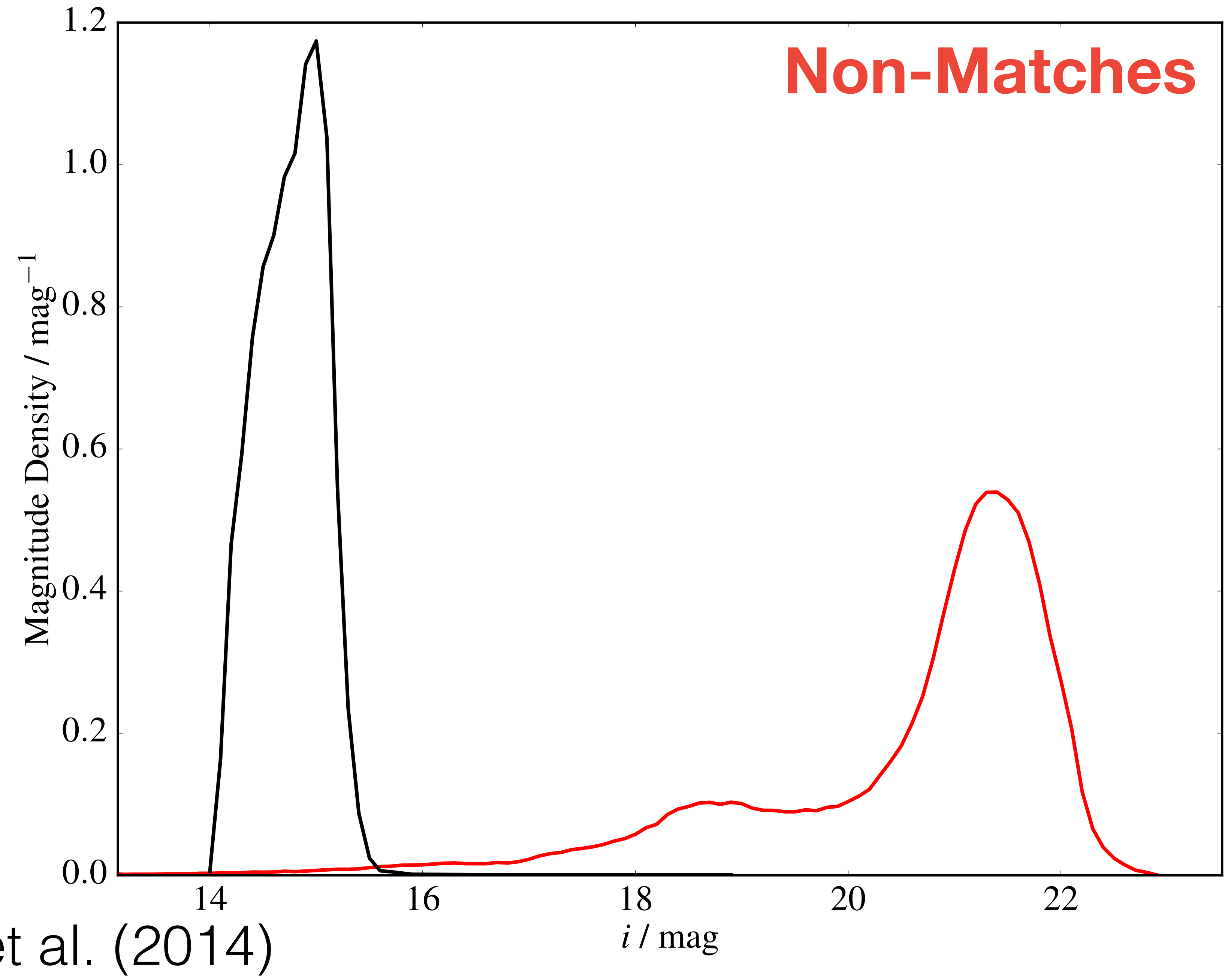
Photometry: The "Field Star" Distribution



Wilson & Naylor (2018a)

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i \lambda_i} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

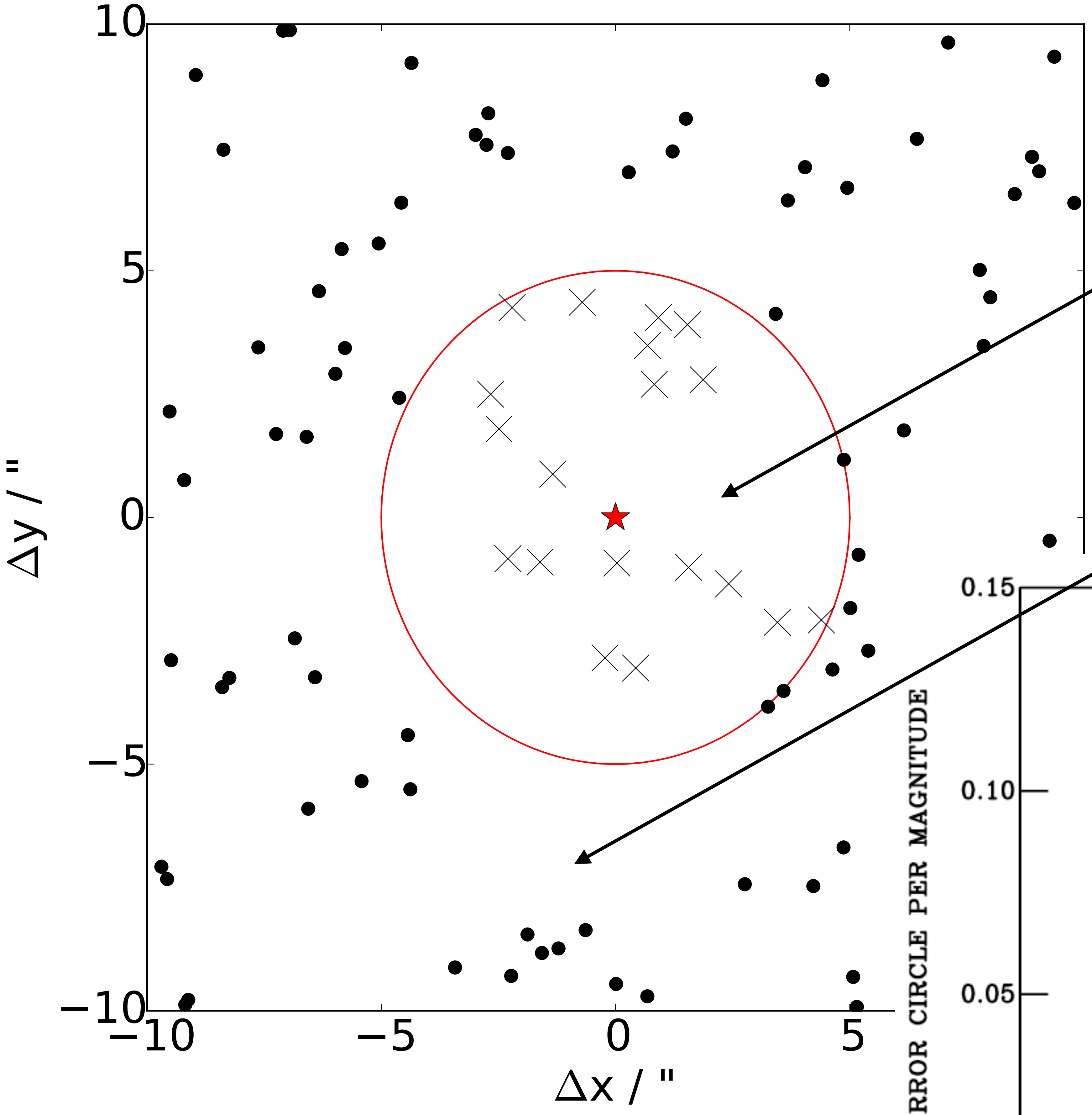
Unmatched **Matched**



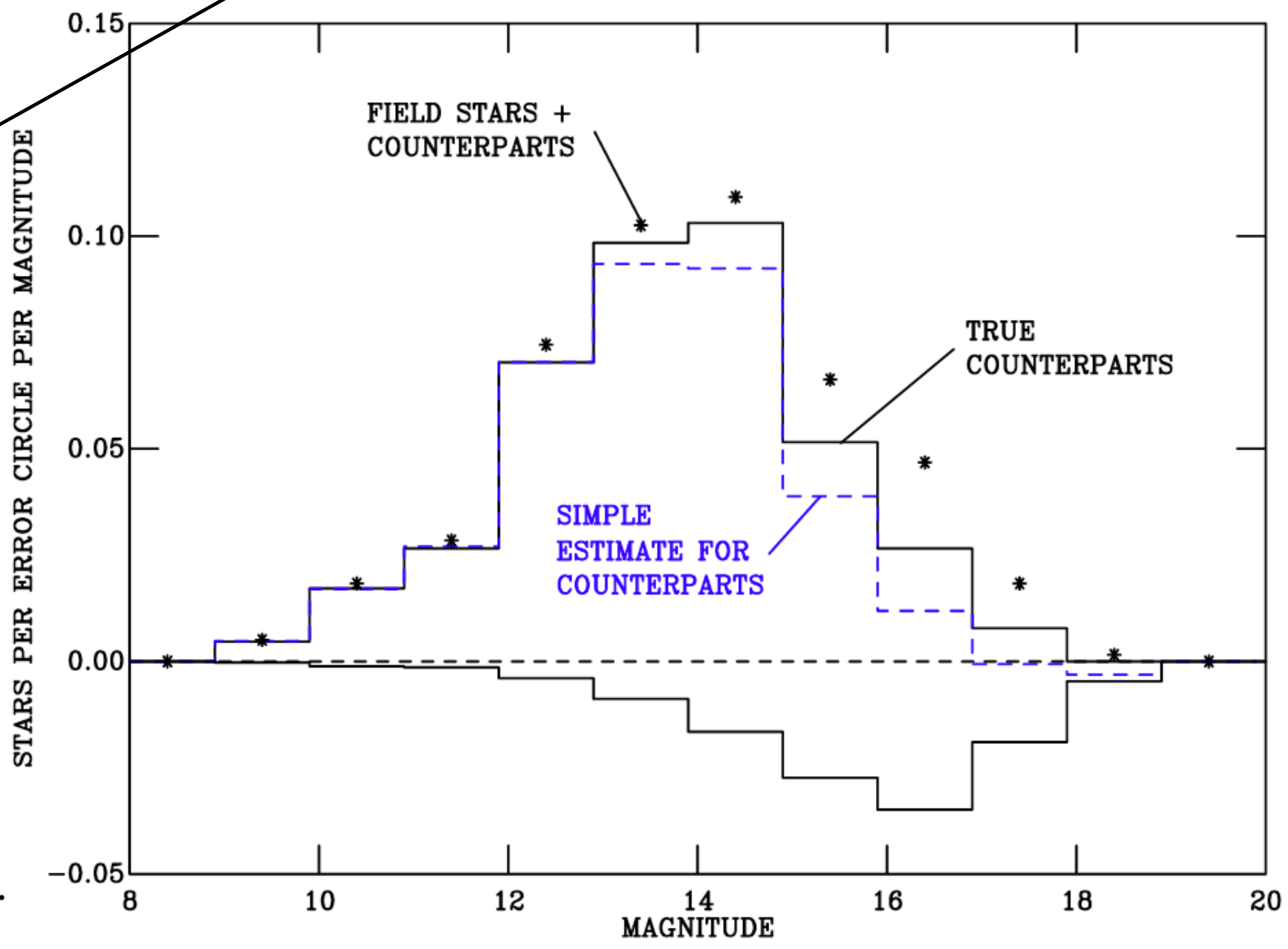
IPHAS - Barentsen et al. (2014)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

Tom J Wilson @onoddil

Photometry: The Counterpart Distribution



Counterparts = (Counterparts + field) - field



$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma f_\gamma^\delta \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i\lambda_i} c_{\gamma\phi}^{\zeta_i\lambda_i}$$

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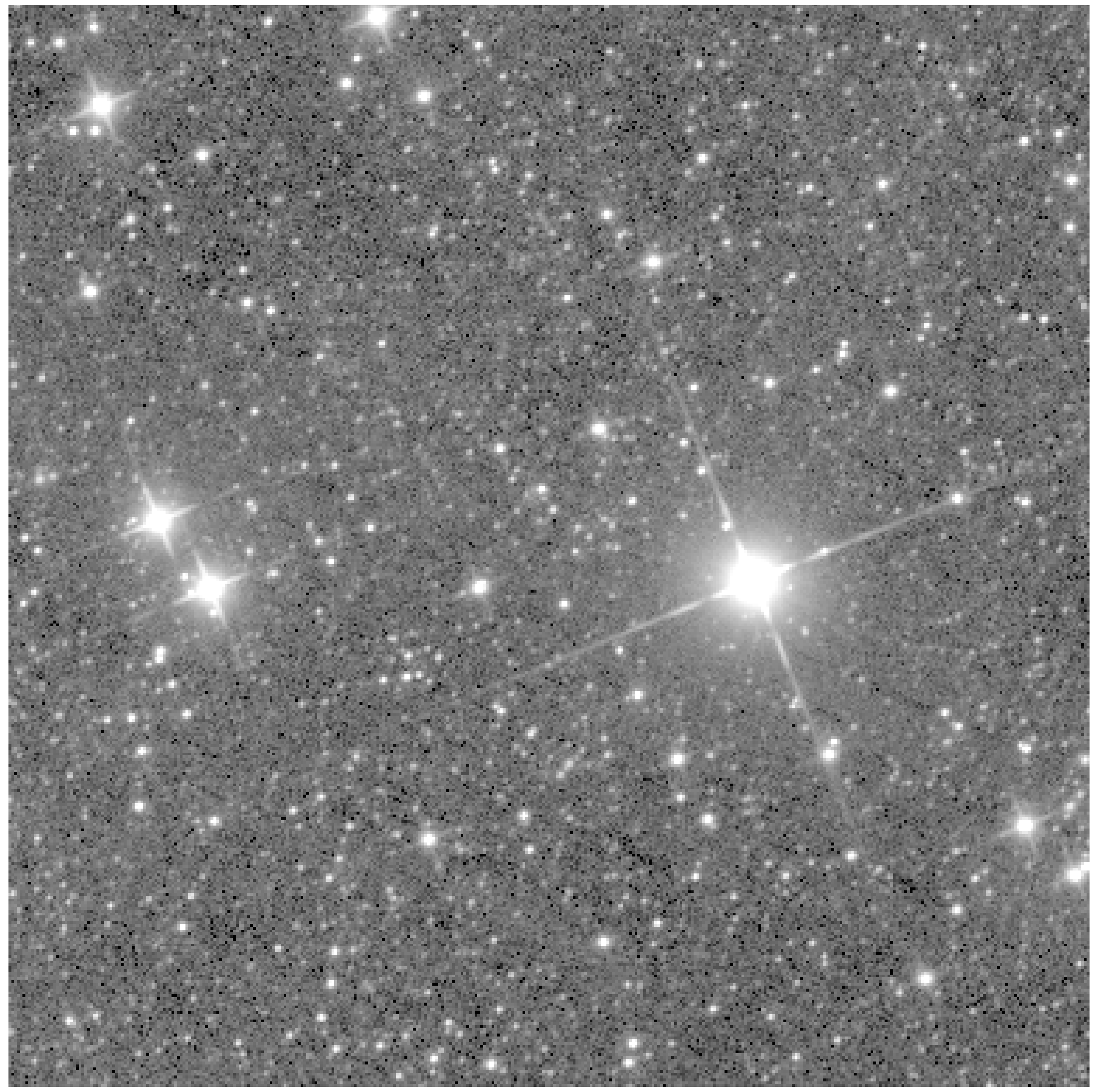
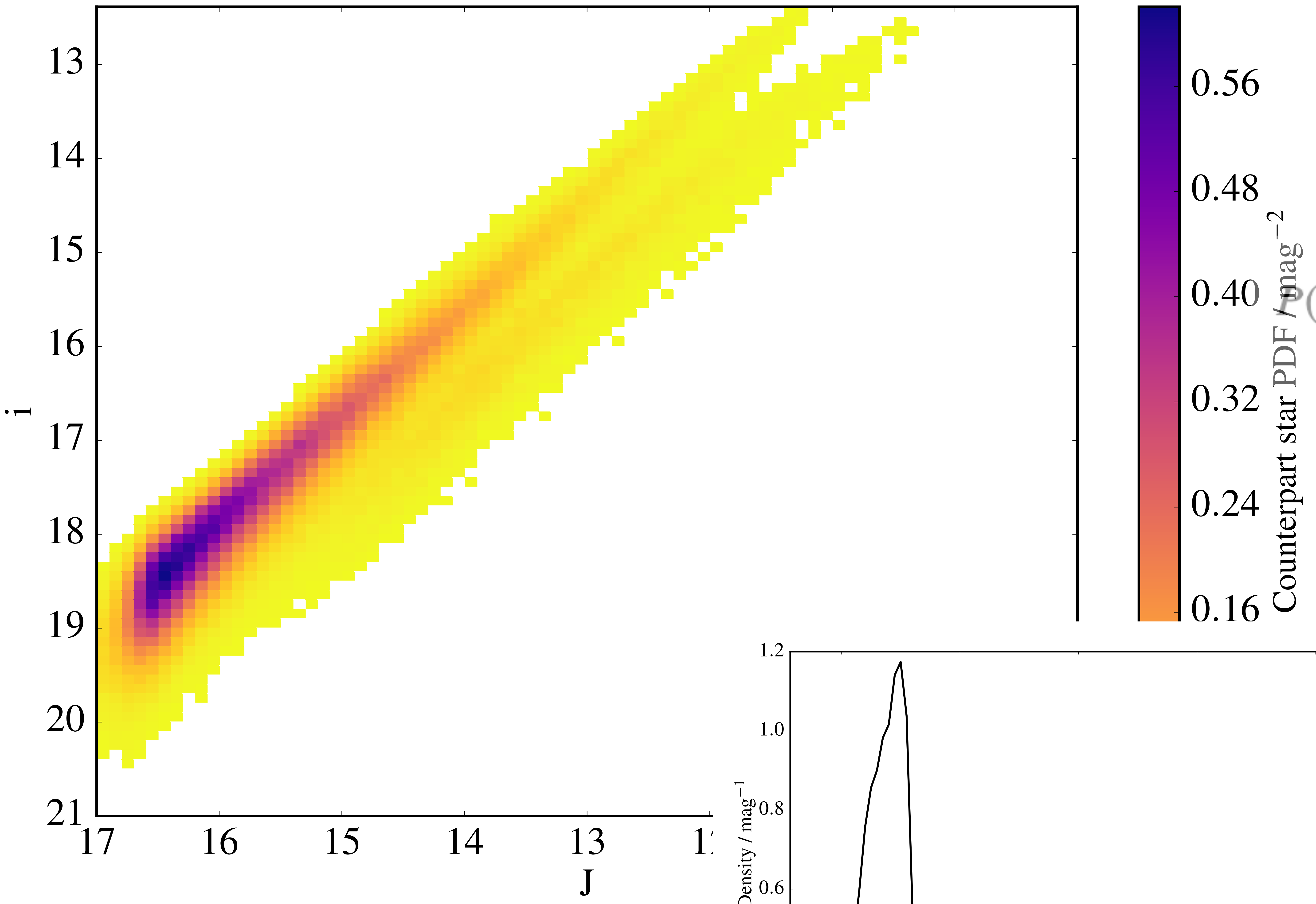


$$Z_{c\gamma} \cdot c_\gamma(m_\phi|m_\gamma) = Z_\gamma b_\gamma(m_\phi|m_\gamma) \exp\left(A_\gamma N_\phi F_\phi(m_\phi)\right) - \left(1 - Z_{c\gamma} C_\gamma(m_\phi|m_\gamma)\right) A_\gamma N_\phi f_\phi(m_\phi).$$

Naylor, Broos, & Feigelson (2013); Wilson & Naylor (2018a)

Tom J Wilson @onoddil

Photometry: The Counterpart Distribution



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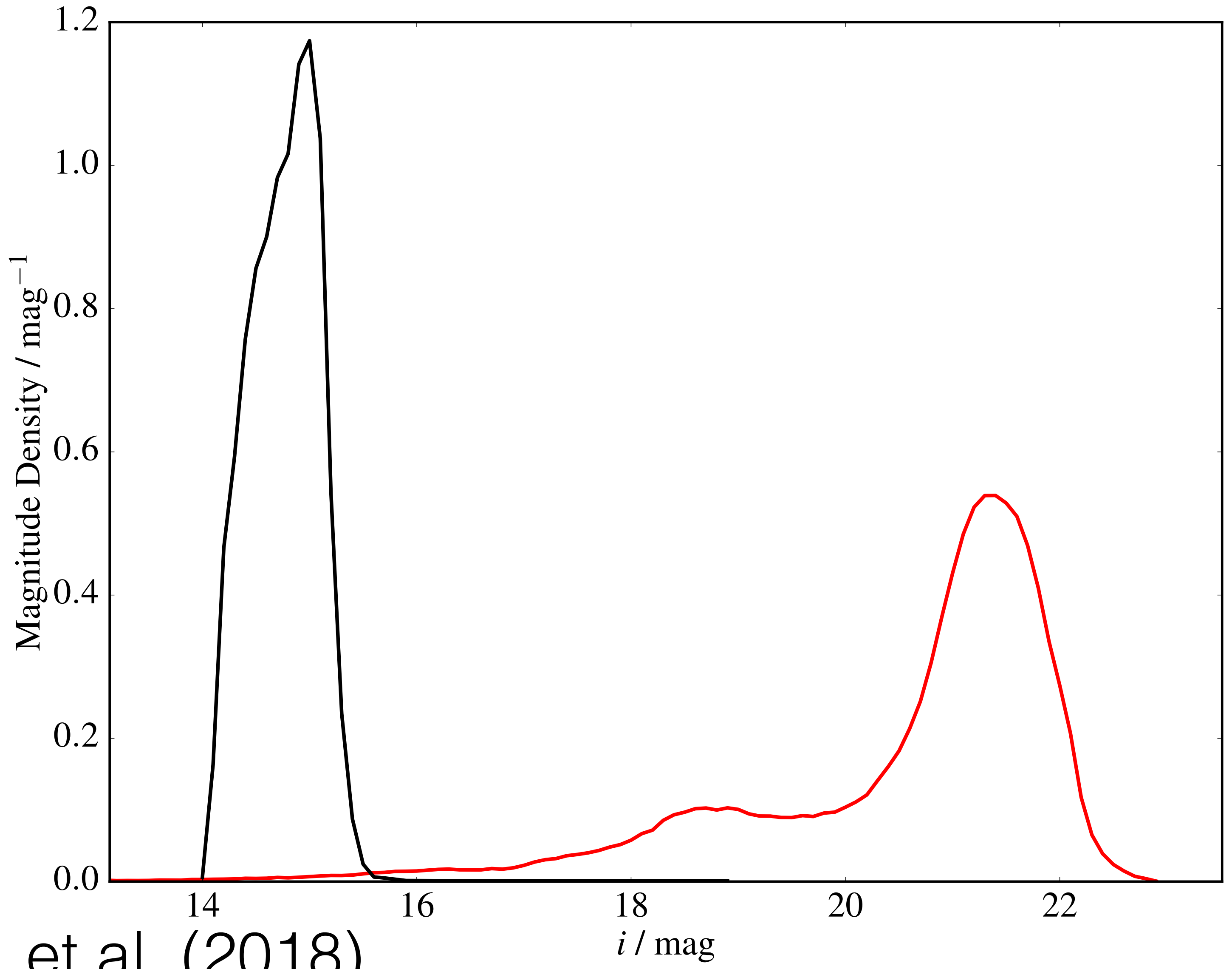
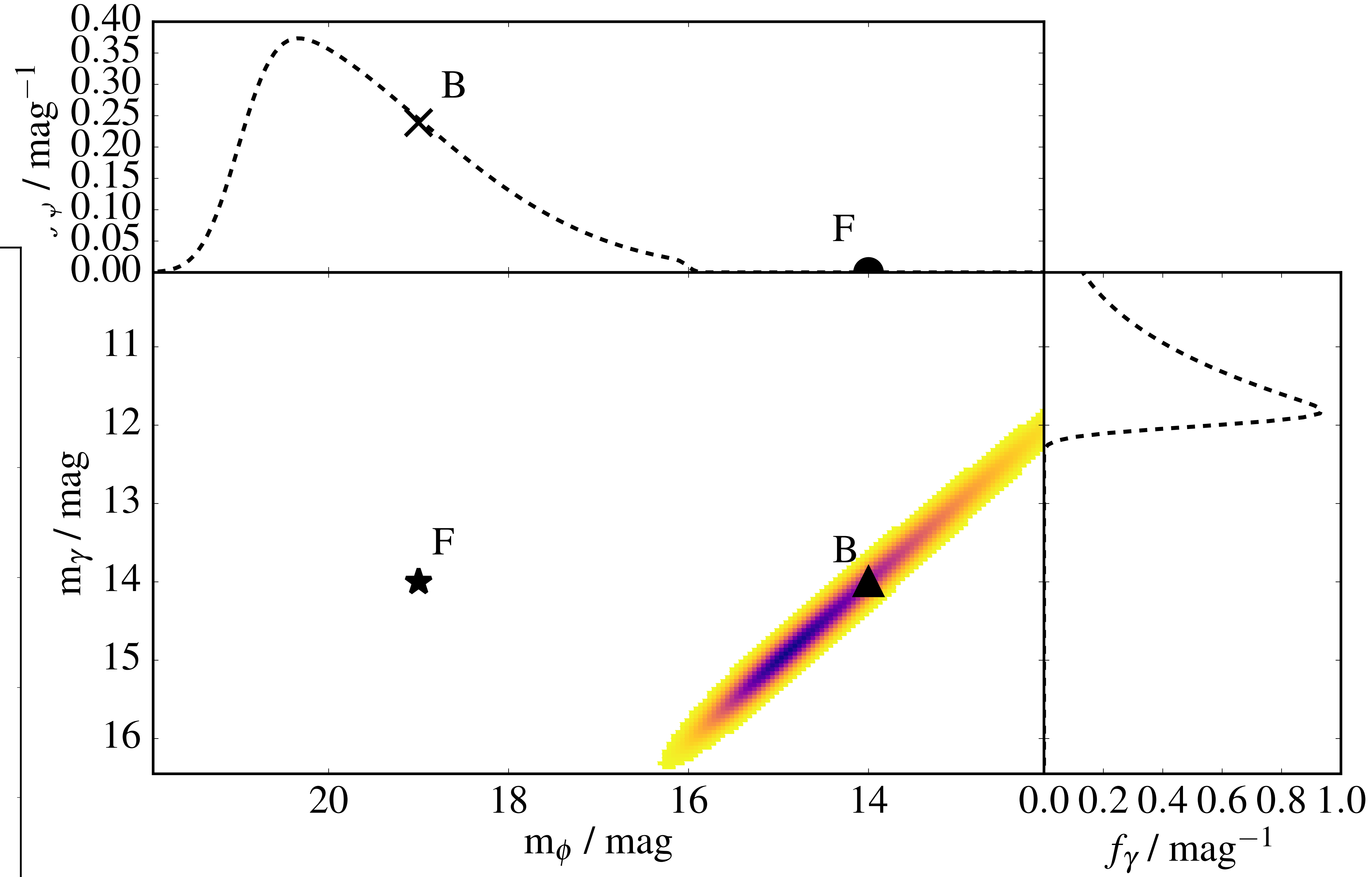




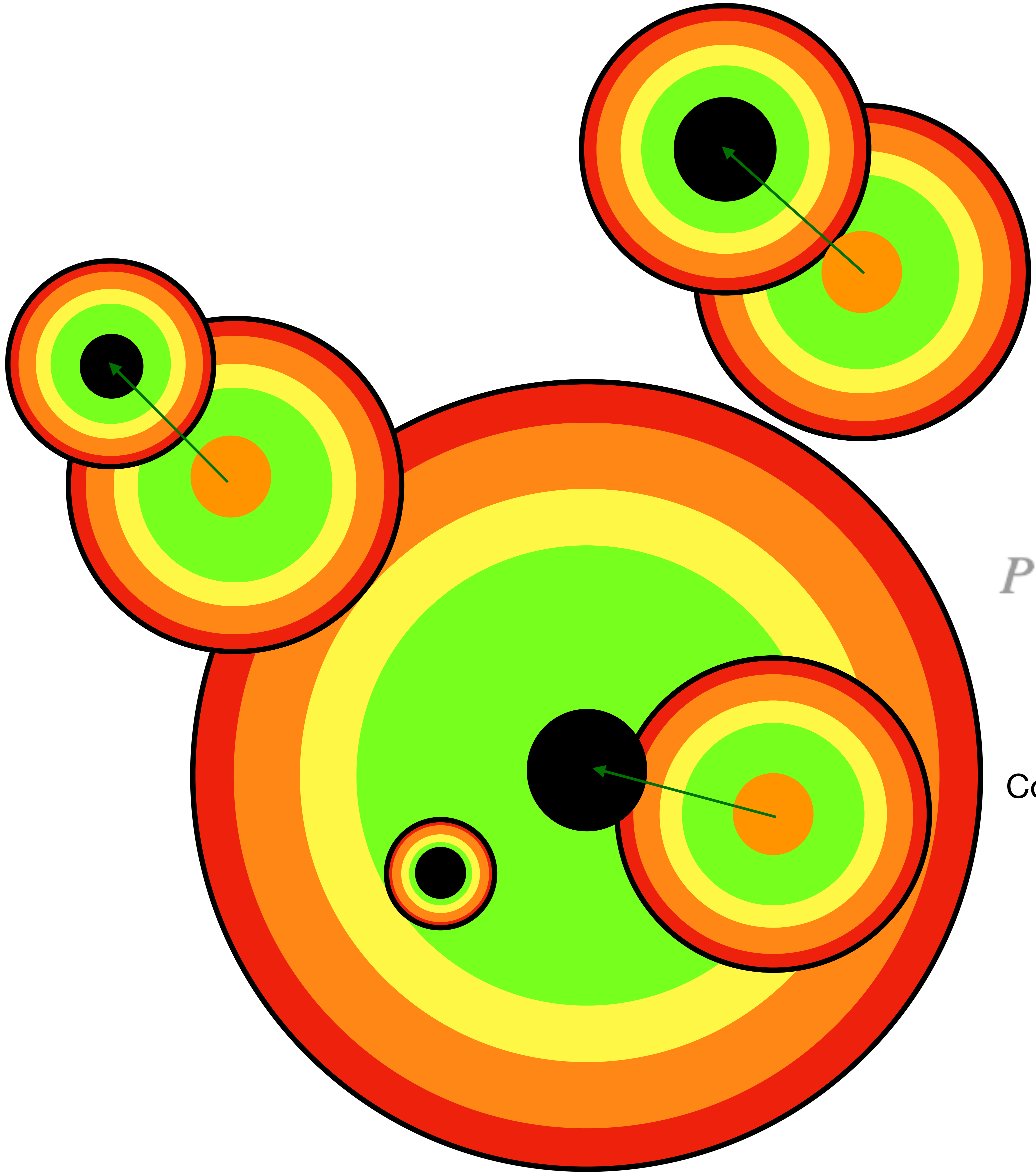
Wilson & Naylor (2018a)

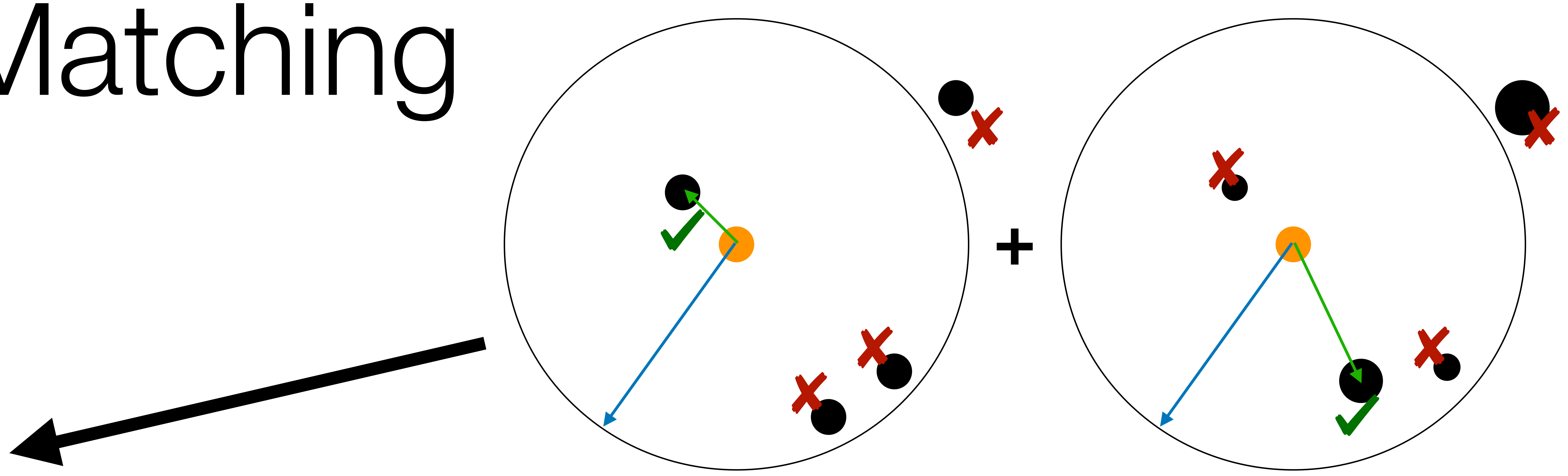
IPHAS - Barentsen et al. (2014)

Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

Tom J Wilson @onoddil

Probabilistic Cross-Matching





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Wilson & Naylor (2018a)

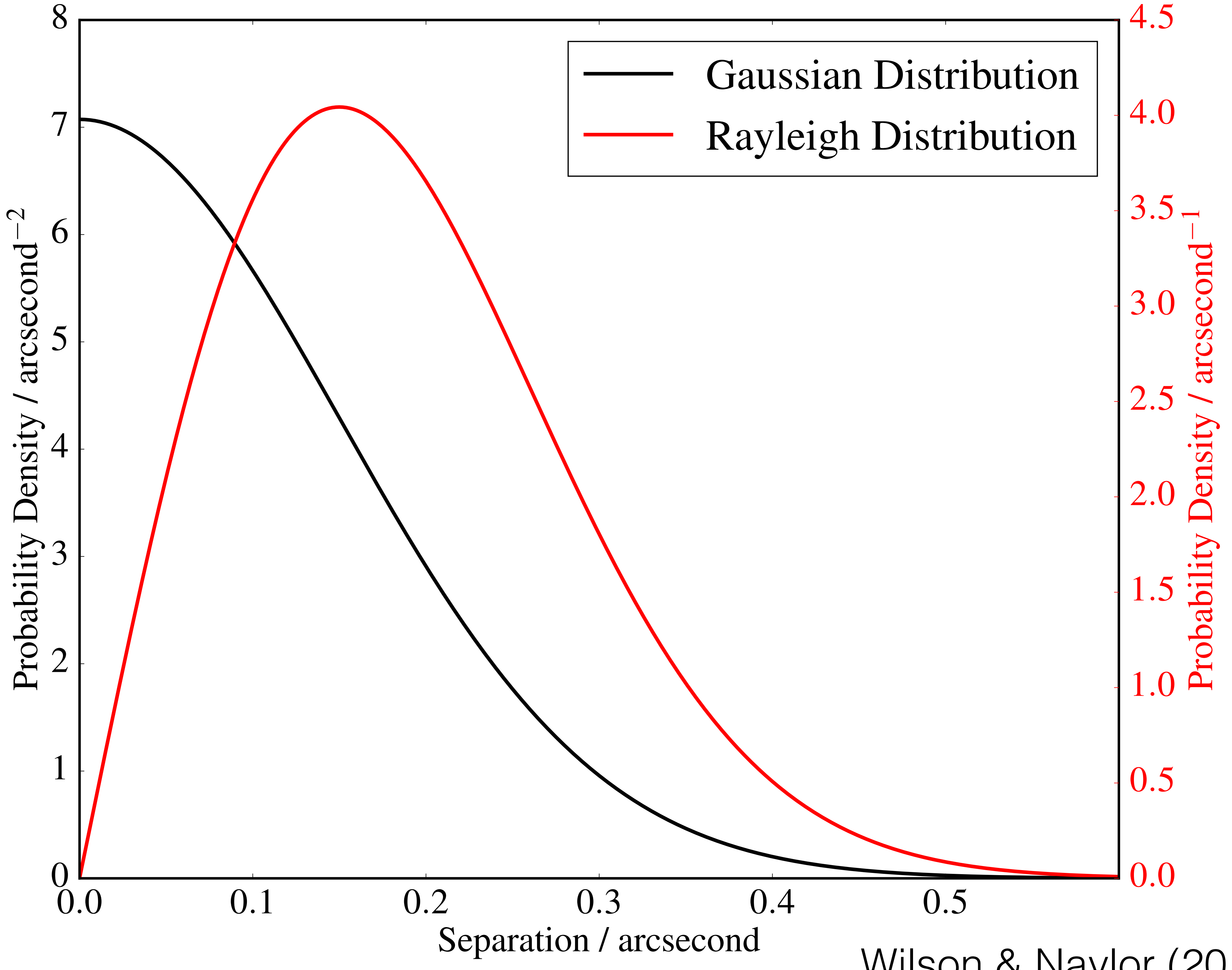
Tom J Wilson @onoddil

The *Astrometric Uncertainty* Function

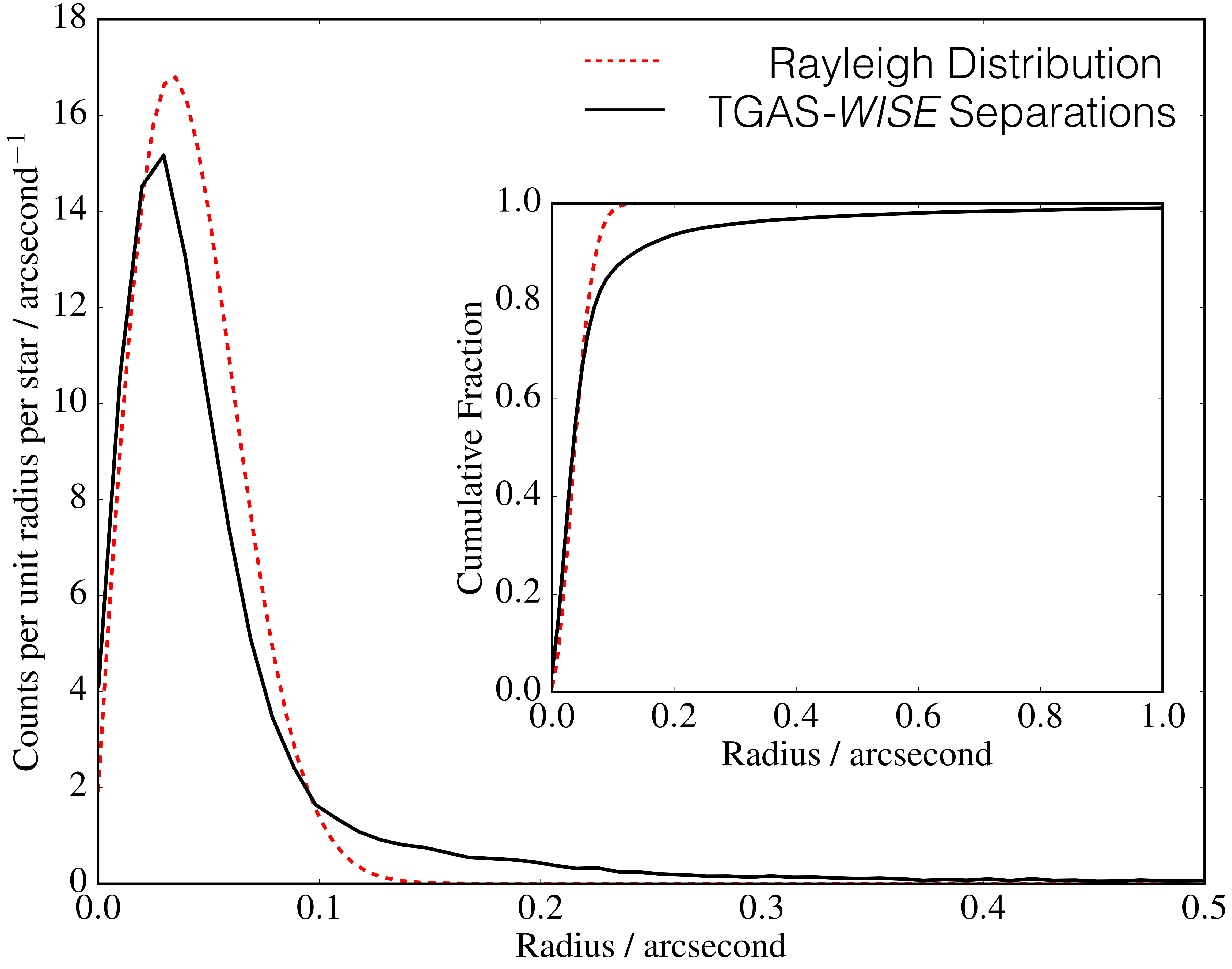
$$g(x, y, \sigma) = (2\pi\sigma^2)^{-1}\exp\left(-\frac{1}{2}\frac{x^2+y^2}{\sigma^2}\right)$$

$$\downarrow$$

$$g(r, \sigma) = \frac{r}{\sigma^2}\exp\left(-\frac{1}{2}\frac{r^2}{\sigma^2}\right)$$



Wilson & Naylor (2017)
WISE - Wright et al. (2010)



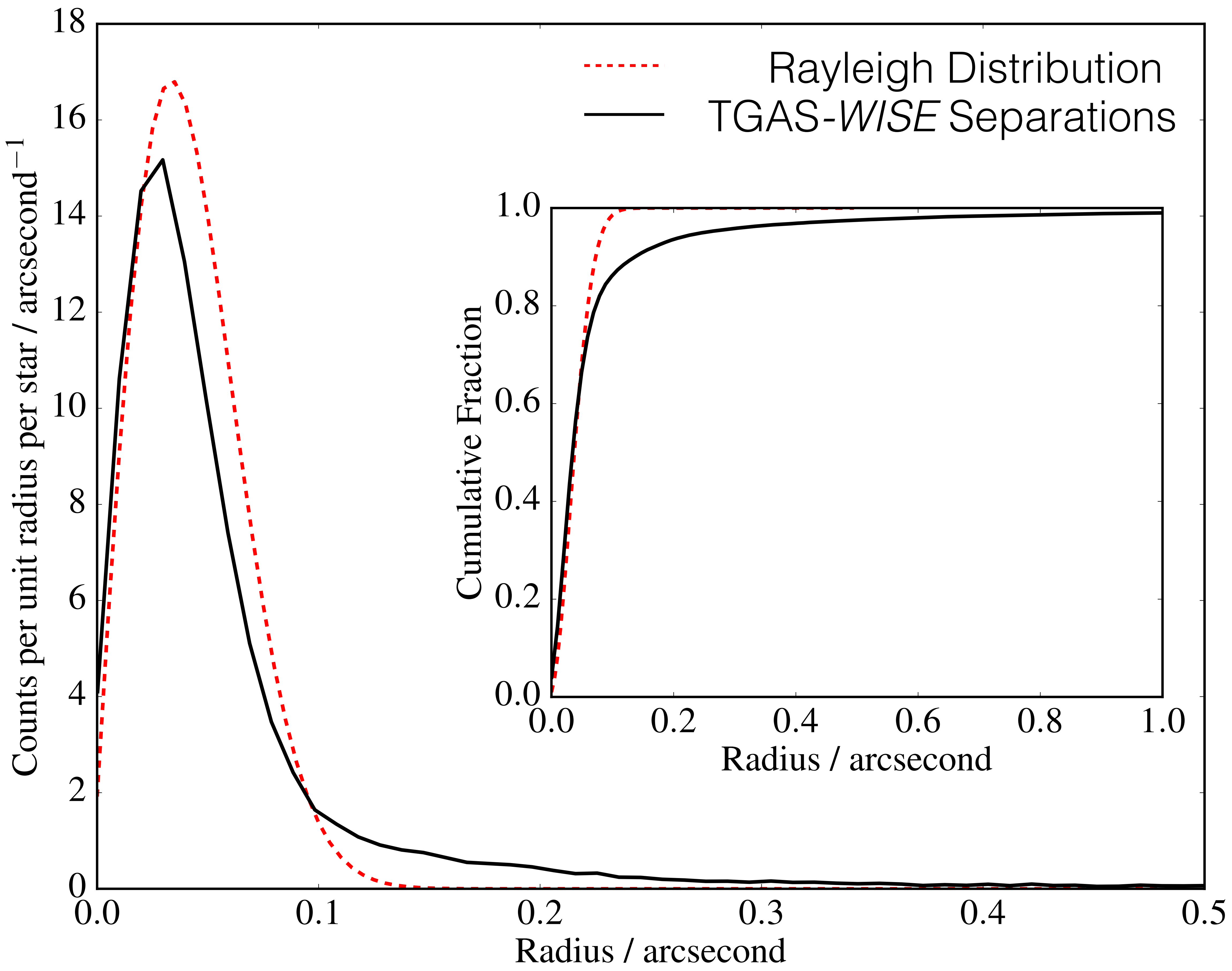
TGAS - Michalik, Lindegren, & Hobbs (2015)
Gaia - Gaia Collaboration, Brown A. G. A., et al. (2016)

Tom J Wilson @onoddil

The *Astrometric Uncertainty* Function

Reasons for large separations:

1) proper motions (e.g. AllWISE Supplement 6.4, Cutri et al. 2012) — no, TGAS provided for all sources
2) false matches — no, 0.1% chance of random match within 0.5 arcseconds
3) What else could it be?



Wilson & Naylor (2017)
WISE - Wright et al. (2010)
TGAS - Michalik, Lindegren, & Hobbs (2015)
Gaia - Gaia Collaboration, Brown A. G. A., et al. (2016)

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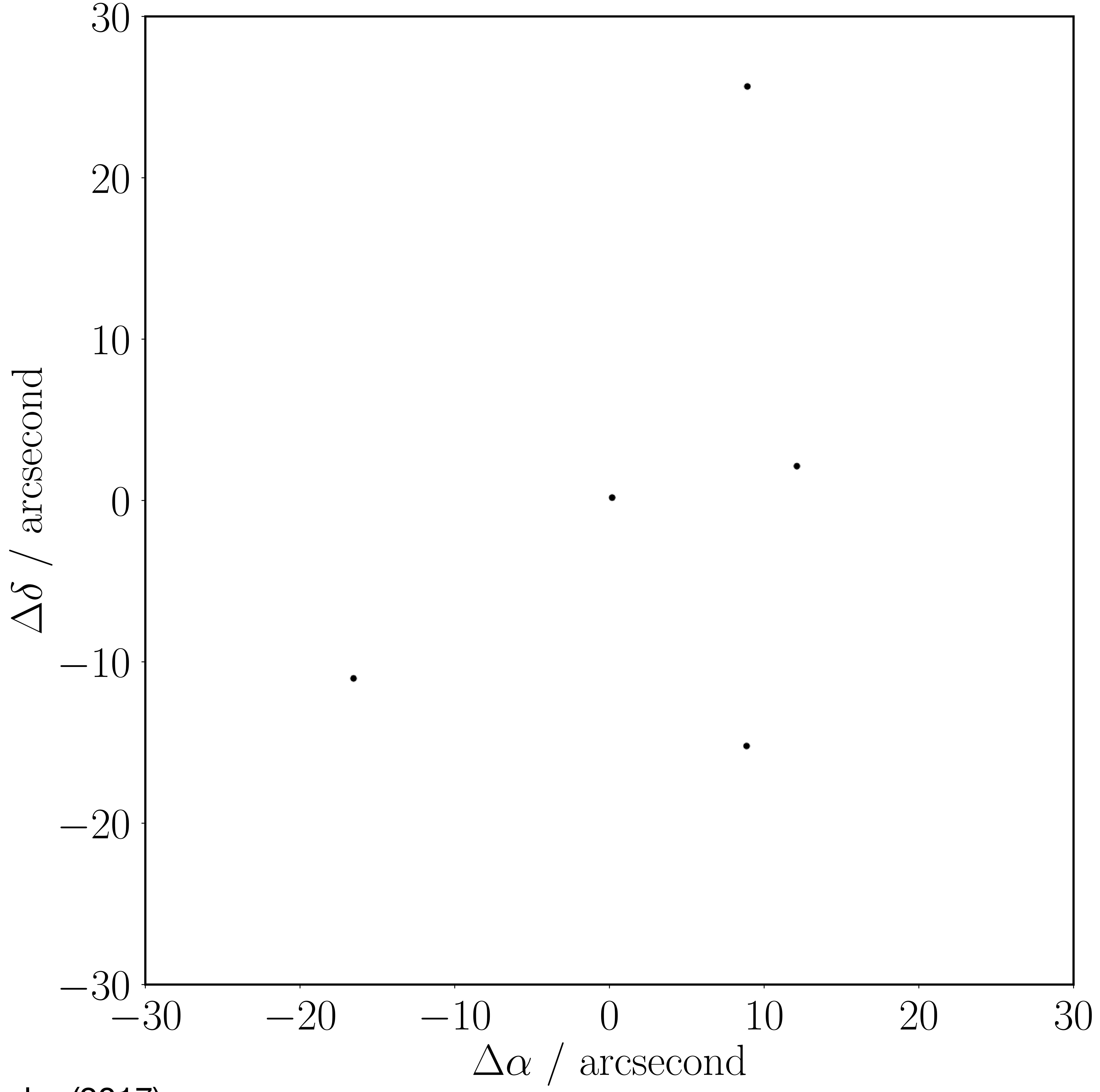
The AUF: Crowding



Wilson & Naylor (2017)

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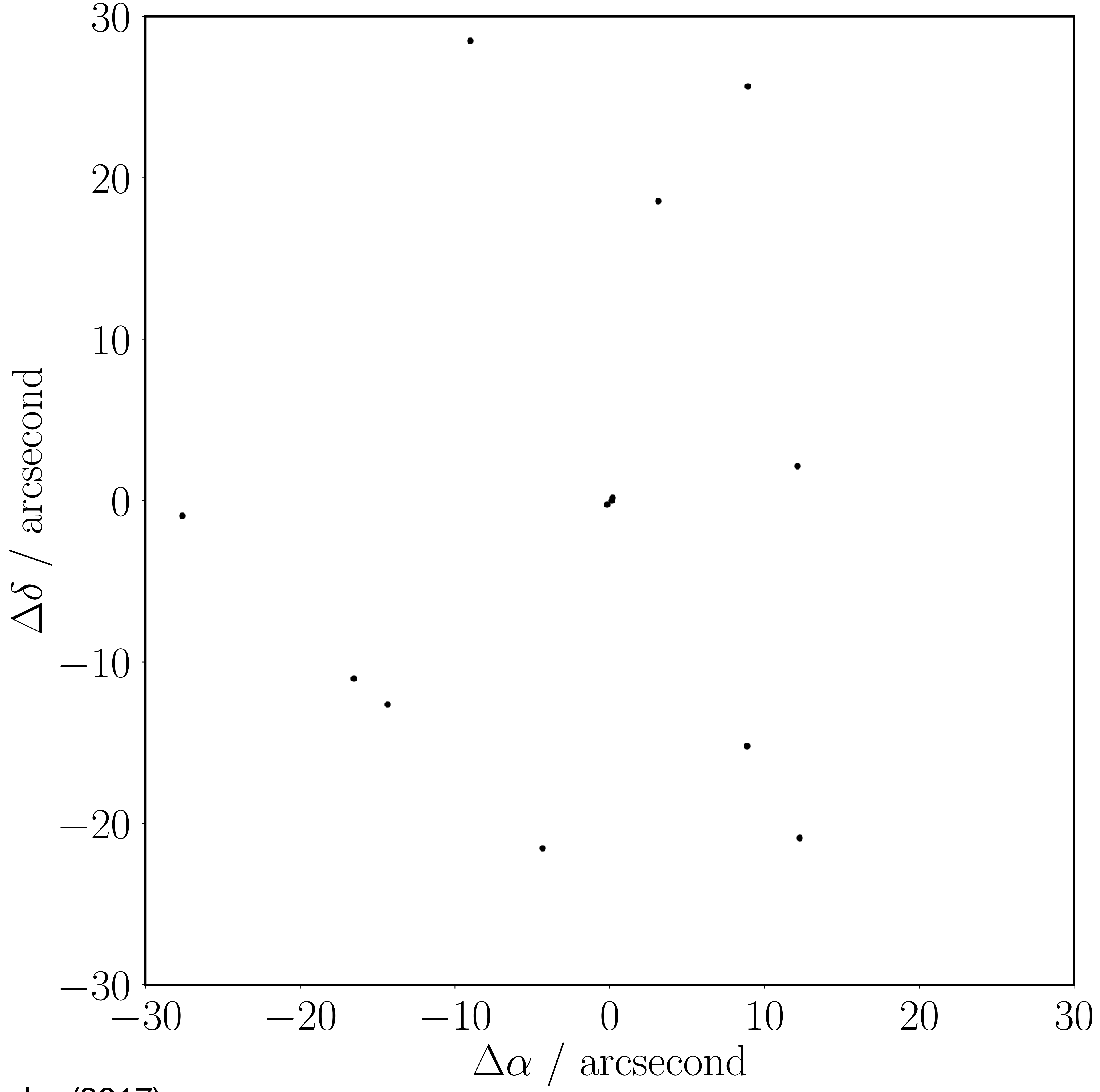

Wilson & Naylor (2017)

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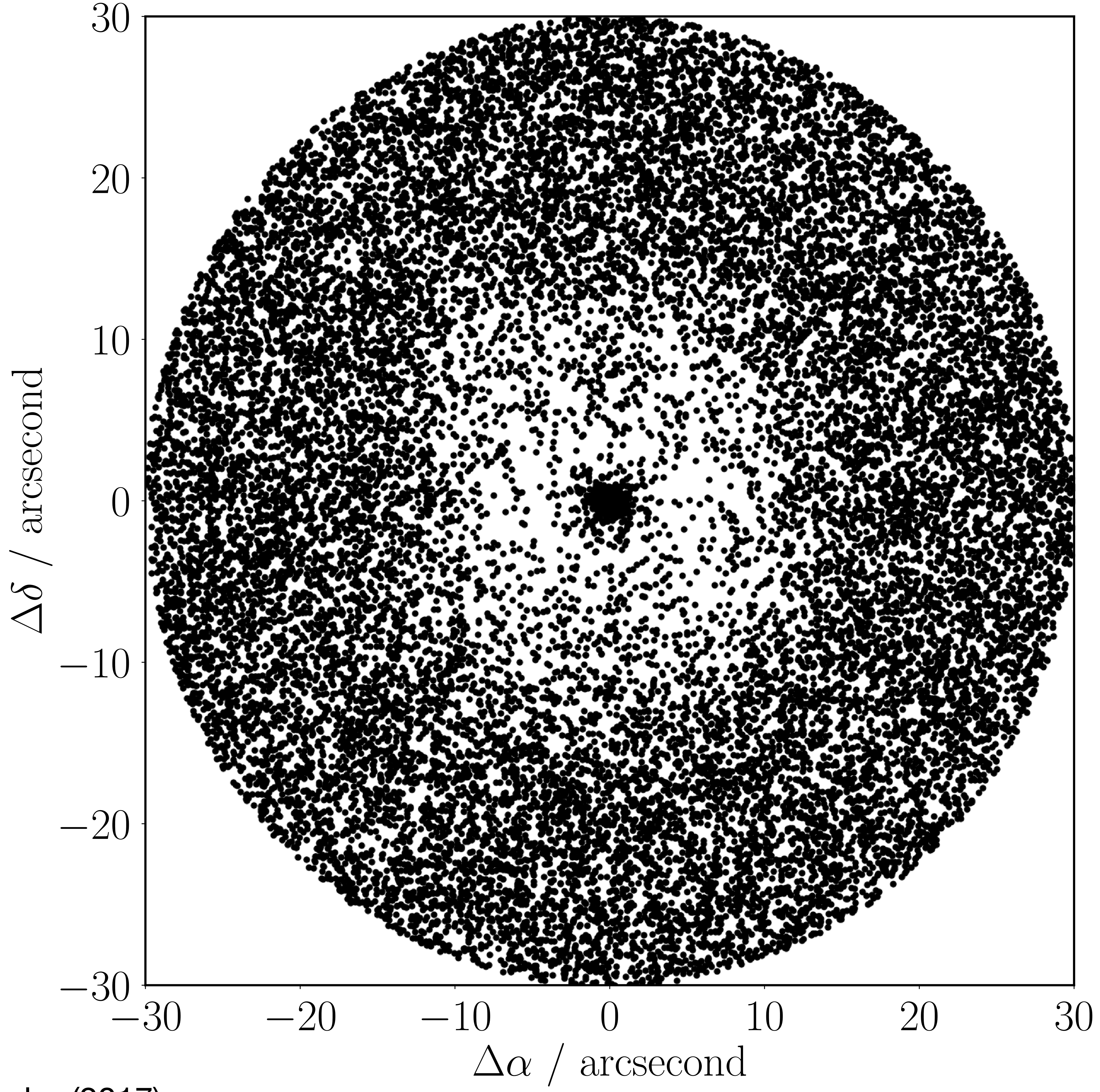

Wilson & Naylor (2017)

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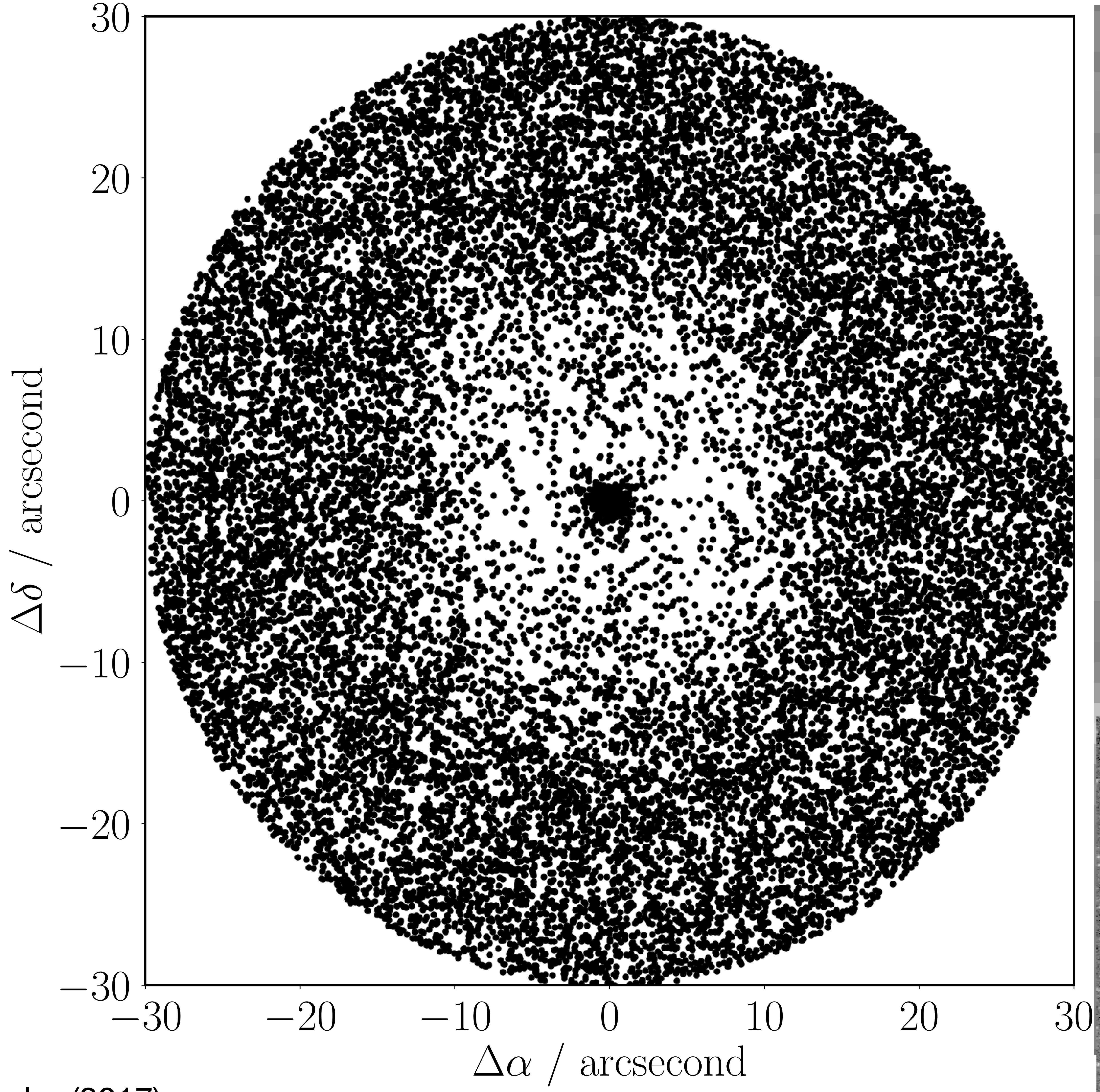
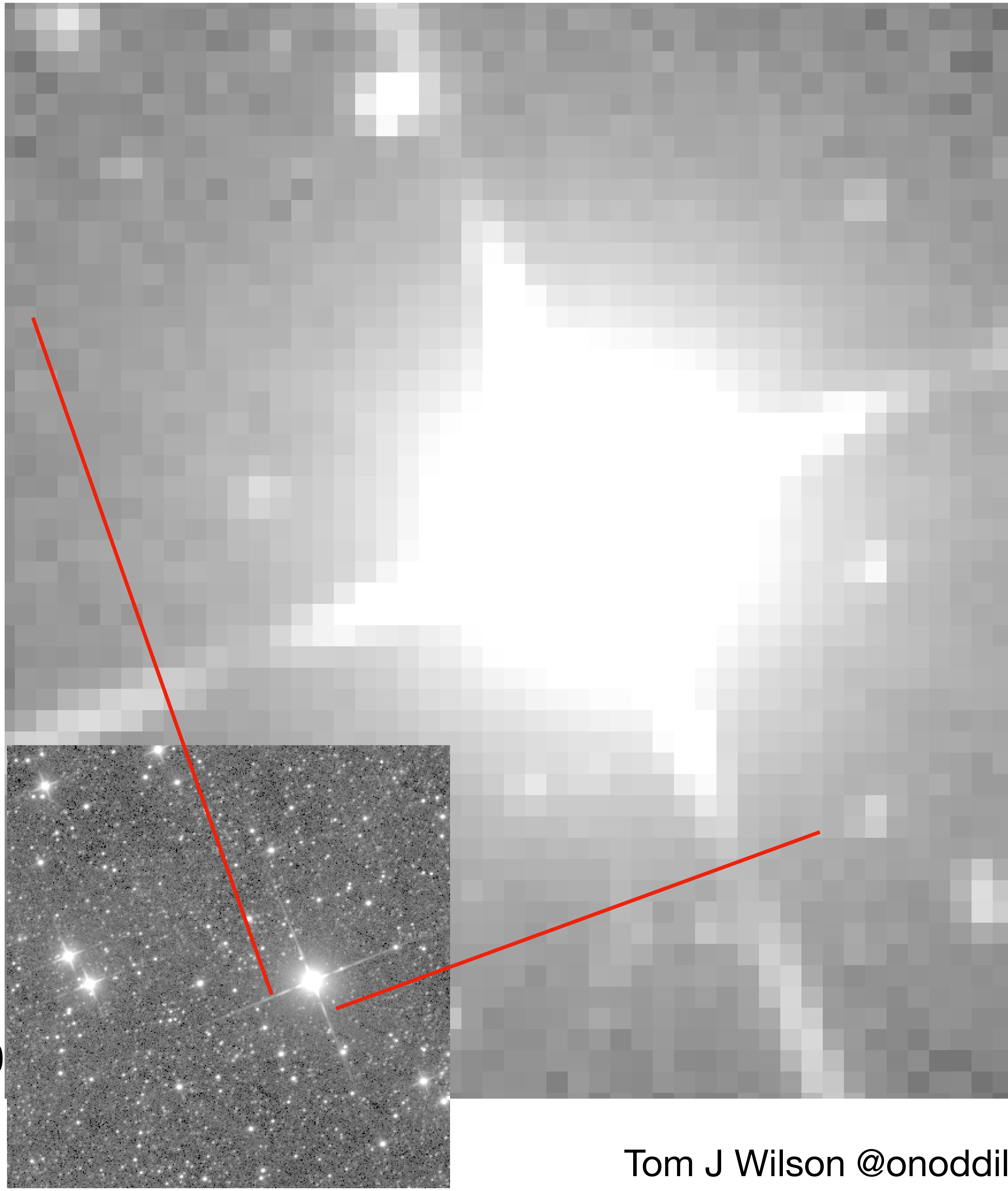


Wilson & Naylor (2017)



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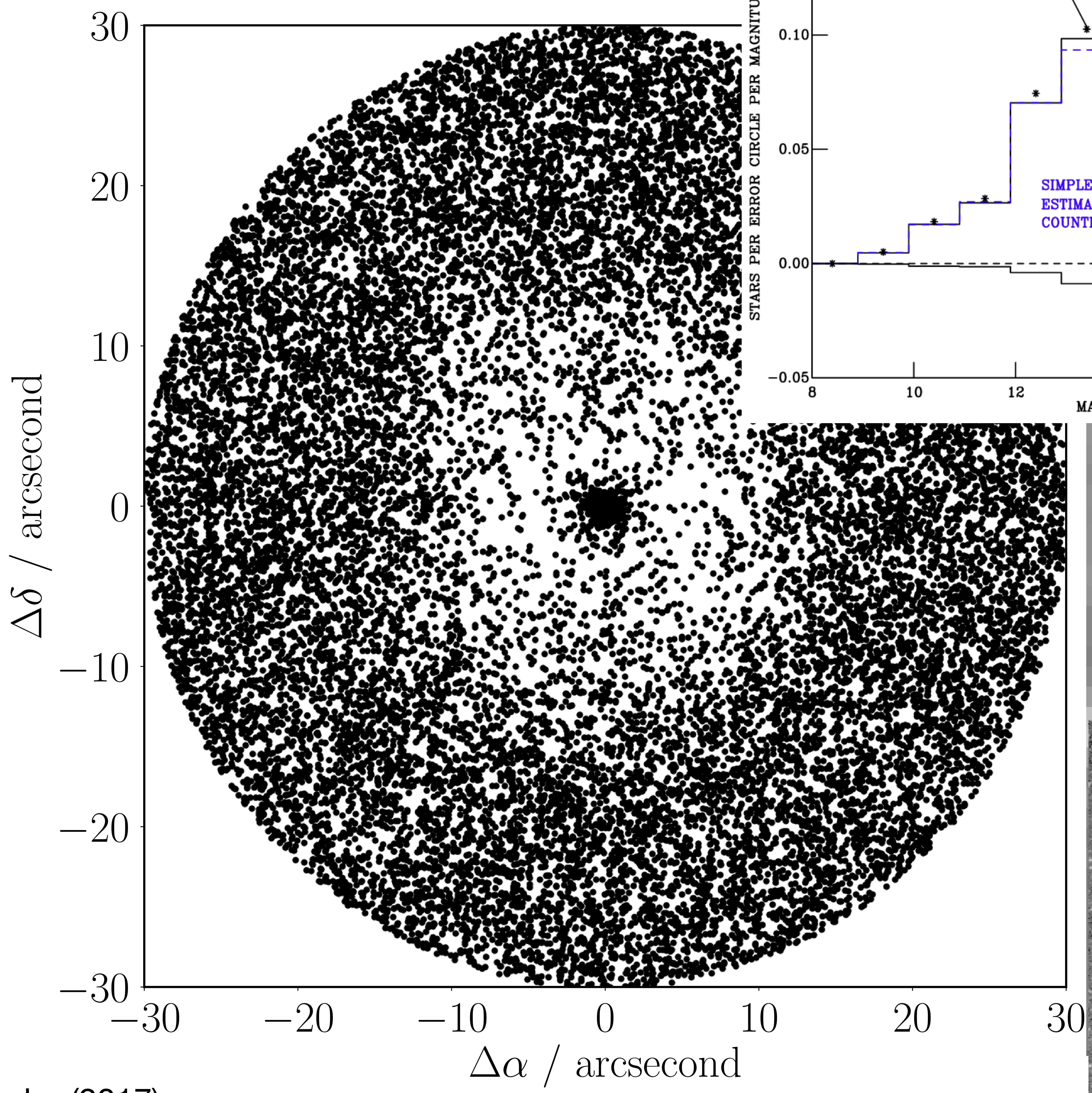

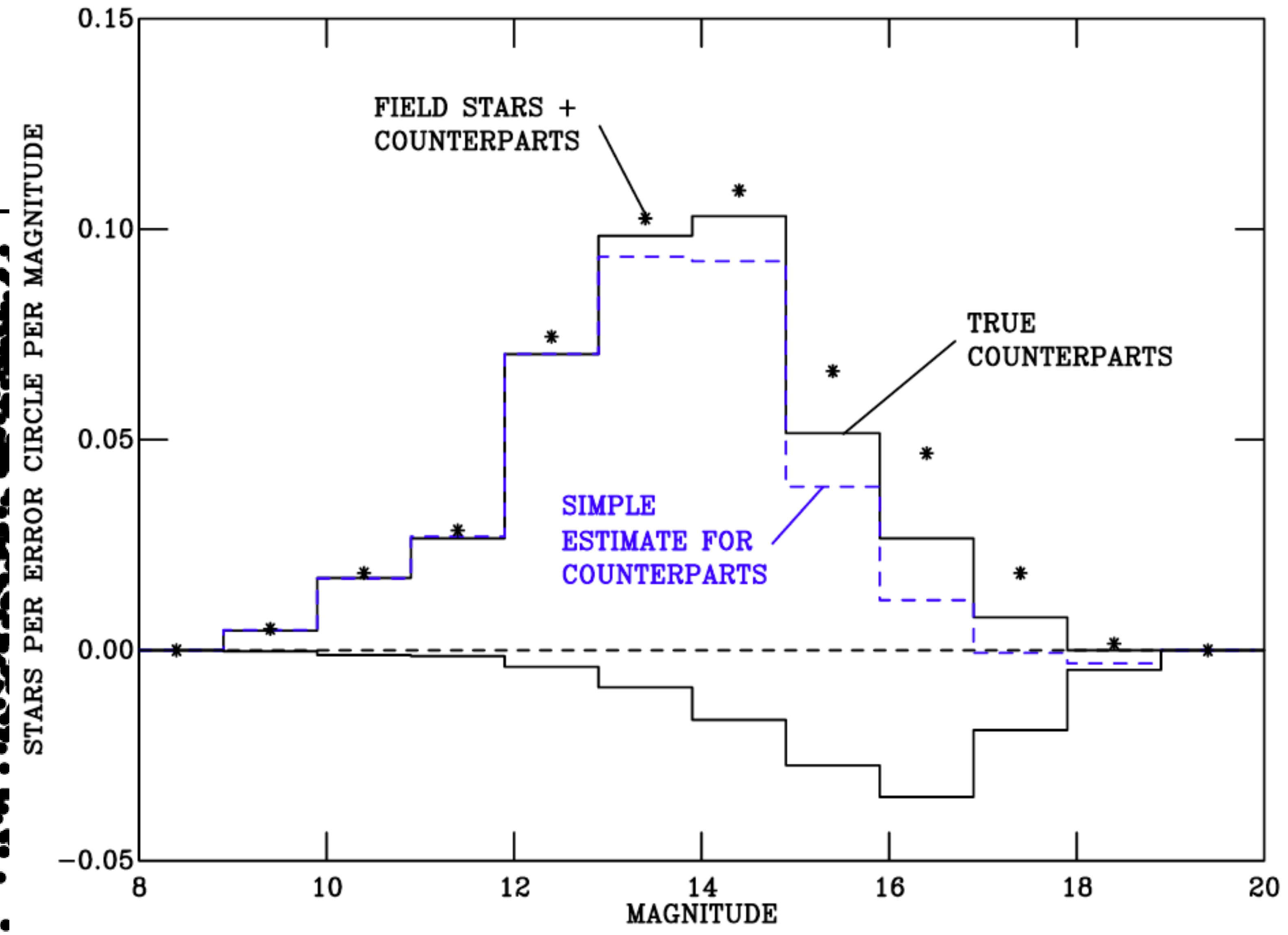

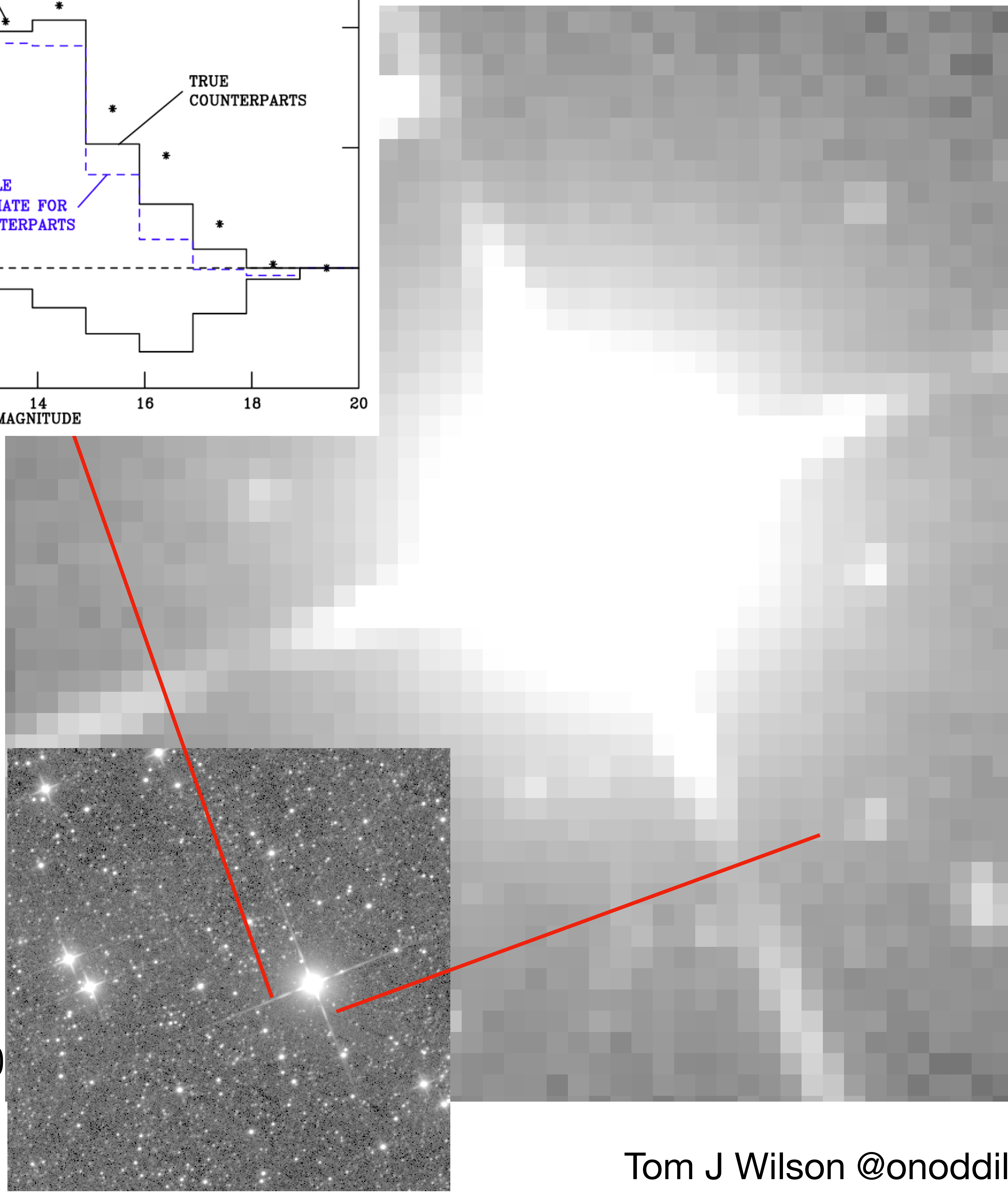

Wilson & Naylor (2017)

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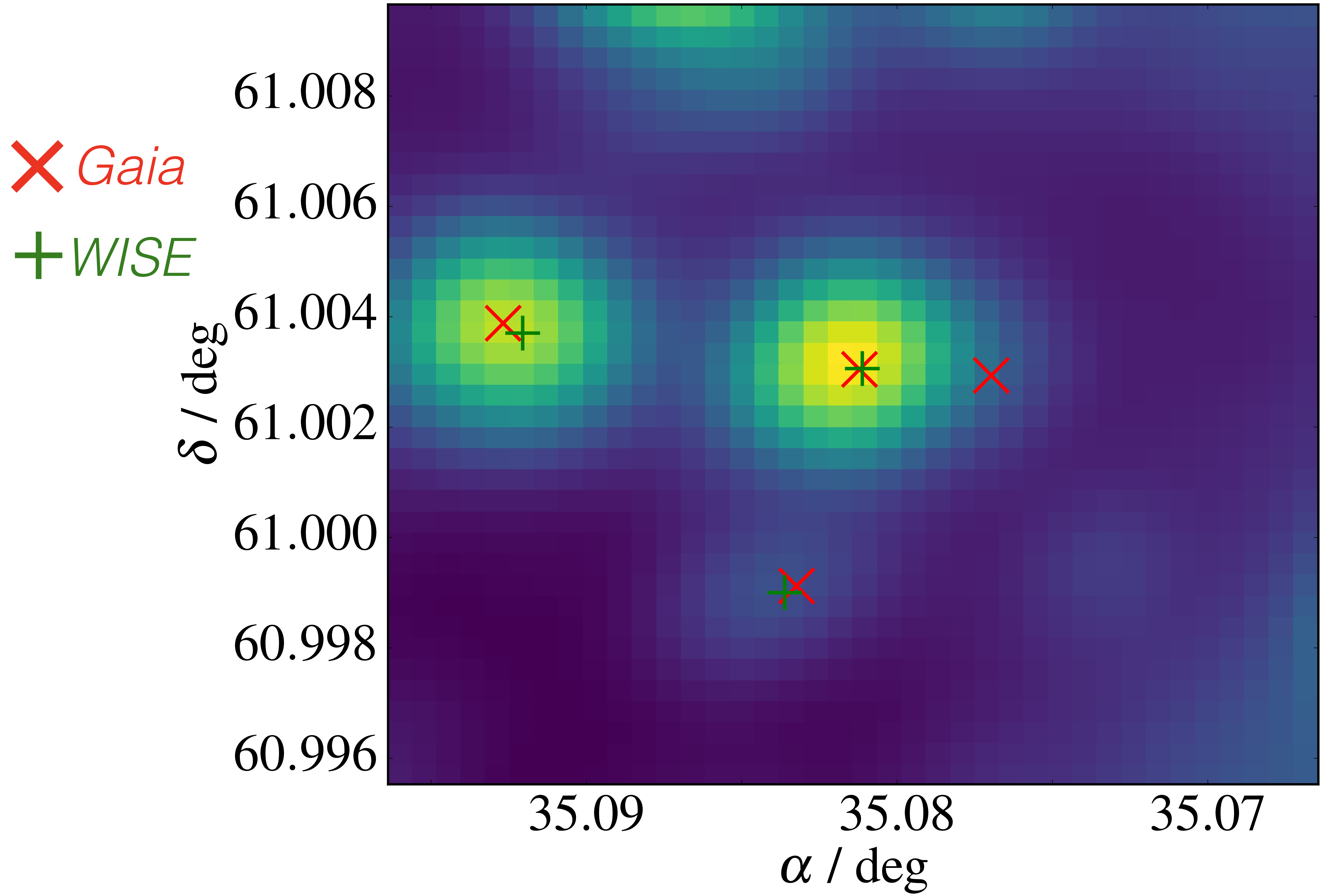
Resolving *Gaia*-*WISE* Blends



“Were the succession of stars endless... there could be absolutely no point, in all that background, at which would not exist a star.”
— Edgar Allan Poe, Eureka (1848)

Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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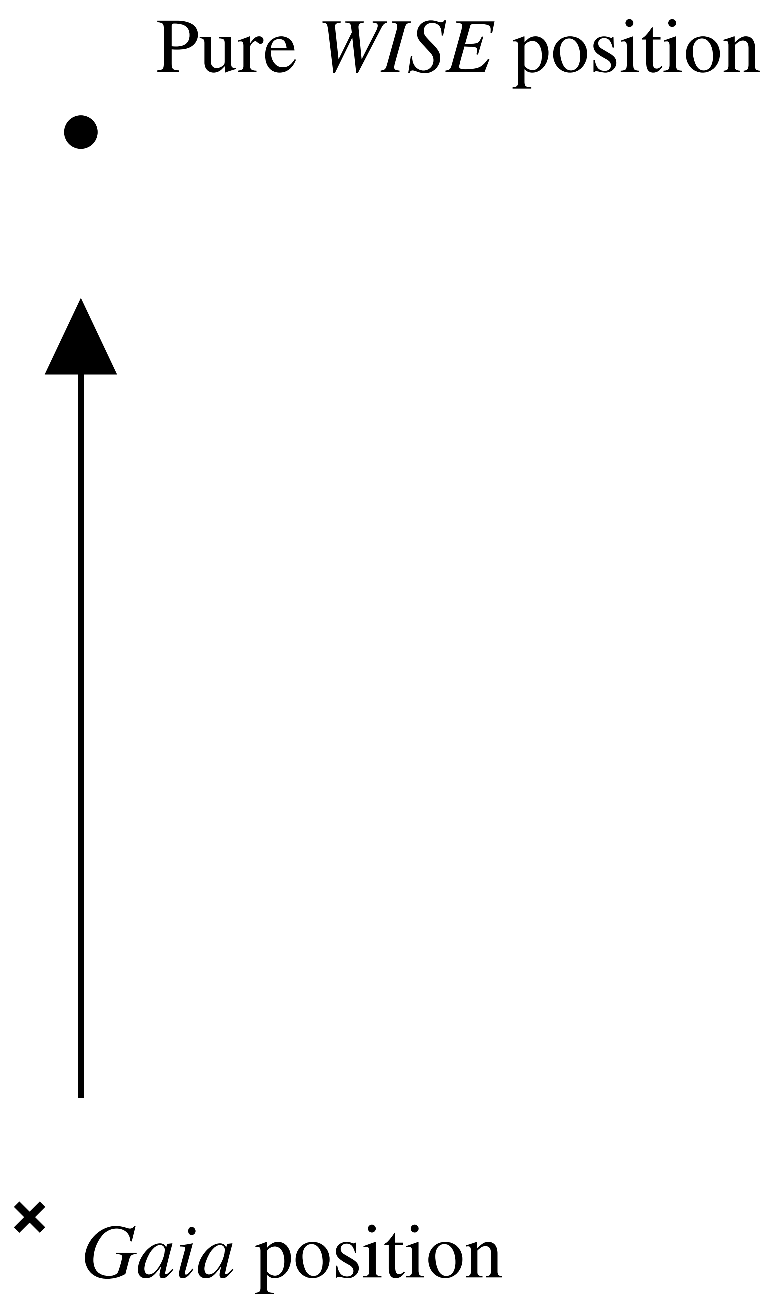
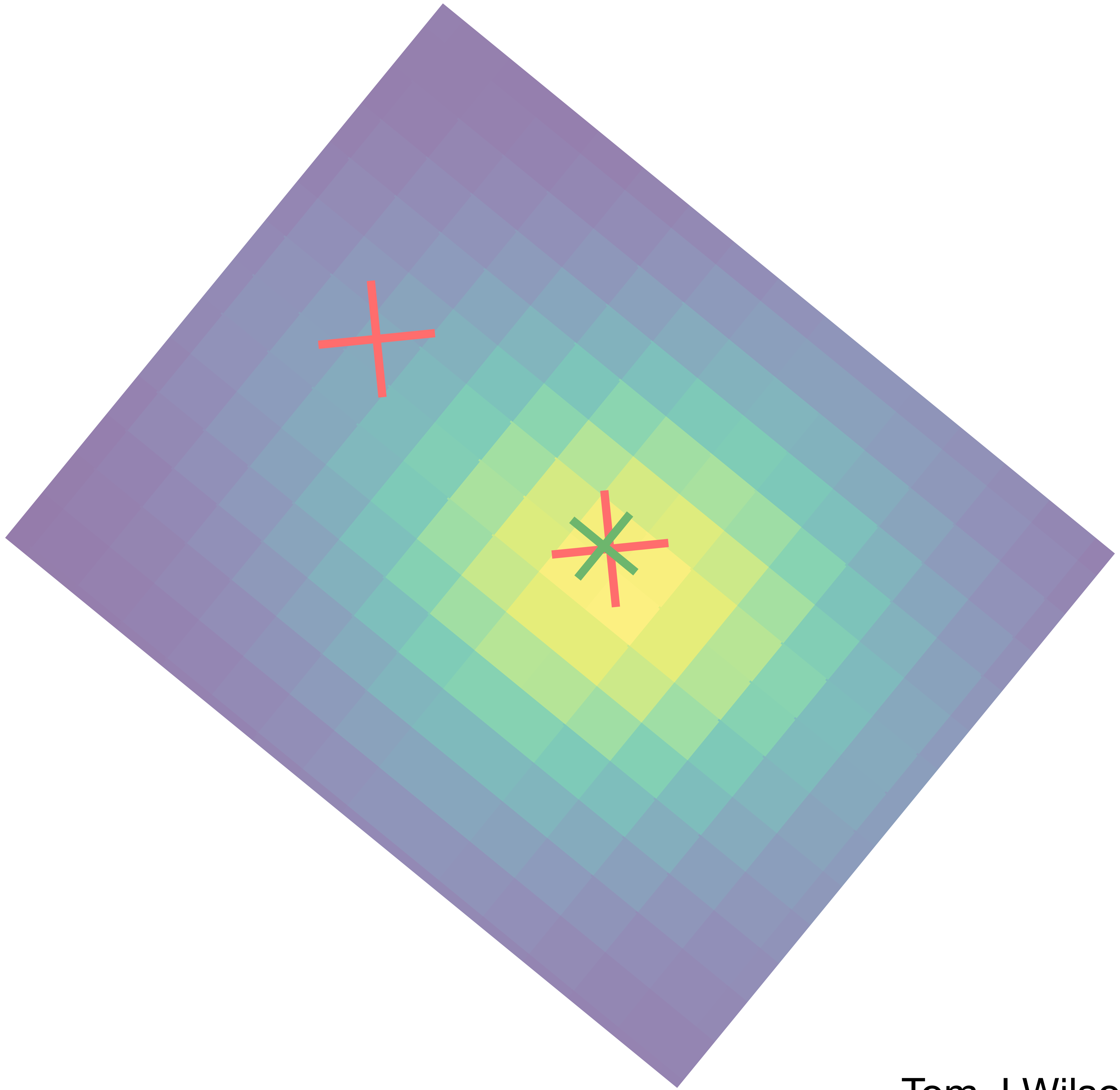
The AUF: Perturbation





Wilson & Naylor (2017)
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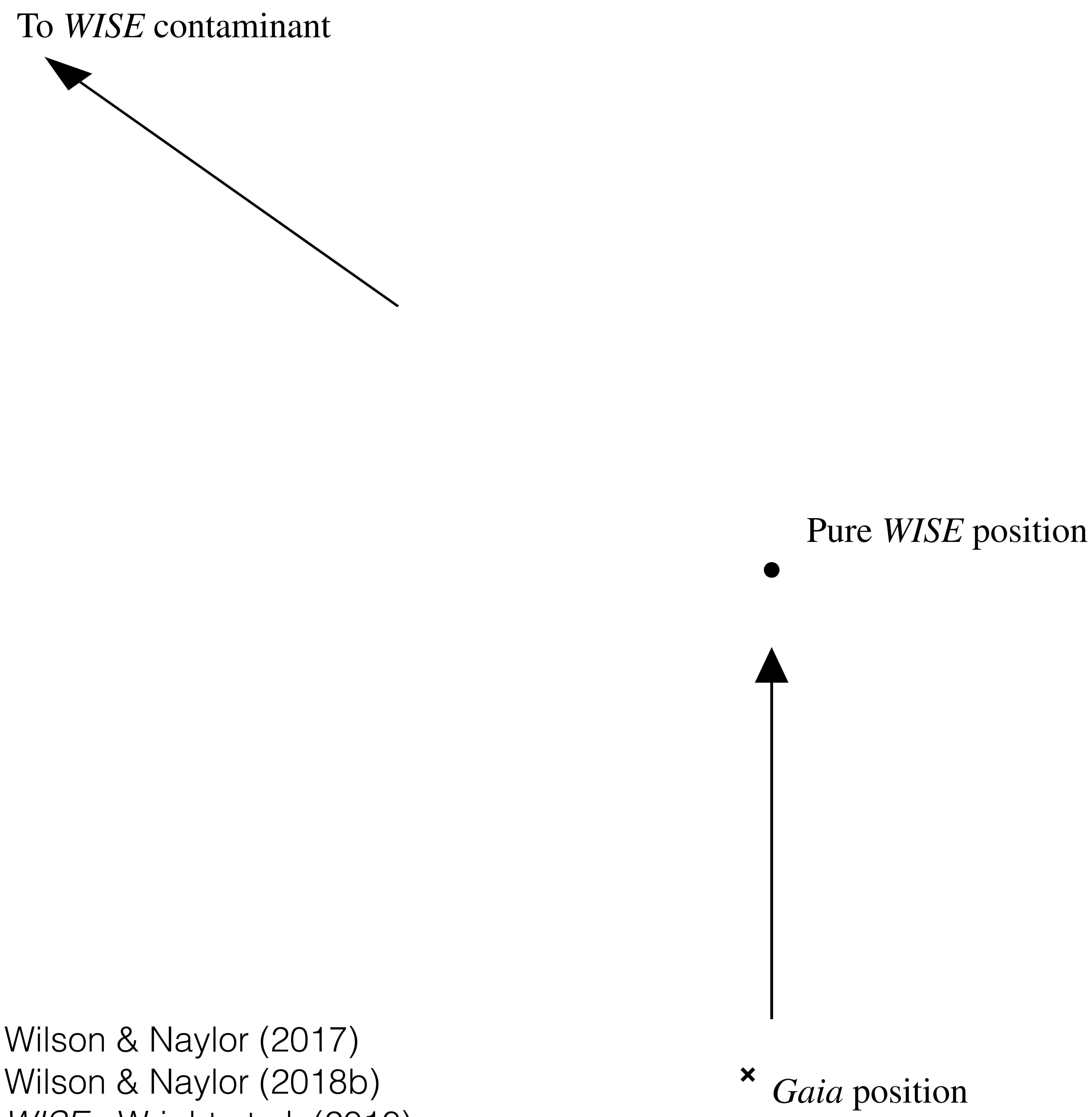

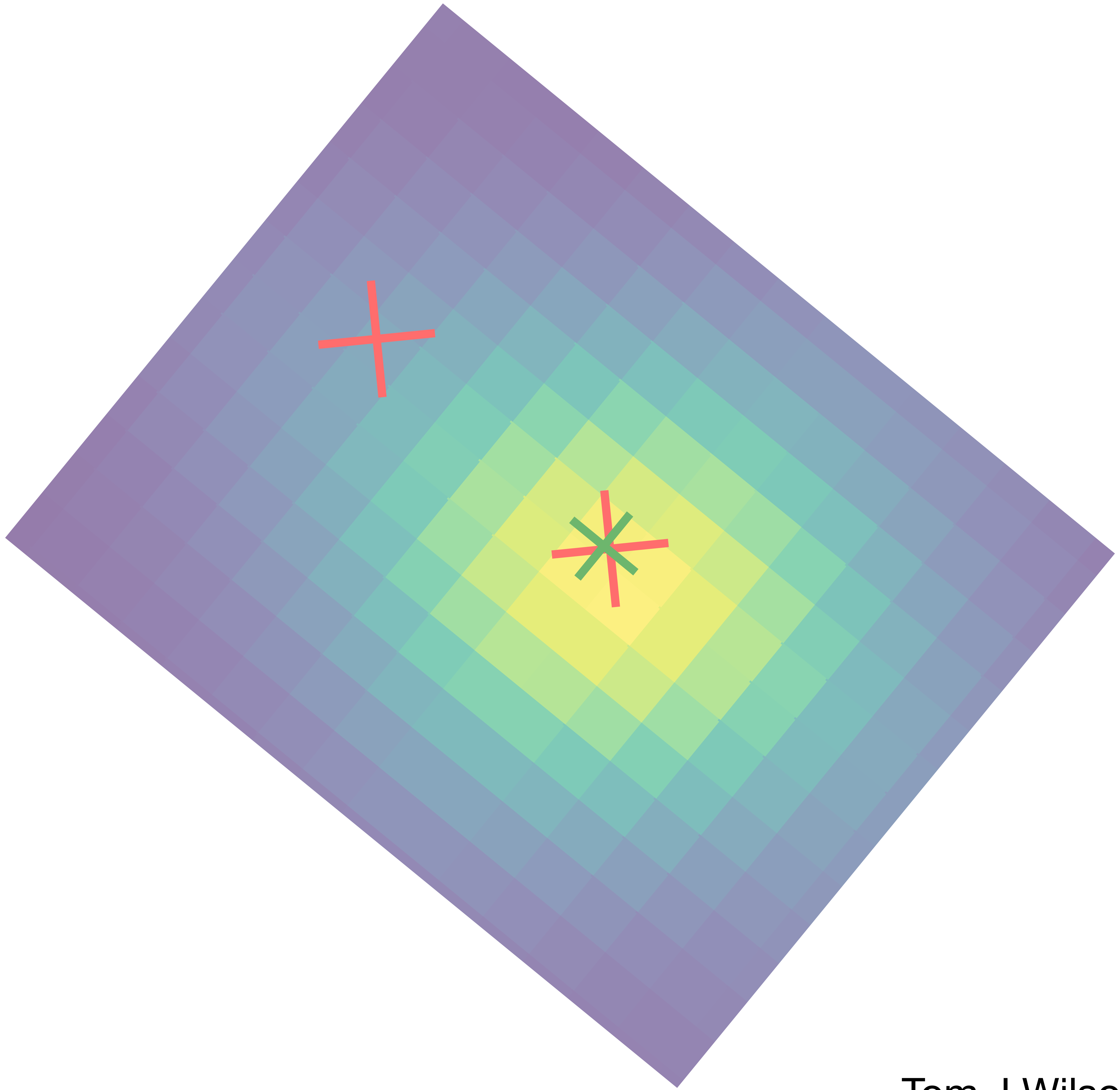

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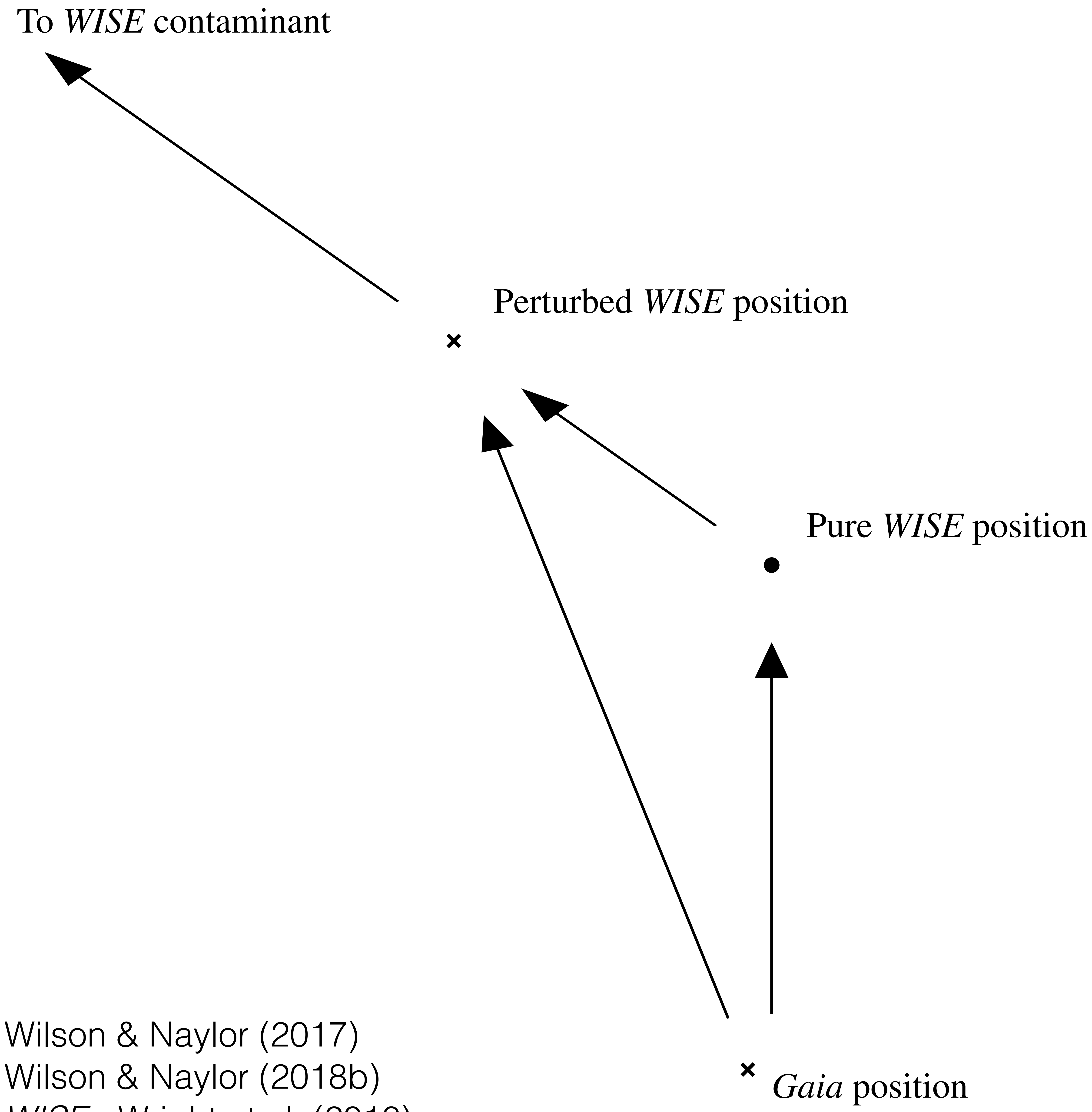

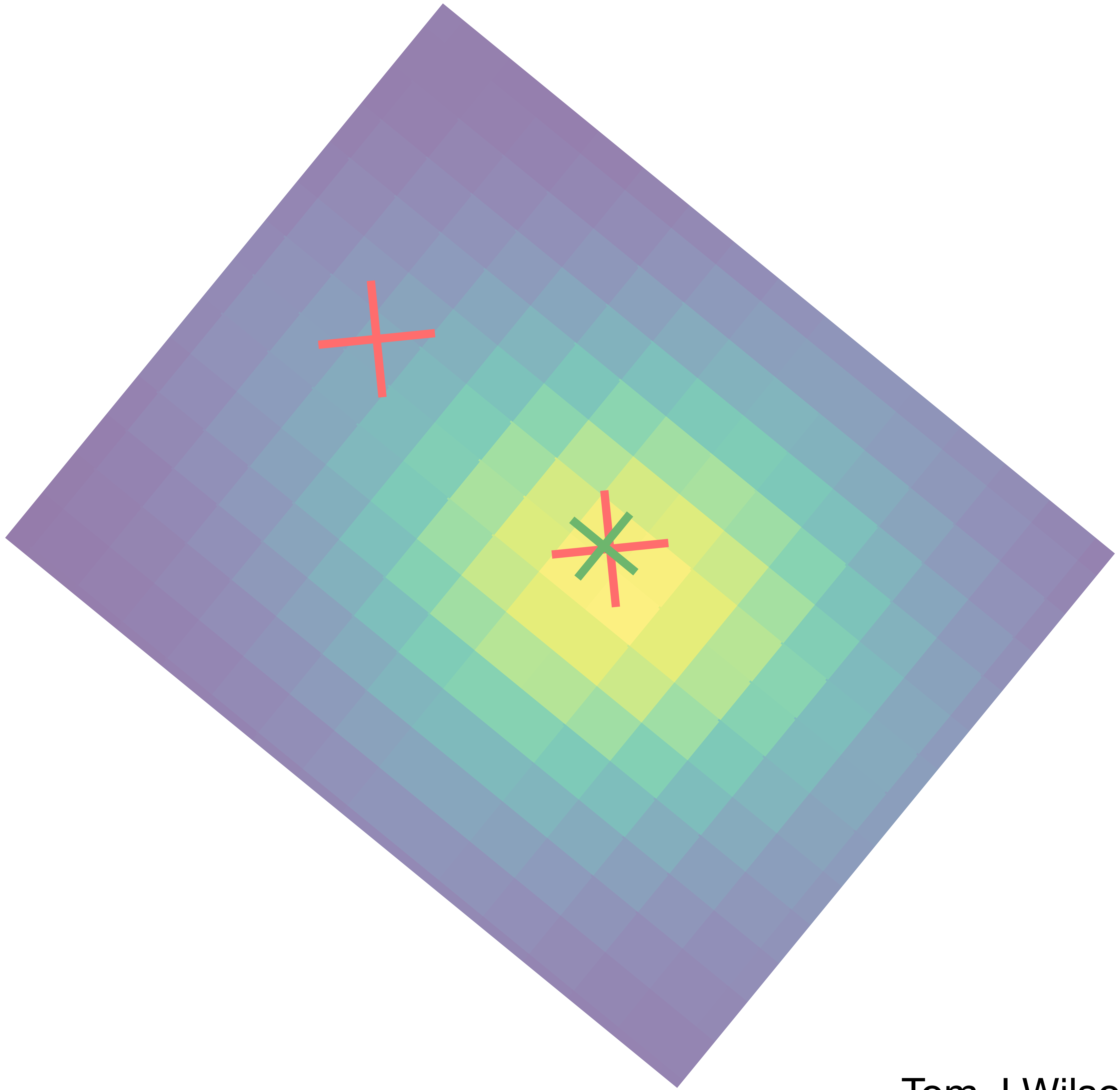

Wilson & Naylor (2017)
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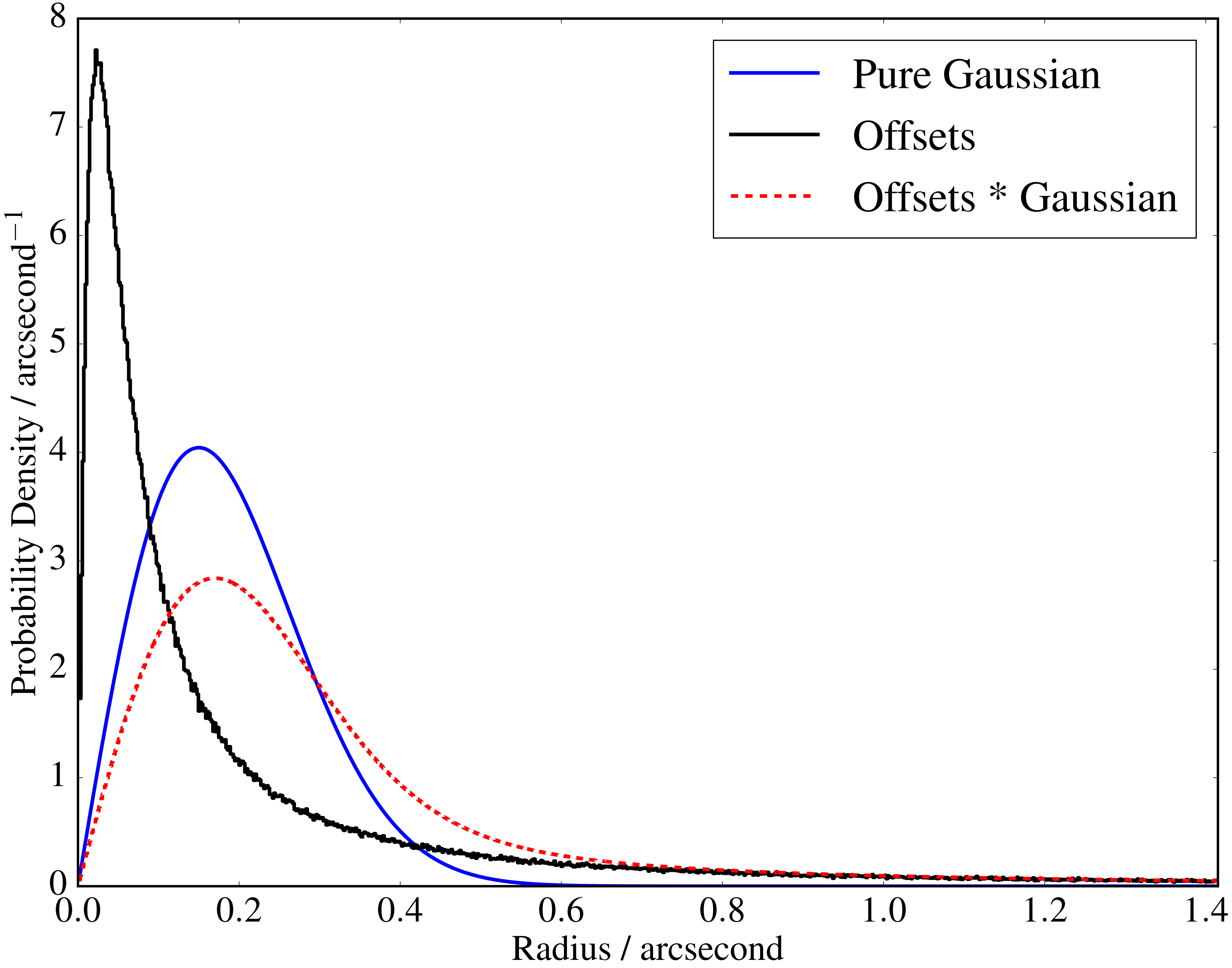

Wilson & Naylor (2018b)

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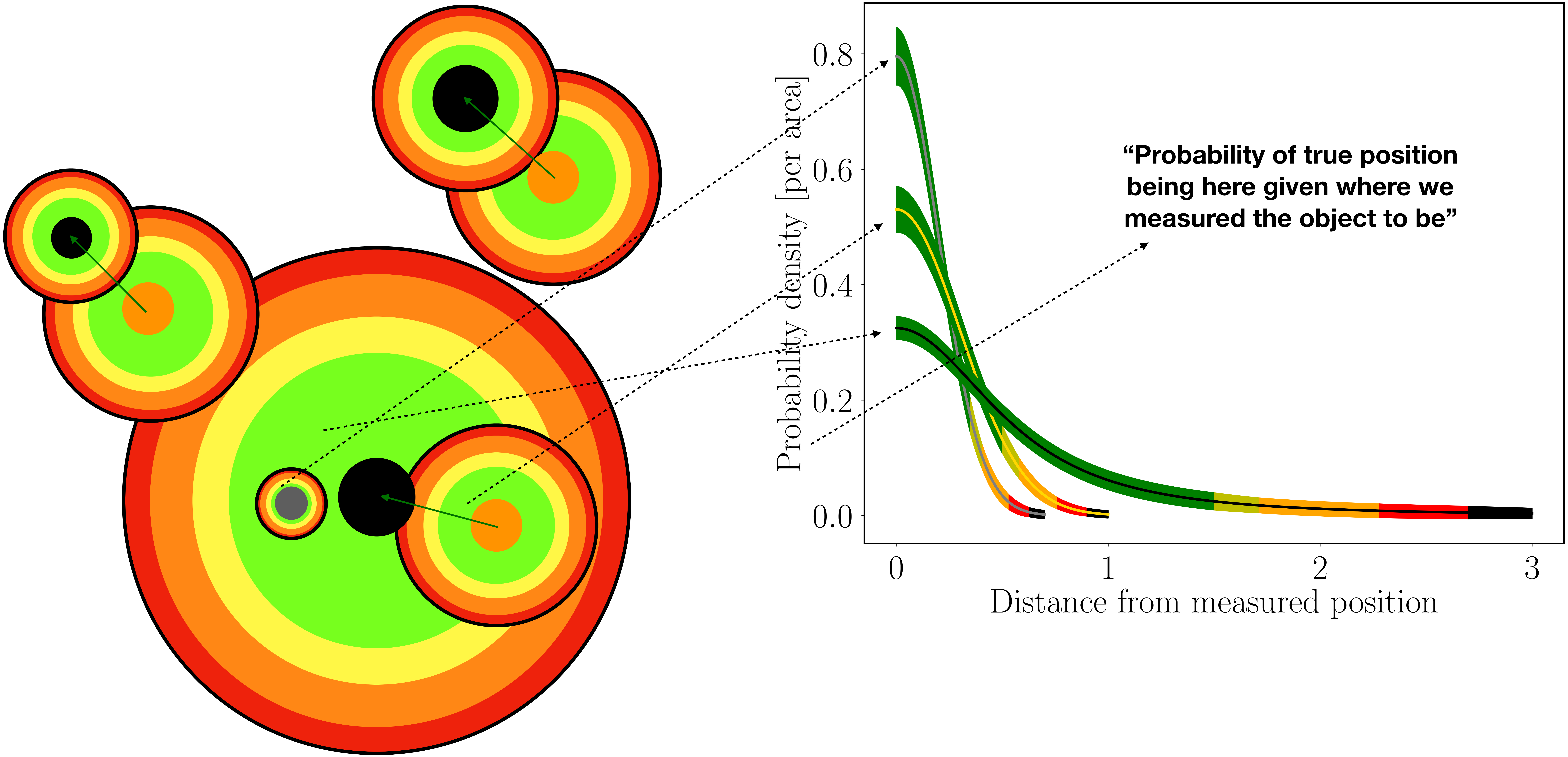
The AUF: Position (Un)Certainty



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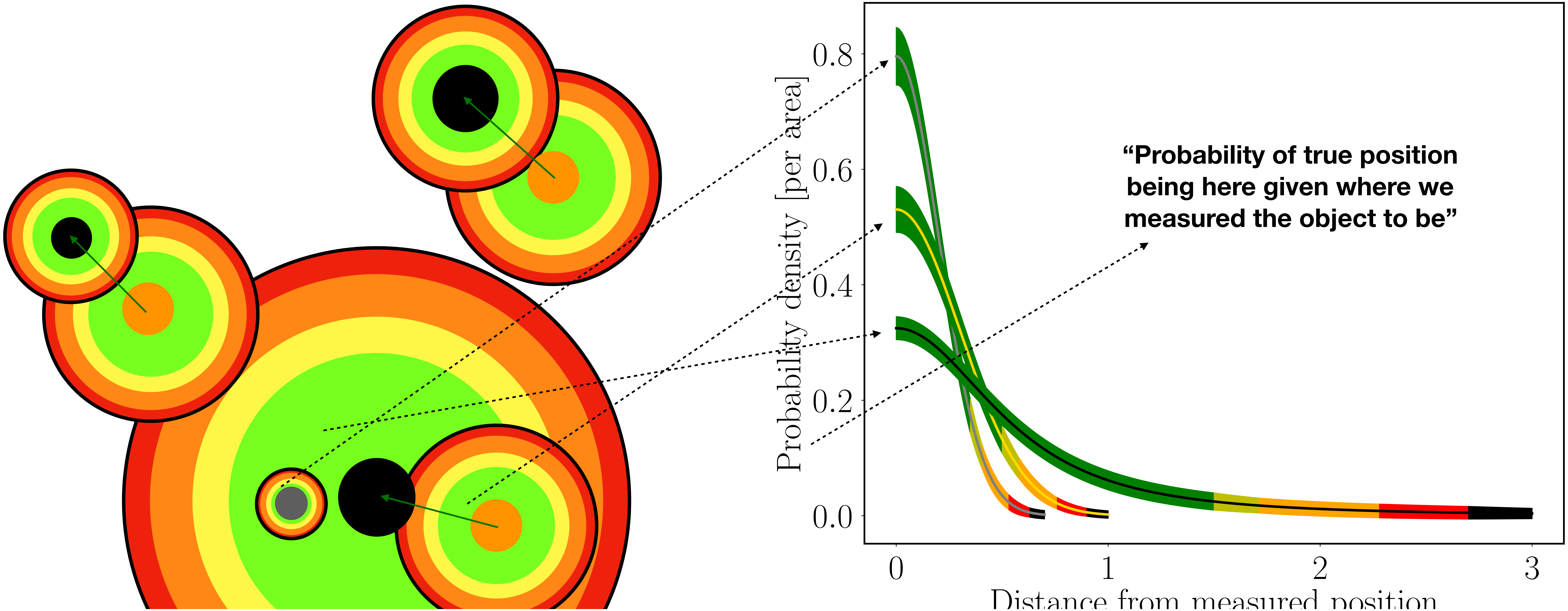
The AUF: Position (Un)Certainty



One assumption made in all of previous works: source positions uncertainties are Gaussian!

$$dp(r|id) = r \times e^{-r^2/2}\, dr.$$

de Ruiter, Willis, & Arp (1977)

$$P(i) = \frac{\frac{Xc(m_i)\,g(\Delta x_i,\Delta y_i)}{Nf(m_i)}}{1 - X + \sum_j \frac{Xc(m_j)\,g(\Delta x_j,\Delta y_j)}{Nf(m_j)}}$$

Naylor, Broos, & Feigelson (2013)

$$p(D|H) = \int p(\boldsymbol{m}|H) \prod_{i=1}^{n} p_i(\boldsymbol{x}_i|\boldsymbol{m}, H)\, d^3m$$

Budavári & Szalay (2008)

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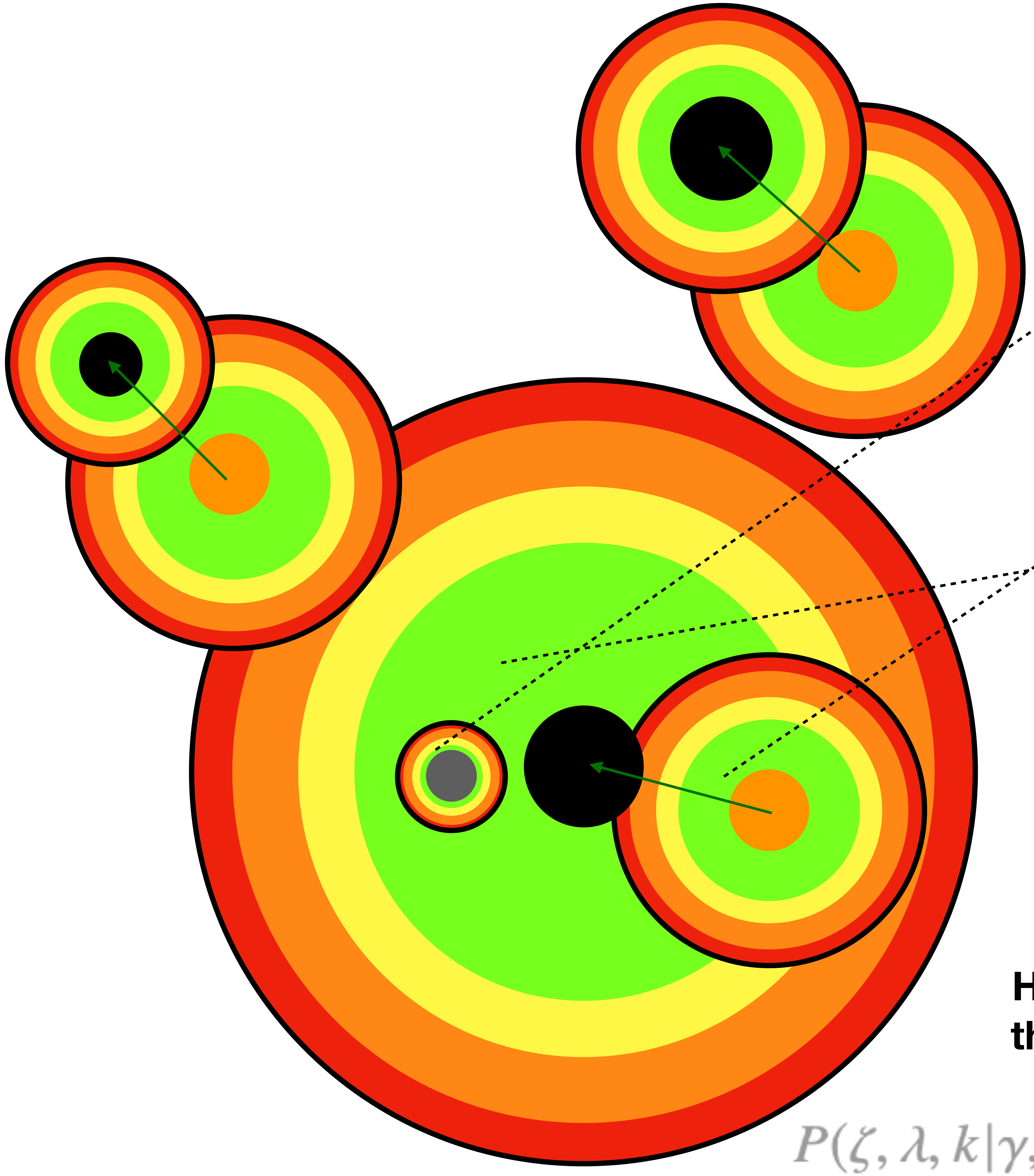
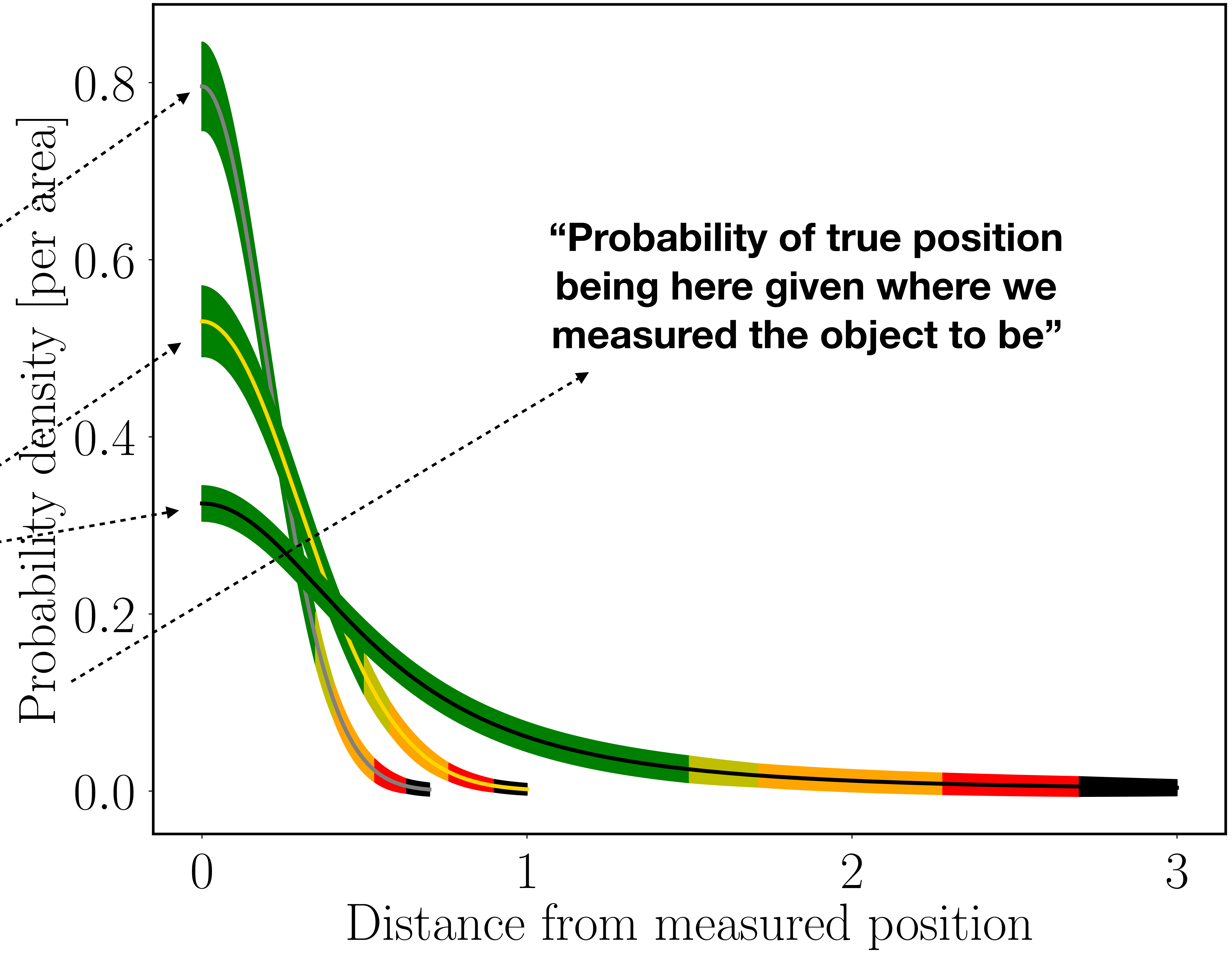
The AUF: Position (Un)Certainty



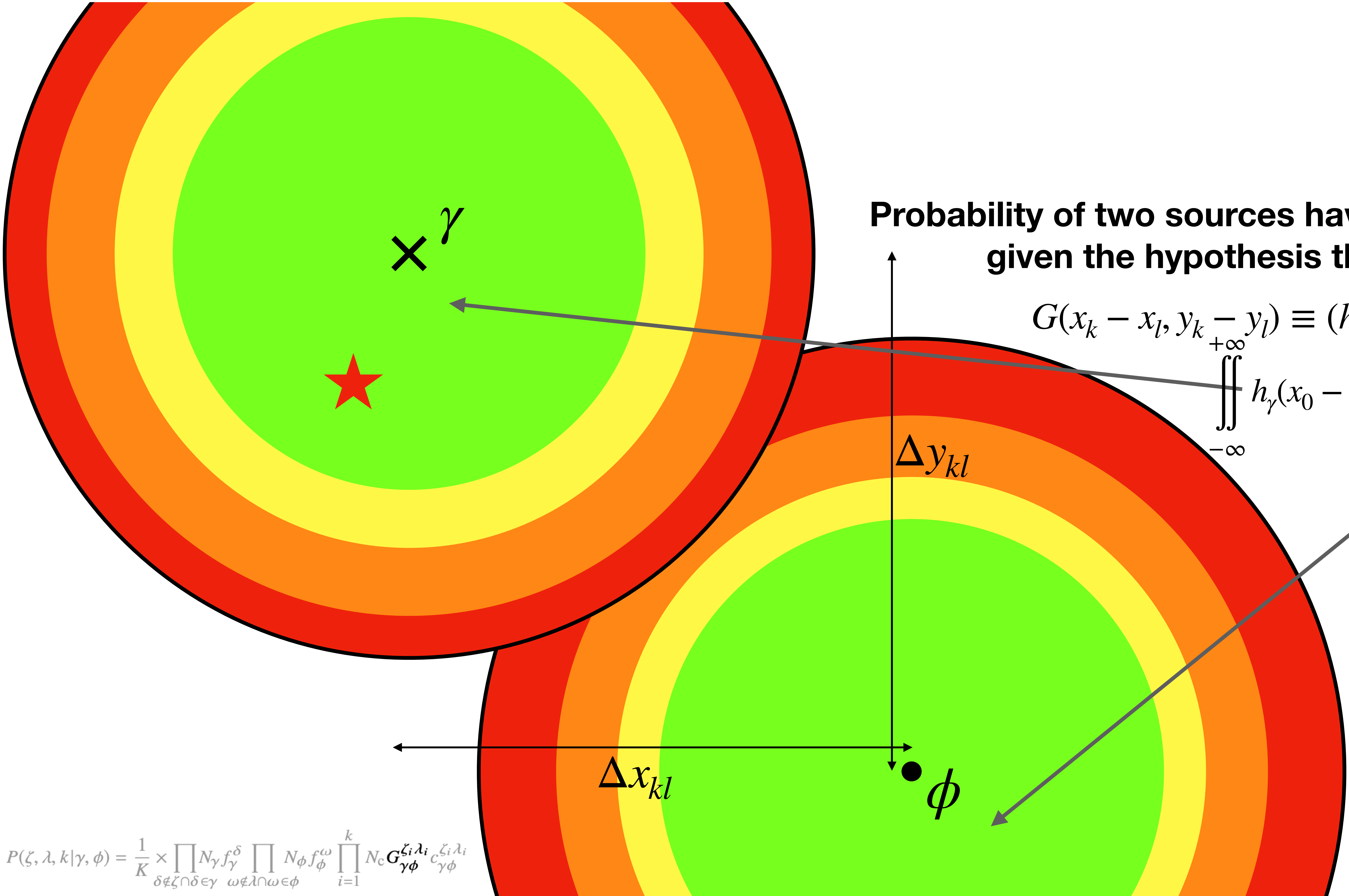


How can we calculate the *general* probability of two sources having their on-sky separation given the hypothesis they are counterparts?

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_\mathrm{c} \boldsymbol{G}_{\boldsymbol{\gamma\phi}}^{\boldsymbol{\zeta_i \lambda_i}} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

Tom J Wilson @onoddil

The AUF: Match Separation Probability



Probability of two sources having their on-sky separation given the hypothesis they are counterparts

$$G(x_k - x_l, y_k - y_l) \equiv (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl}) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0$$

Wilson & Naylor (2018a)

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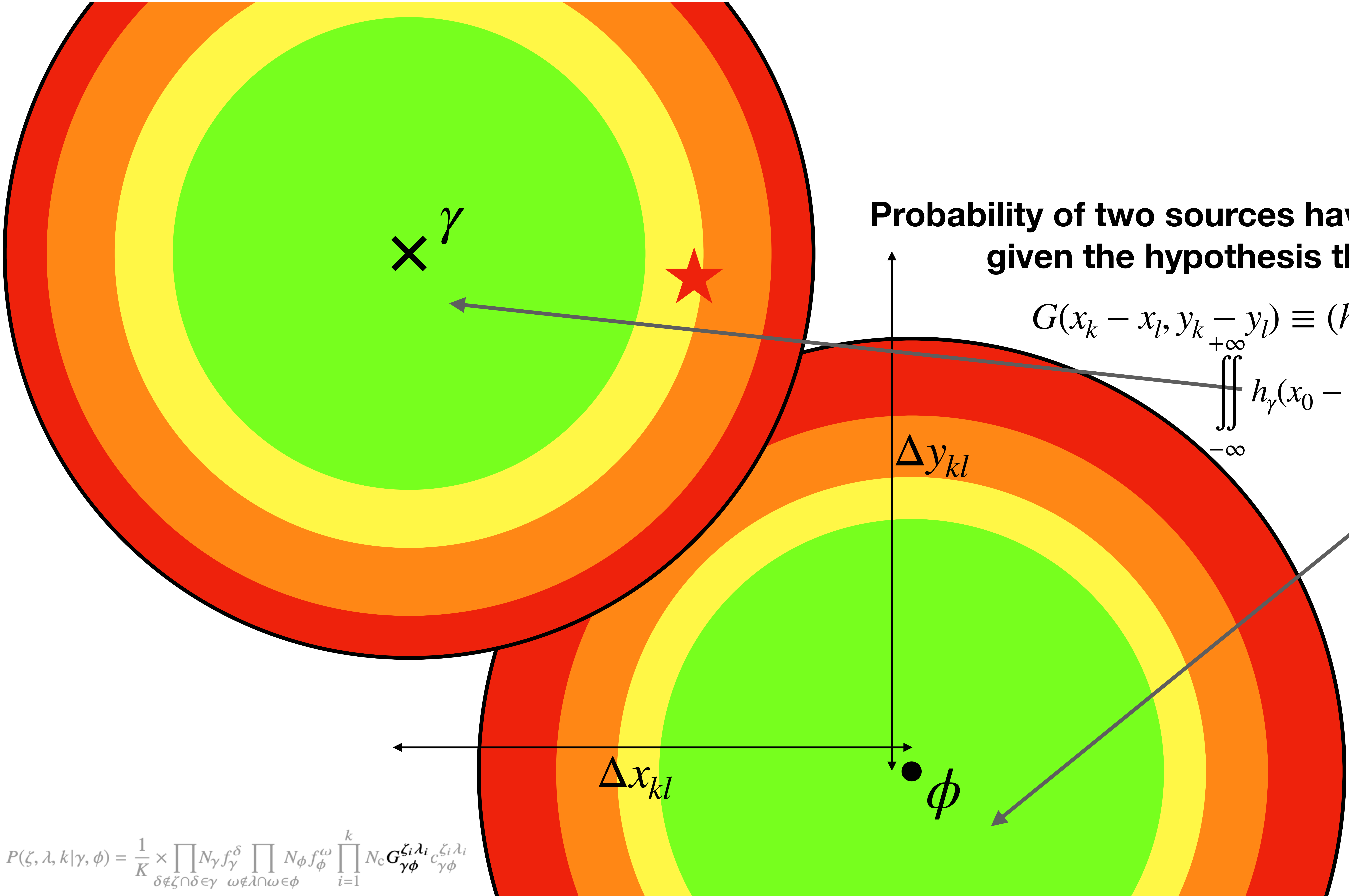

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Wilson & Naylor (2018a)

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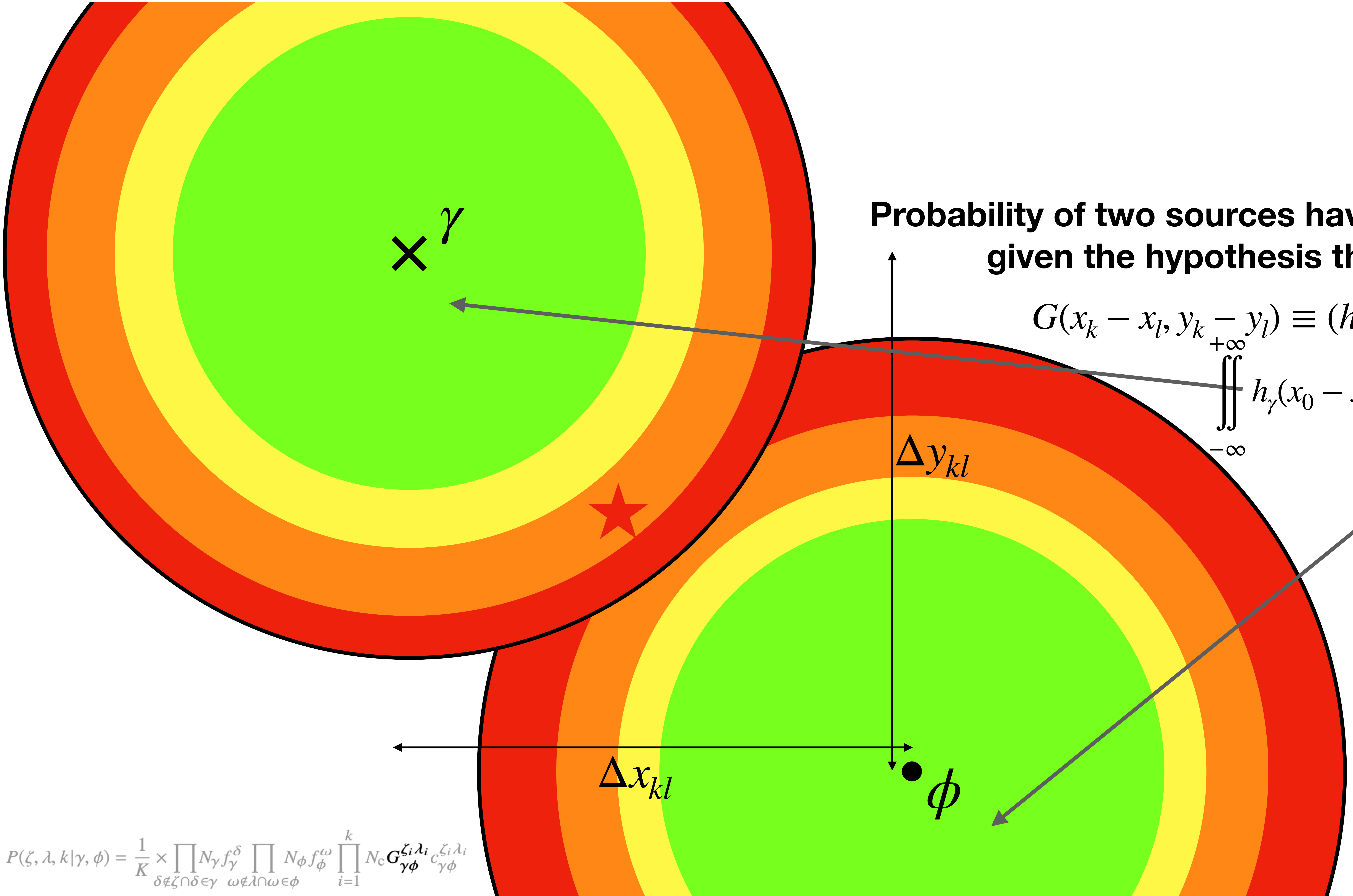

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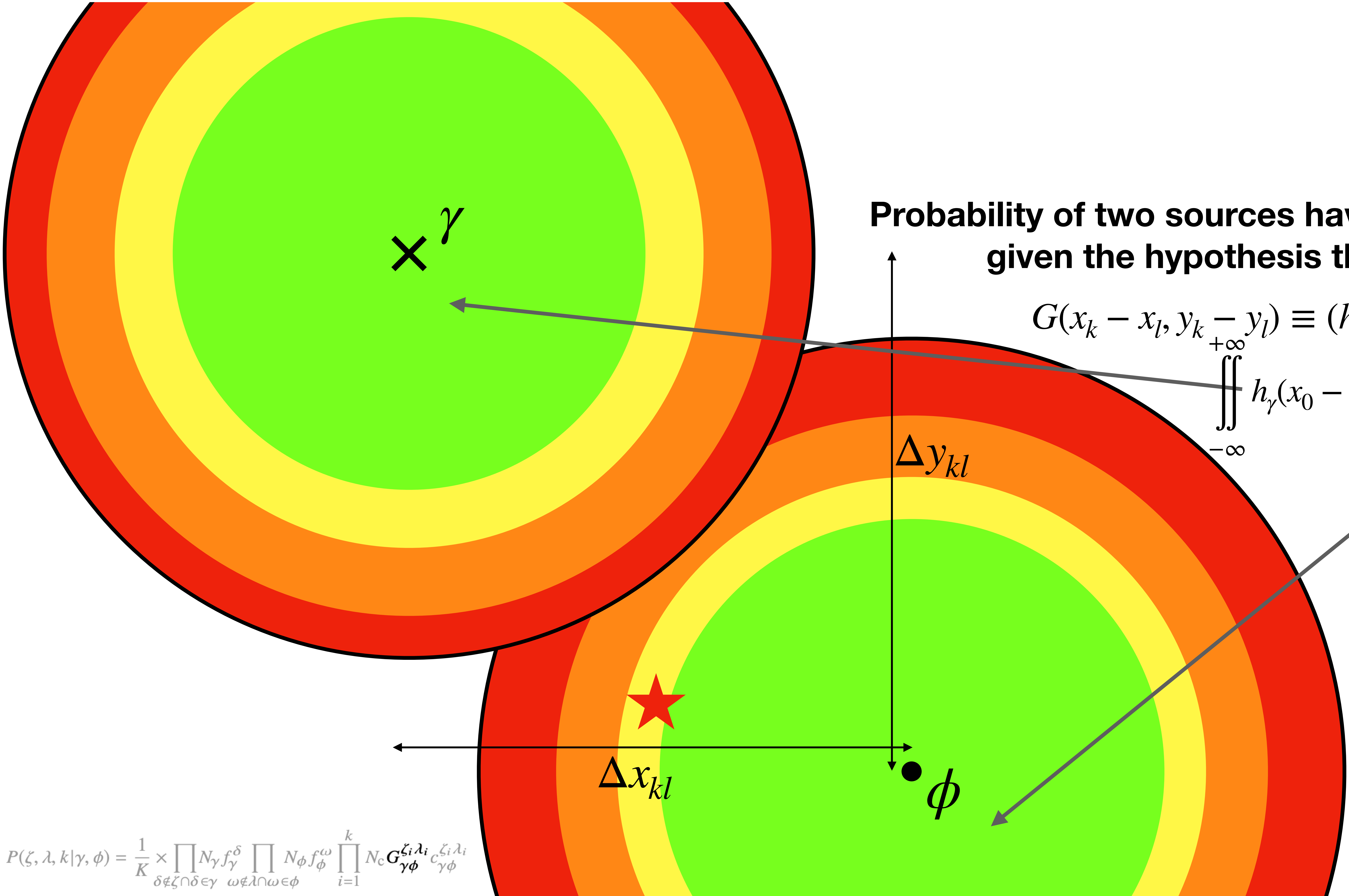

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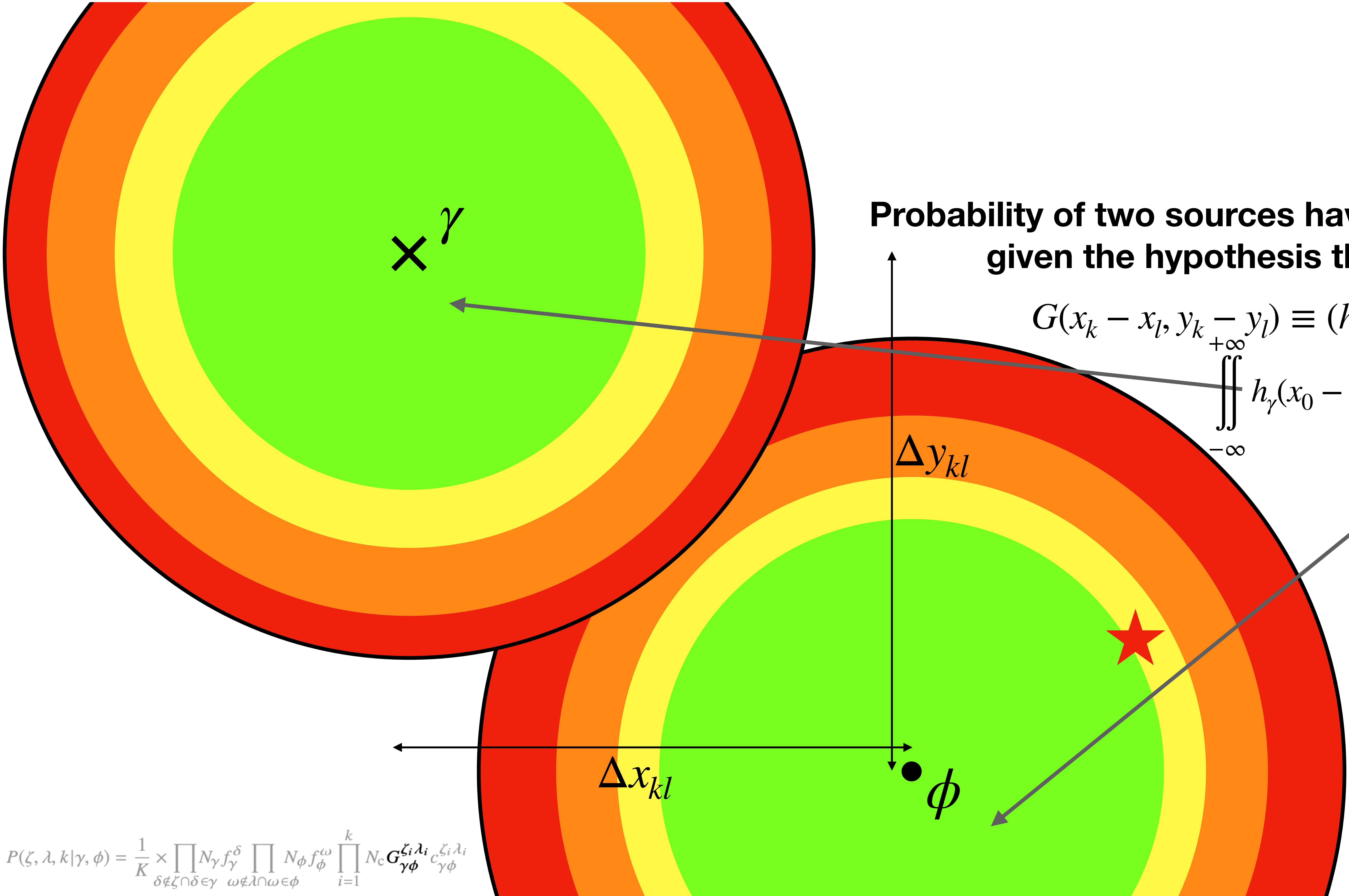

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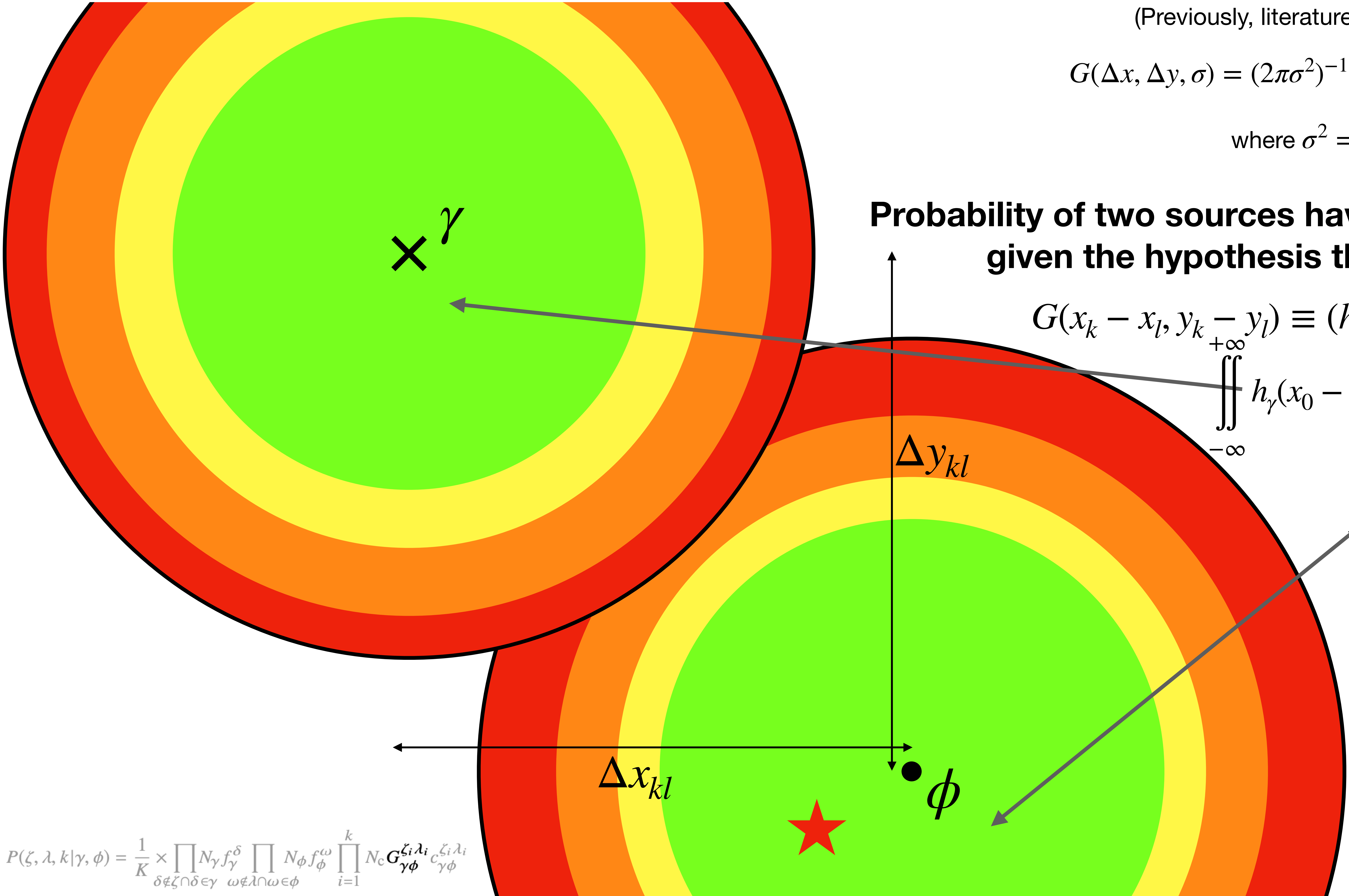
Wilson & Naylor (2018a)

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c \boldsymbol{G}_{\gamma\phi}^{\zeta_i \lambda_i} c_{\gamma\phi}^{\zeta_i \lambda_i}$$

The AUF: Match Separation Probability



(Previously, literature assumed that e.g.

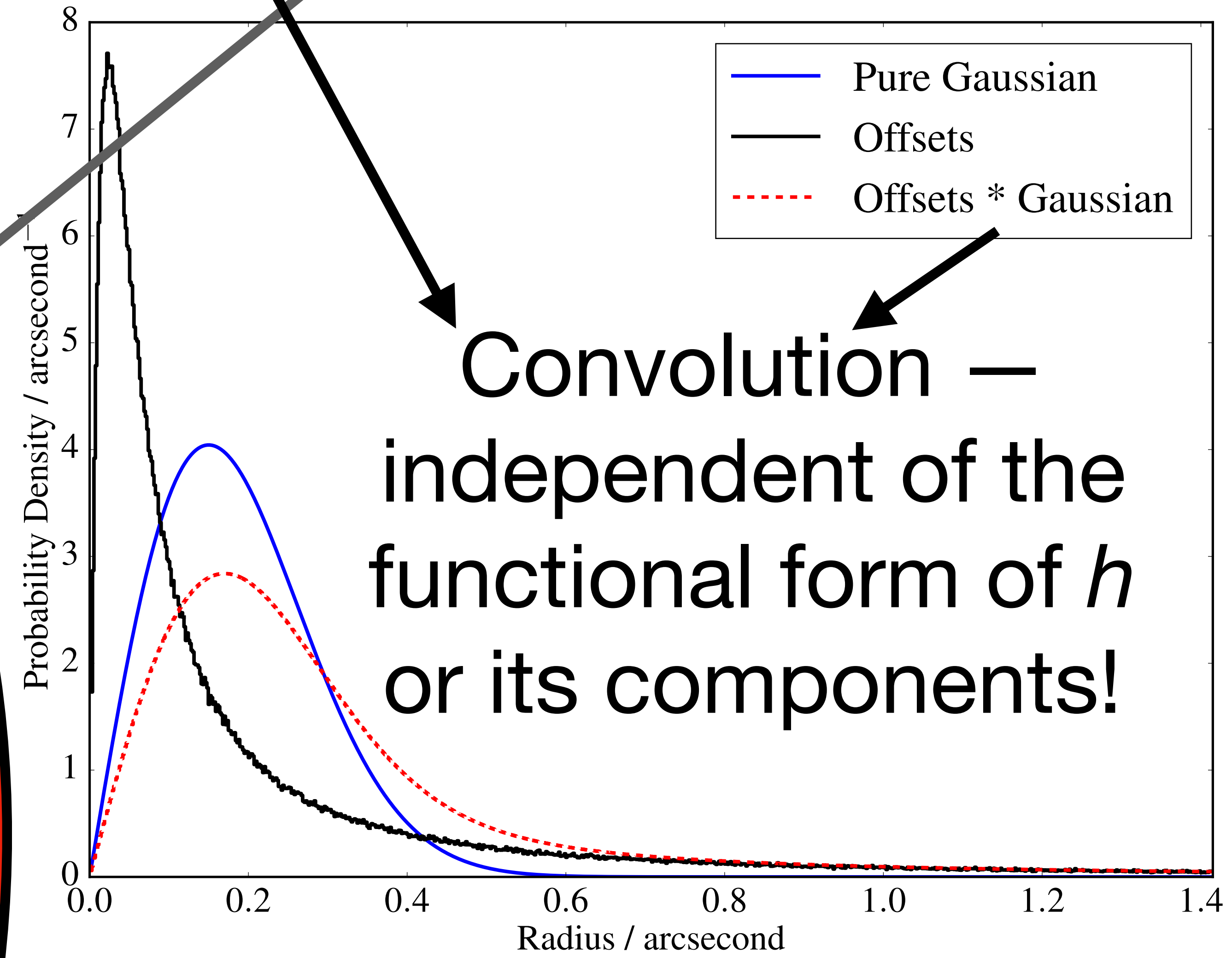
$$G(\Delta x, \Delta y, \sigma) = (2\pi\sigma^2)^{-1}\exp\left(-\frac{1}{2}\frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right)$$

where $\sigma^2 = \sigma_\gamma^2 + \sigma_\phi^2$)

Probability of two sources having their on-sky separation given the hypothesis they are counterparts

$$G(x_k - x_l, y_k - y_l) \equiv (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl}) = \iint_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0$$

Wilson & Naylor (2018a)



Convolution — independent of the functional form of *h* or its components!

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma f_\gamma^\delta \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c \boldsymbol{G}_{\gamma\phi}^{\zeta_i\lambda_i} c_{\gamma\phi}^{\zeta_i\lambda_i}$$

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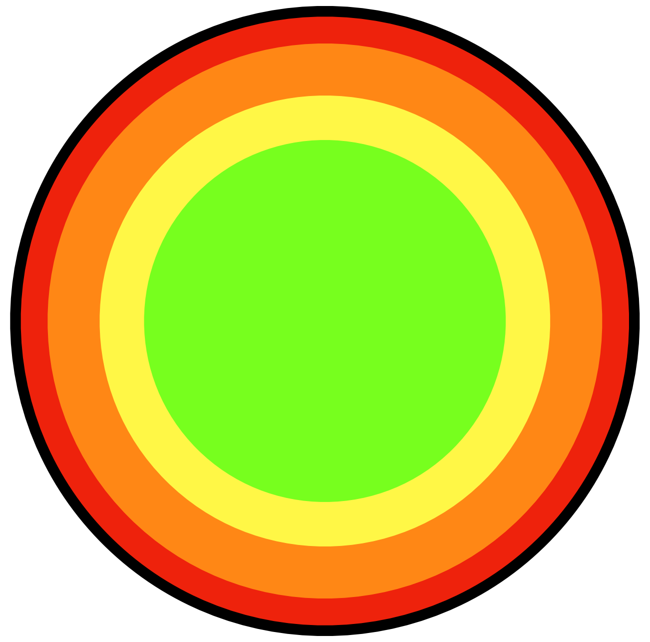
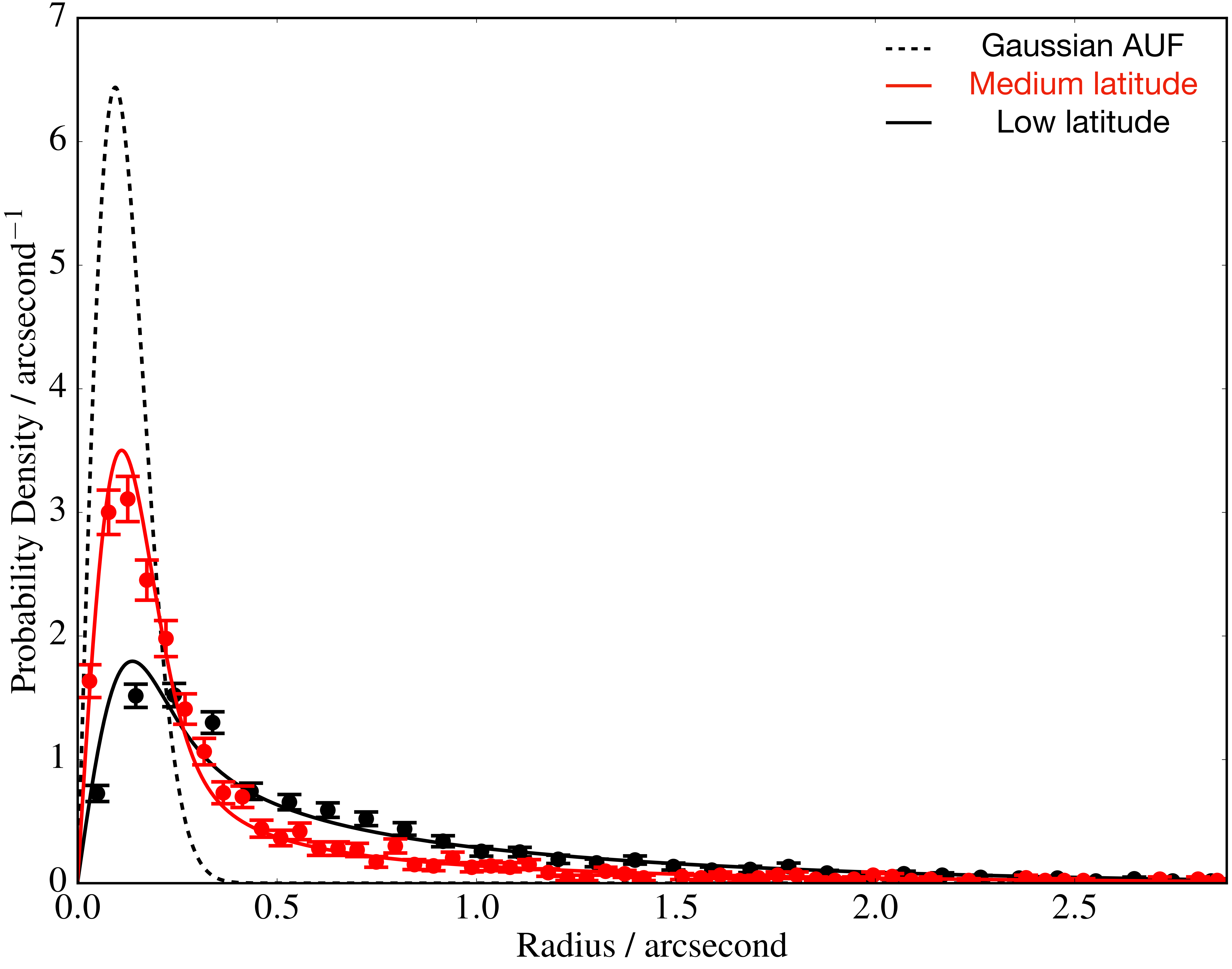
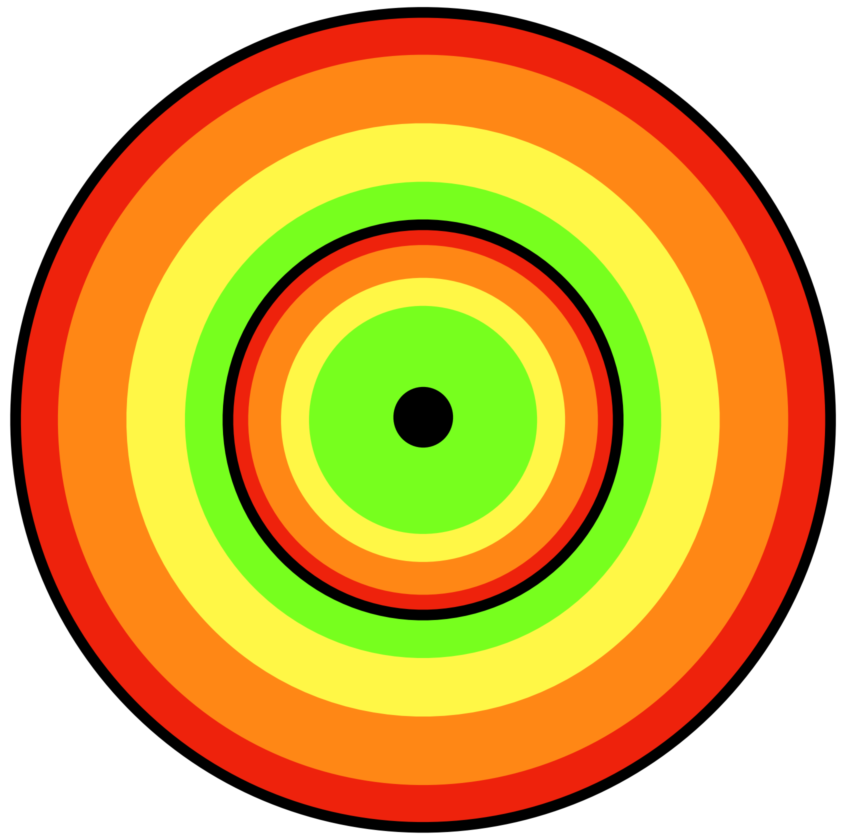
The Astrometric Uncertainty Function



The AUF does not need to, and in fact quite often <u>should *not*</u>, be Gaussian!

The Effect of Crowding on *Gaia*-*WISE* Matches





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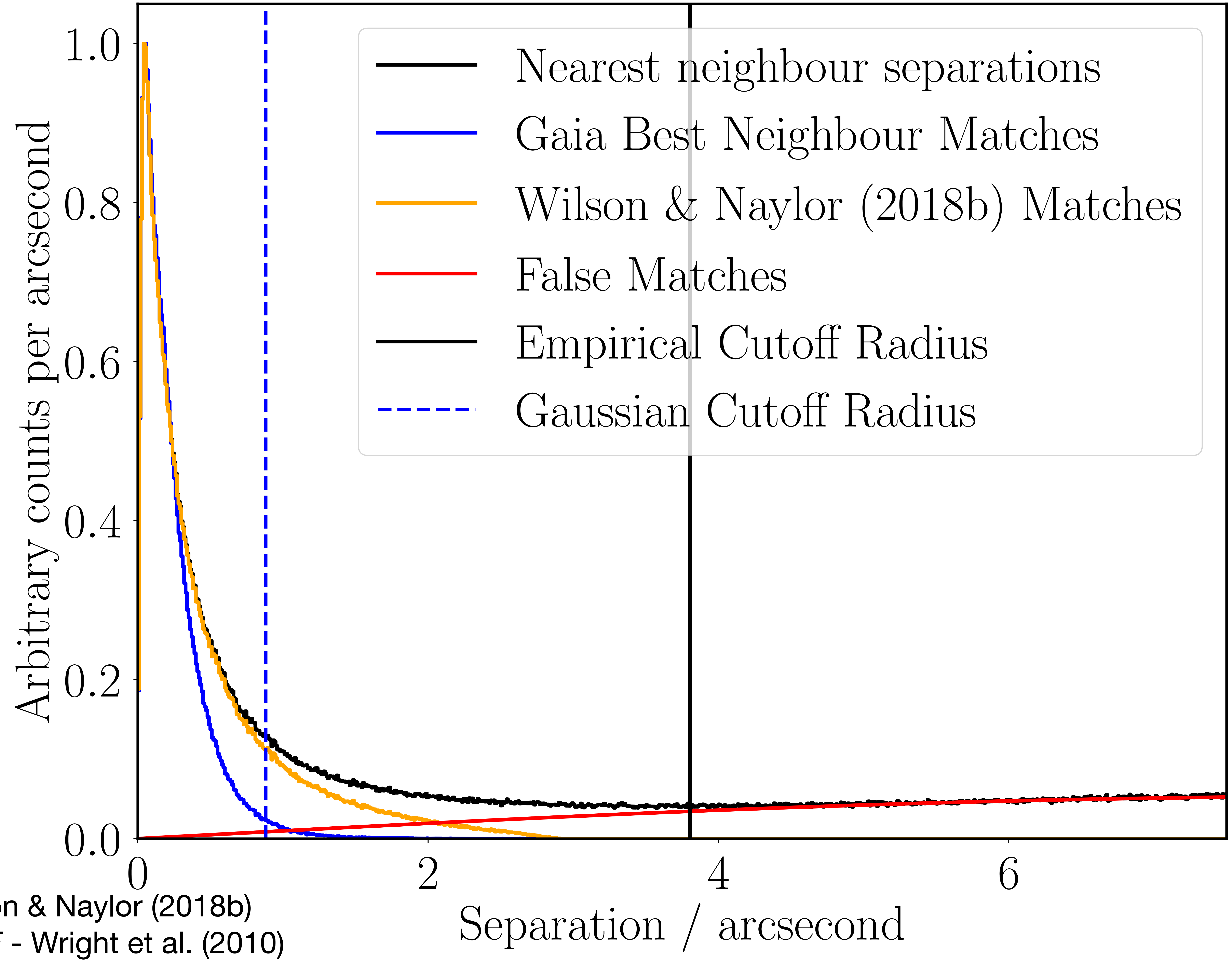
The Effect of Crowding on *Gaia*-*WISE* Matches



If this effect was not taken into account, we would be incorrectly led to believe 50% of *Gaia*-*WISE** sources were not matches!

*"*Euclid*-Rubin"

Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia matches - Marrese et al. (2019)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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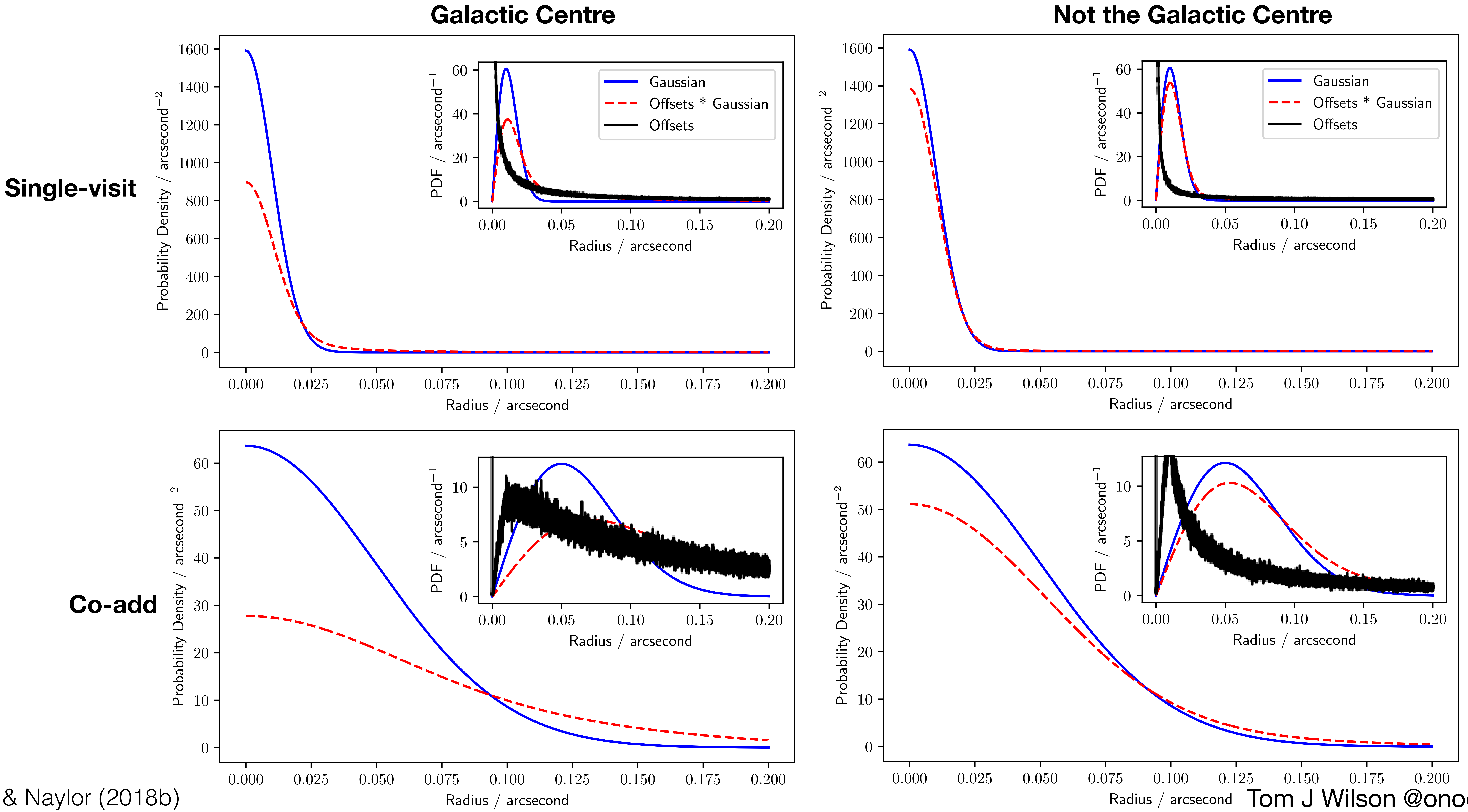
The Rubin AUF: Galactic Plane



Wilson & Naylor (2018b)

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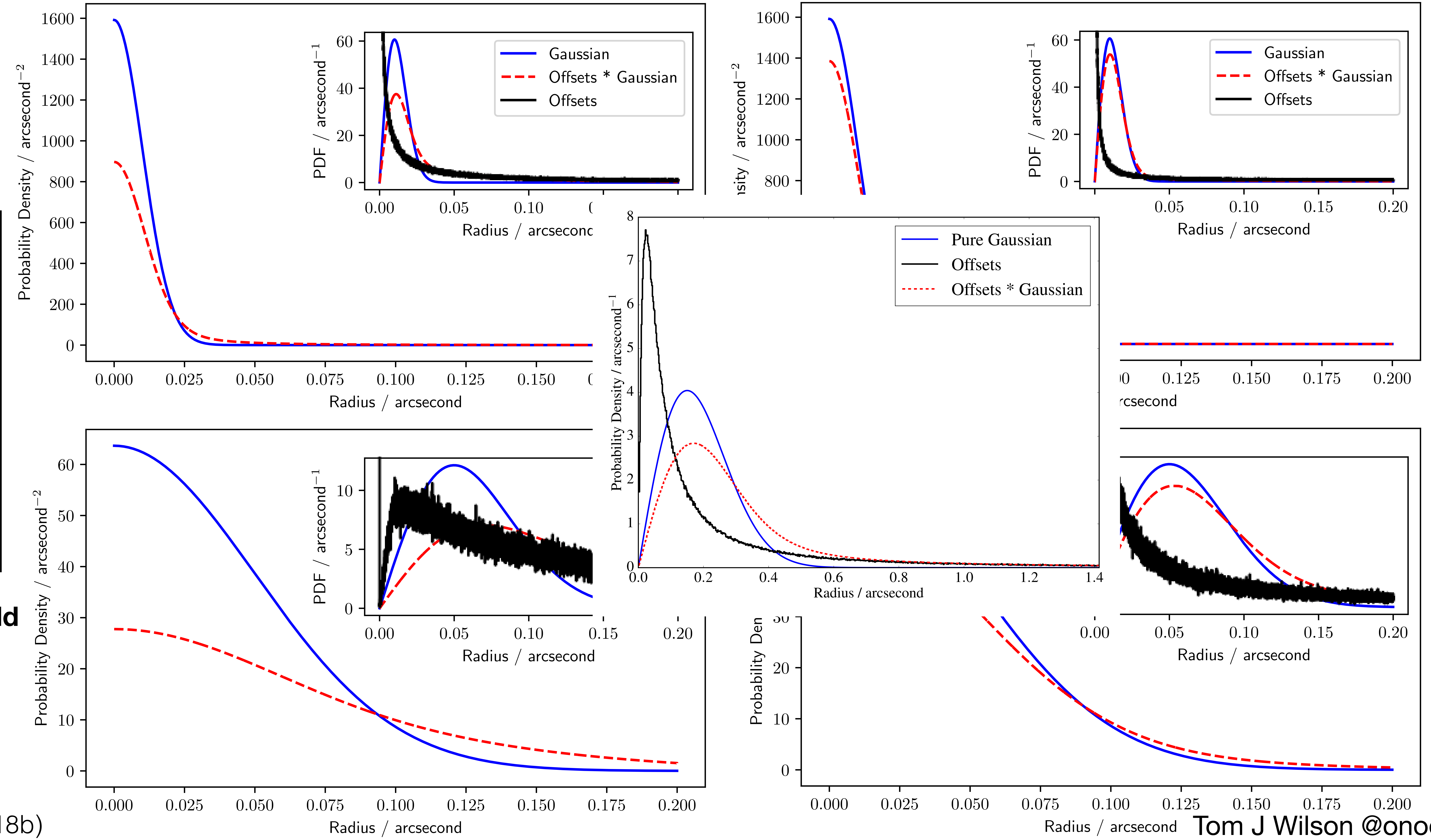
The Rubin AUF: Galactic Plane

Galactic Centre | **Not the Galactic Centre**

Single-visit

Co-add

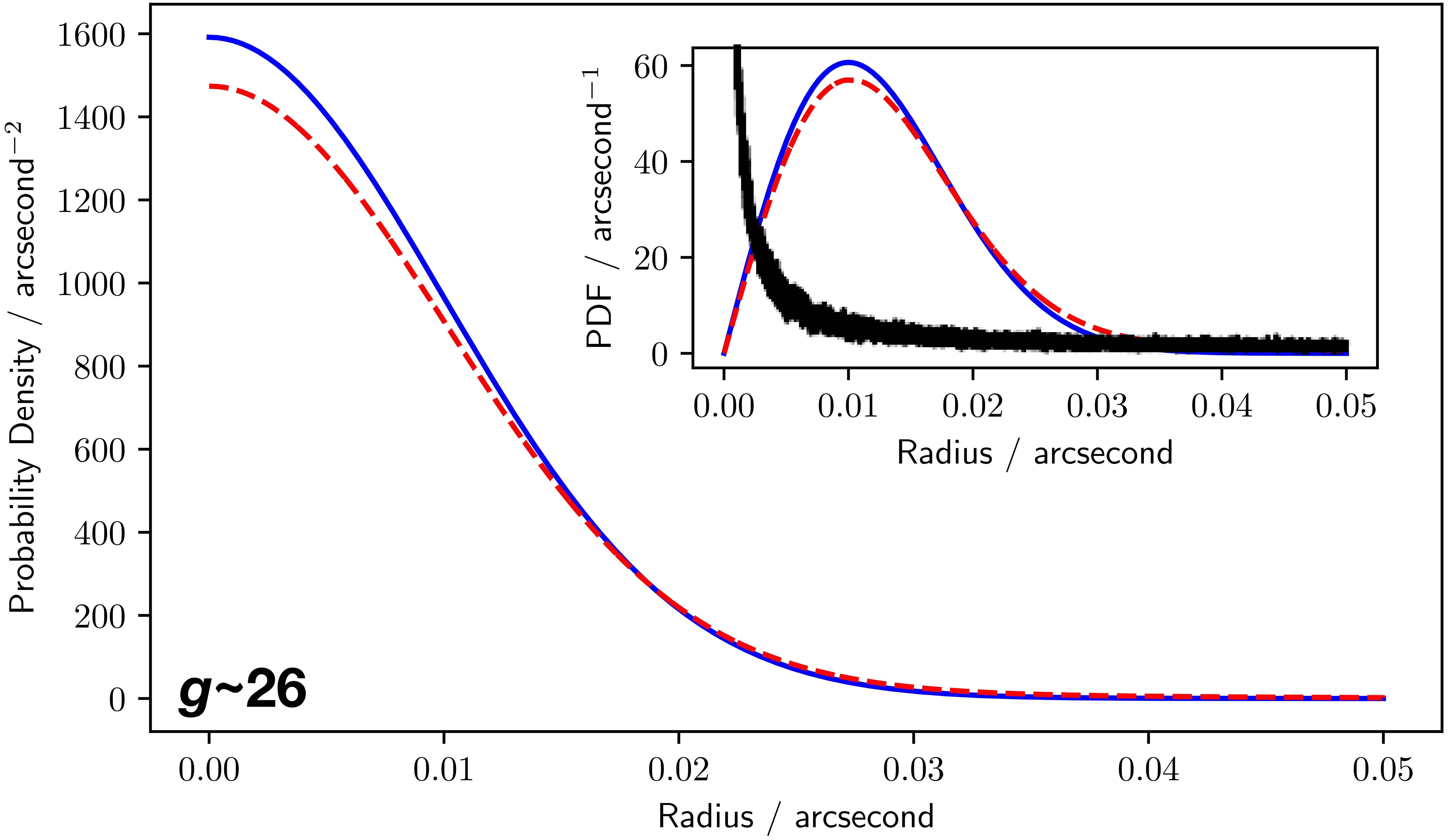


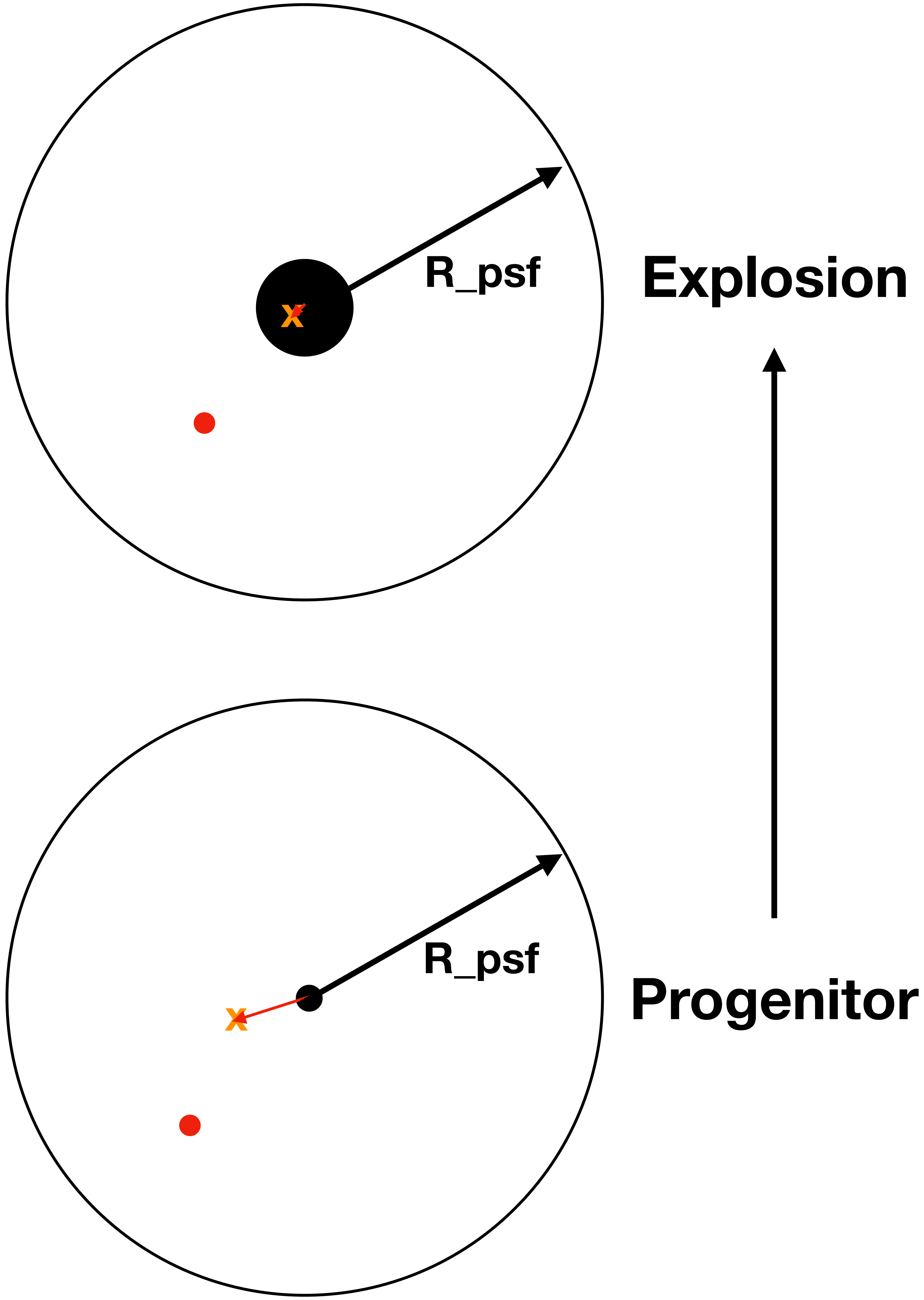
LSST will suffer approximately the same number of unresolved contaminants per PSF area as *WISE*! The Perturbation component of the AUF will overwhelm the Centroid component for a large % of the Galactic Plane.

Wilson & Naylor (2018b)

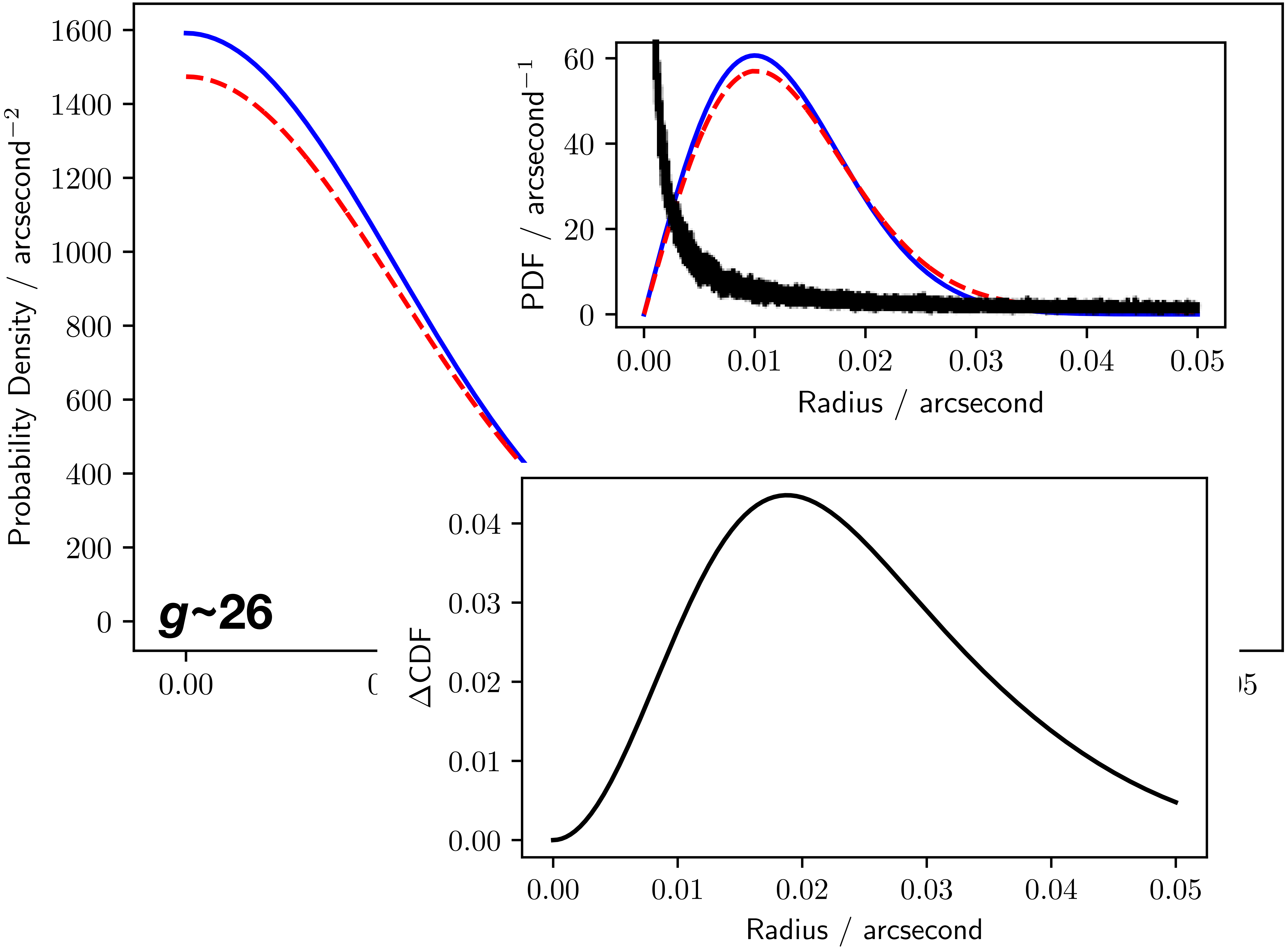
Tom J Wilson @onoddil

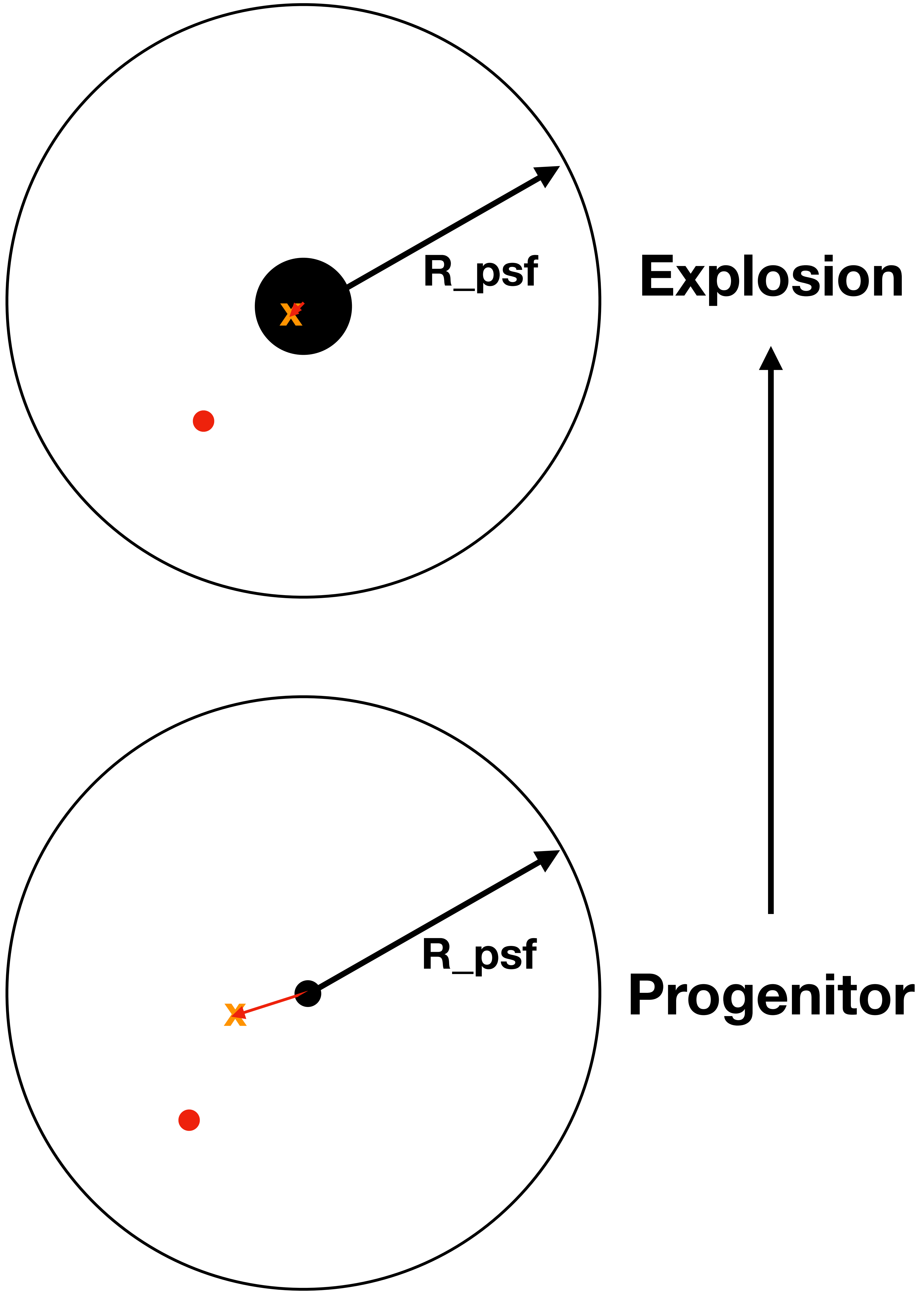
The Rubin AUF: Extra-Galactic, Transients





The Rubin AUF: Extra-Galactic, Transients





Wilson & Naylor (2018b); also see Wilson (2022, RNAAS)

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The Rubin AUF: Extra-Galactic, Transients

Wilson & Naylor (2018b); also see Wilson (2022, RNAAS)

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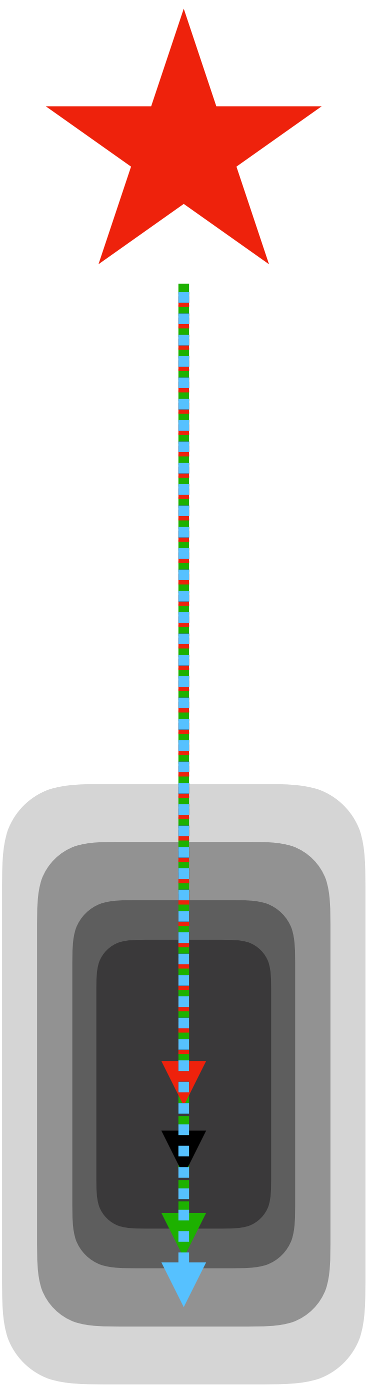
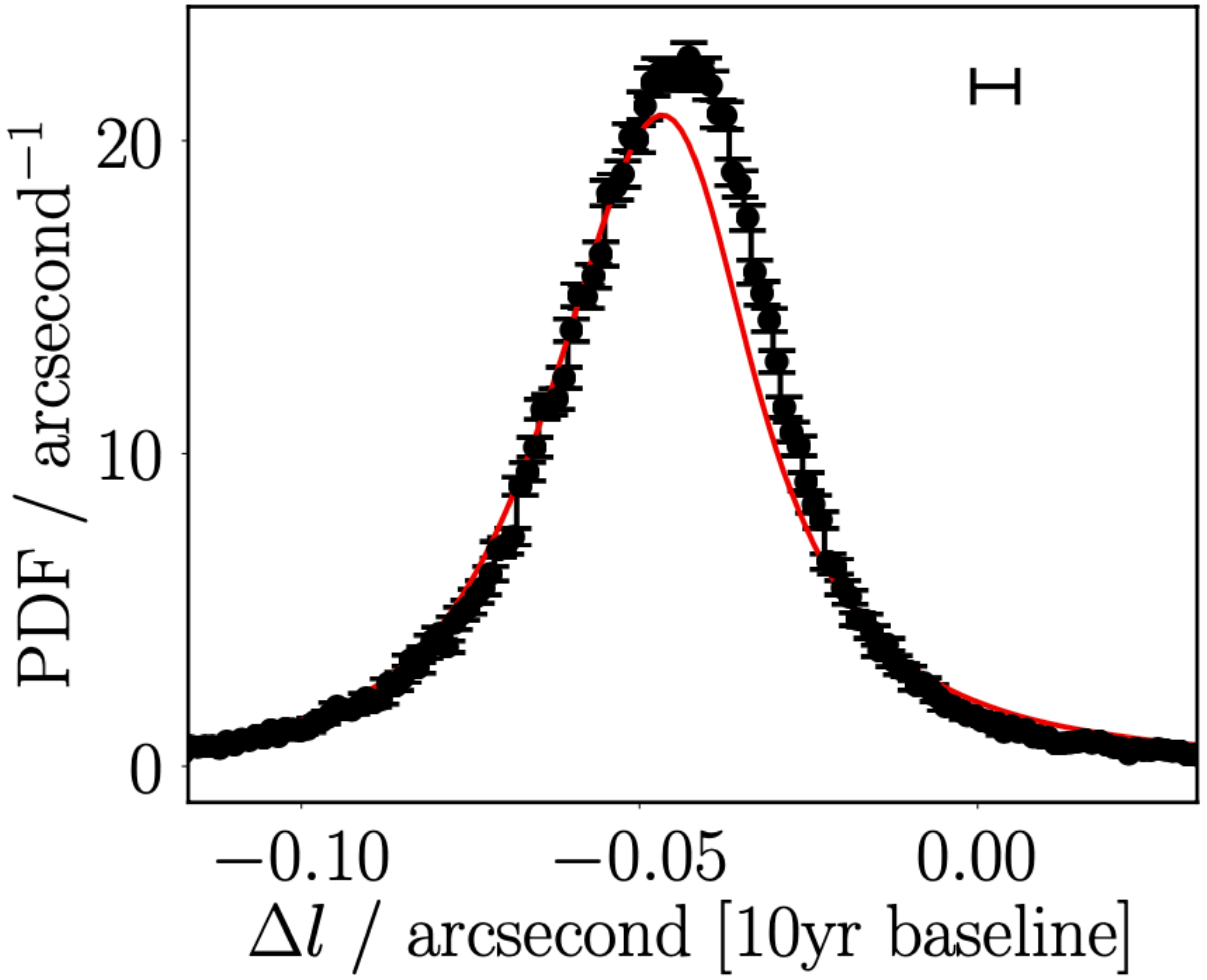
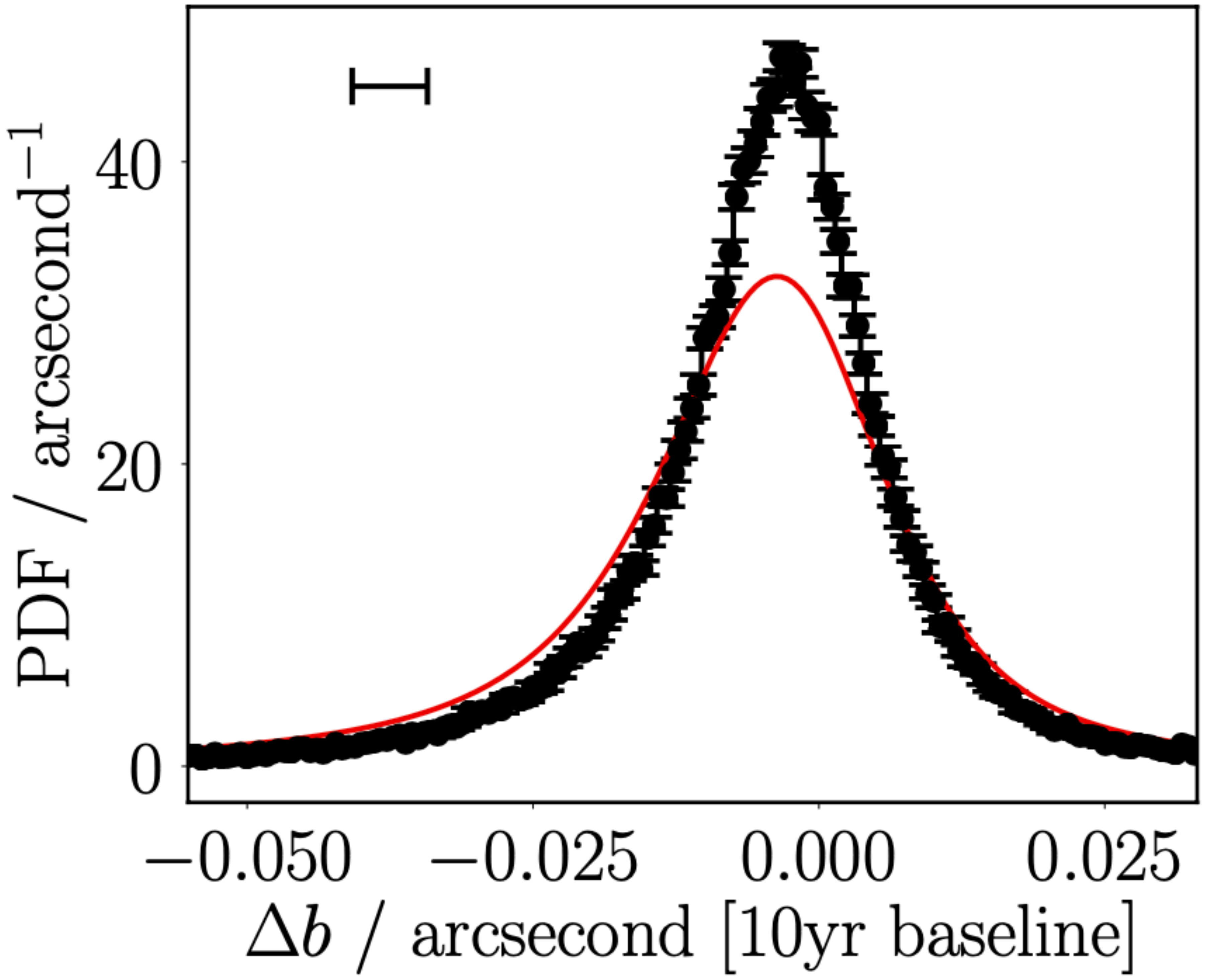
Unknown Proper Motions

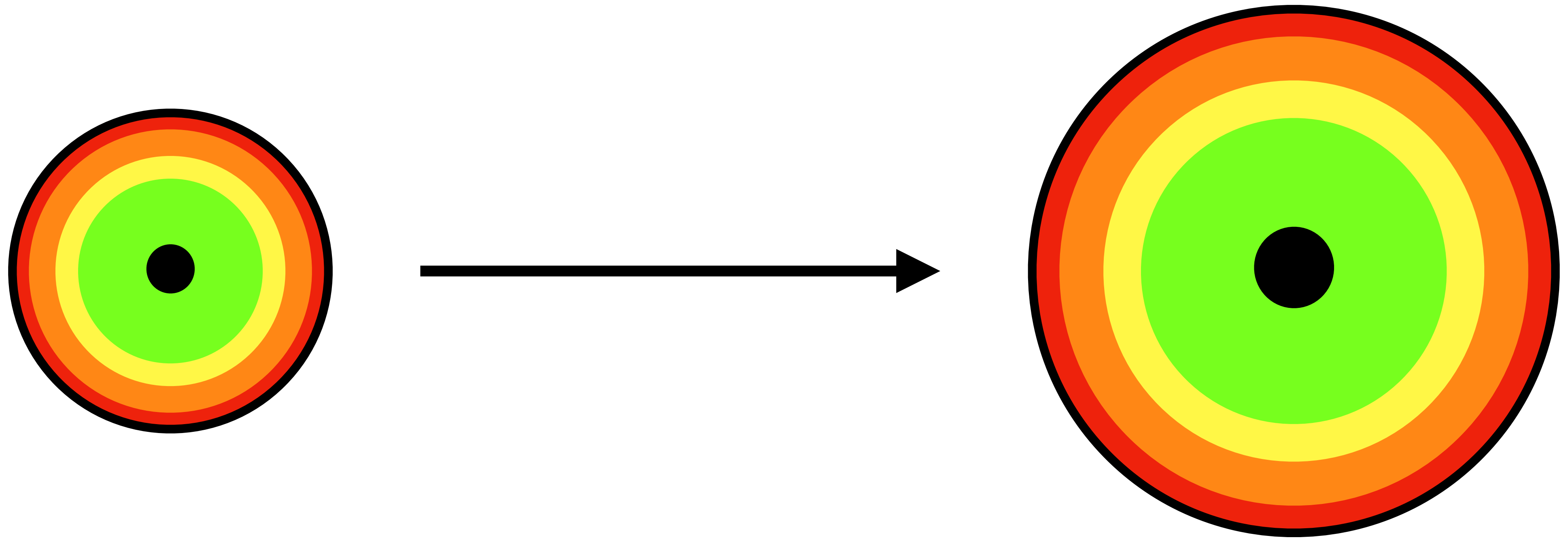
Object in 2015



Projected to 2025



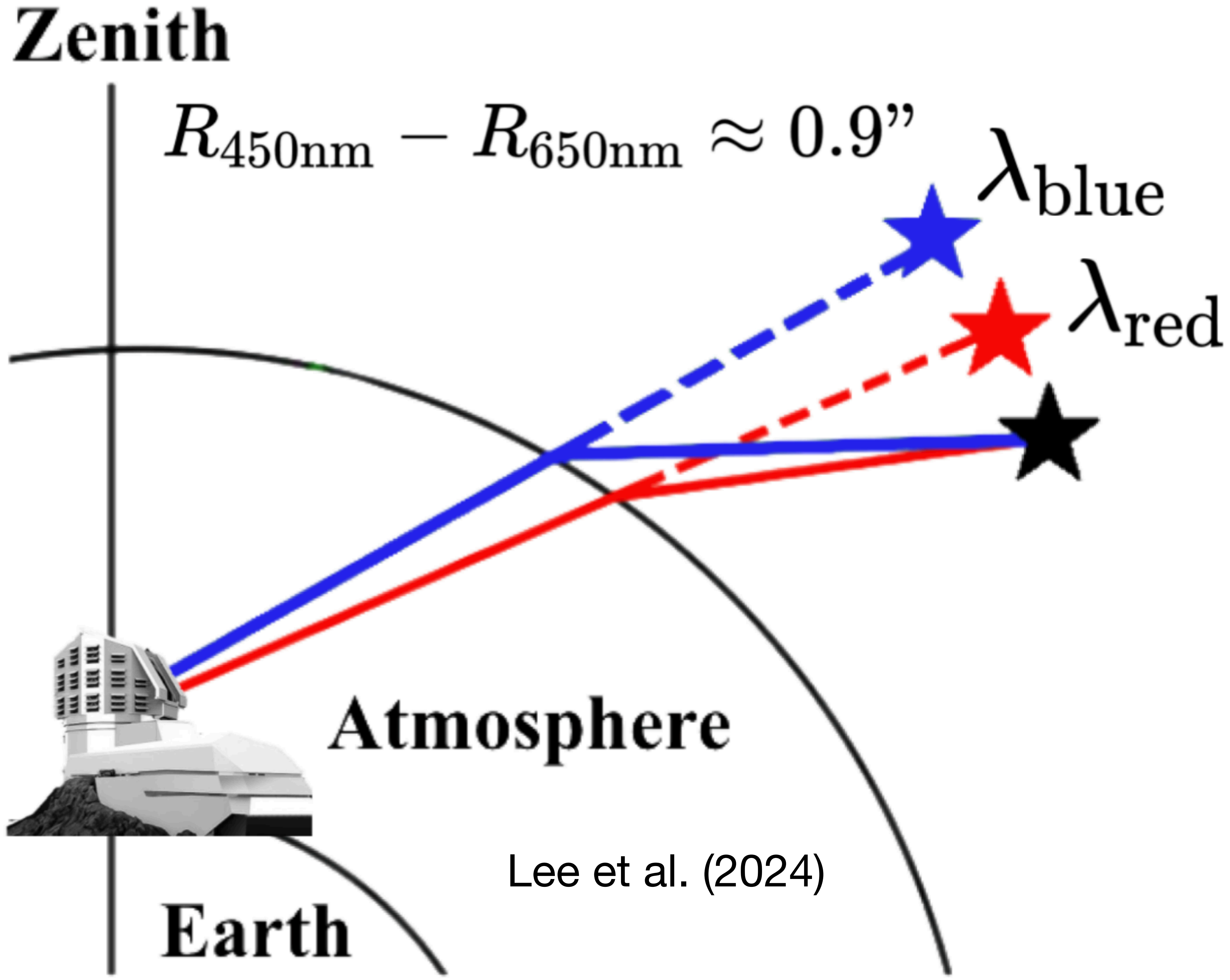




(This also applies to *uncertain* proper motions, where we can incorporate the covariance matrix of weakly-constrained proper motions, e.g. just above the single-visit LSST limit)

Wilson (2023, RASTI, 2, 1)
Gaia eDR3 - Gaia Collaboration et al. (2021, A&A, 649, A1)

Differential Chromatic Refraction



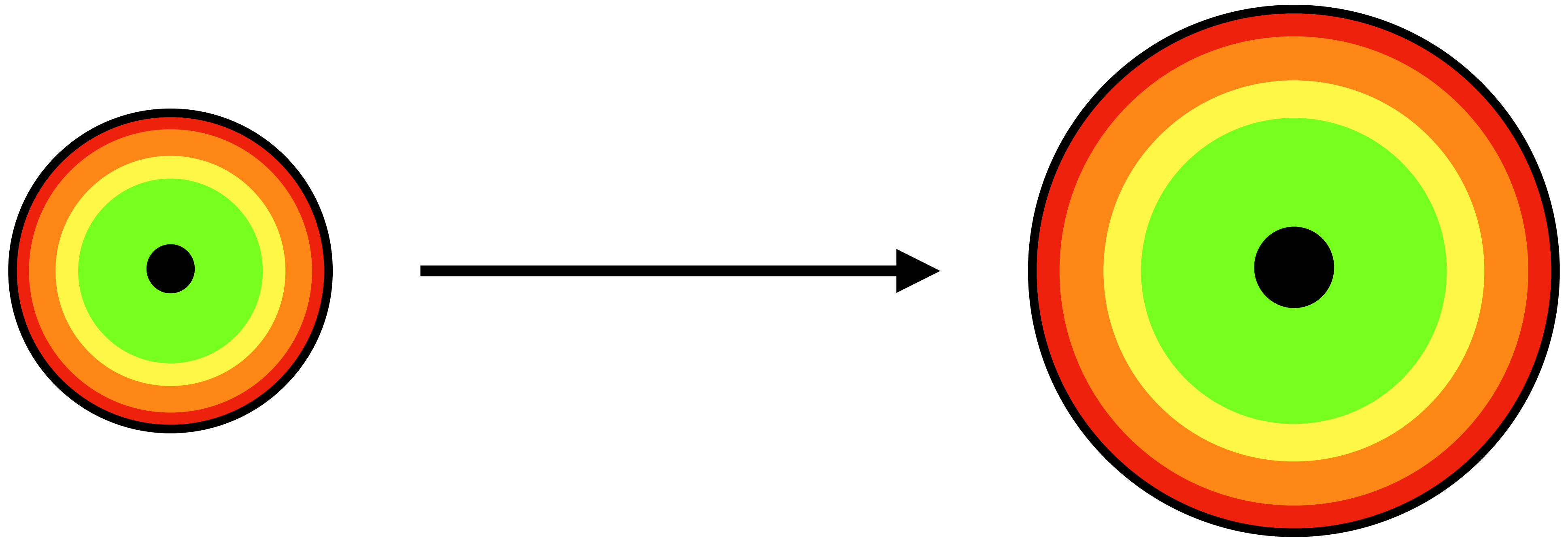
Lee et al. (2024)

e.g. gbdes, Bernstein et al. (2017)

$$\Delta \mathbf{x}^w = K_b c \tan z \, \hat{\mathbf{p}}$$

Unknown/uncertain per-band (b) scaling factor

Unknown/uncertain photometric colour c



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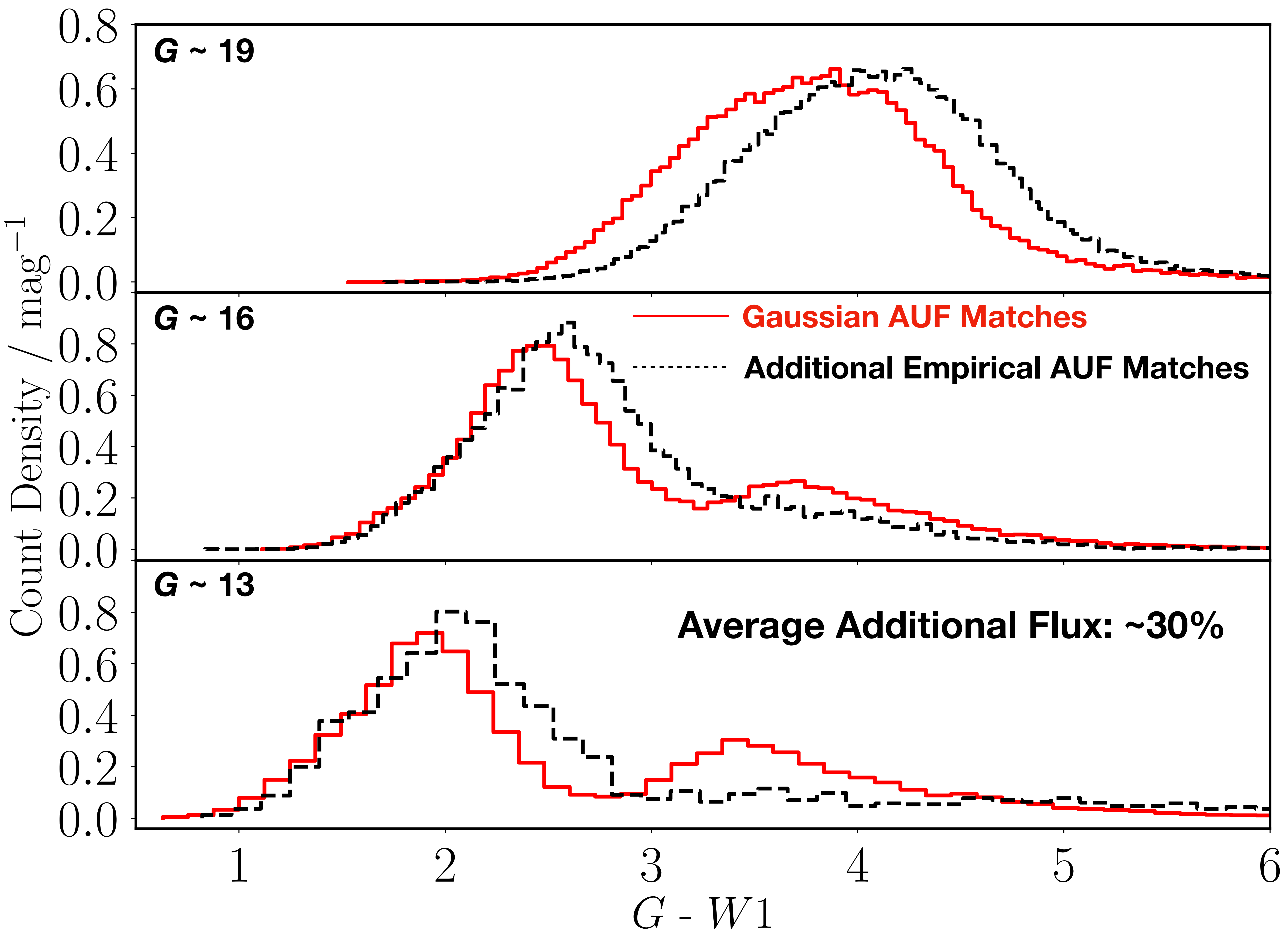
Photometric Effects of Crowding



"Extra flux" has an impact on derived proper motions and parallaxes, and IR excesses!

Wilson & Naylor (2018b)

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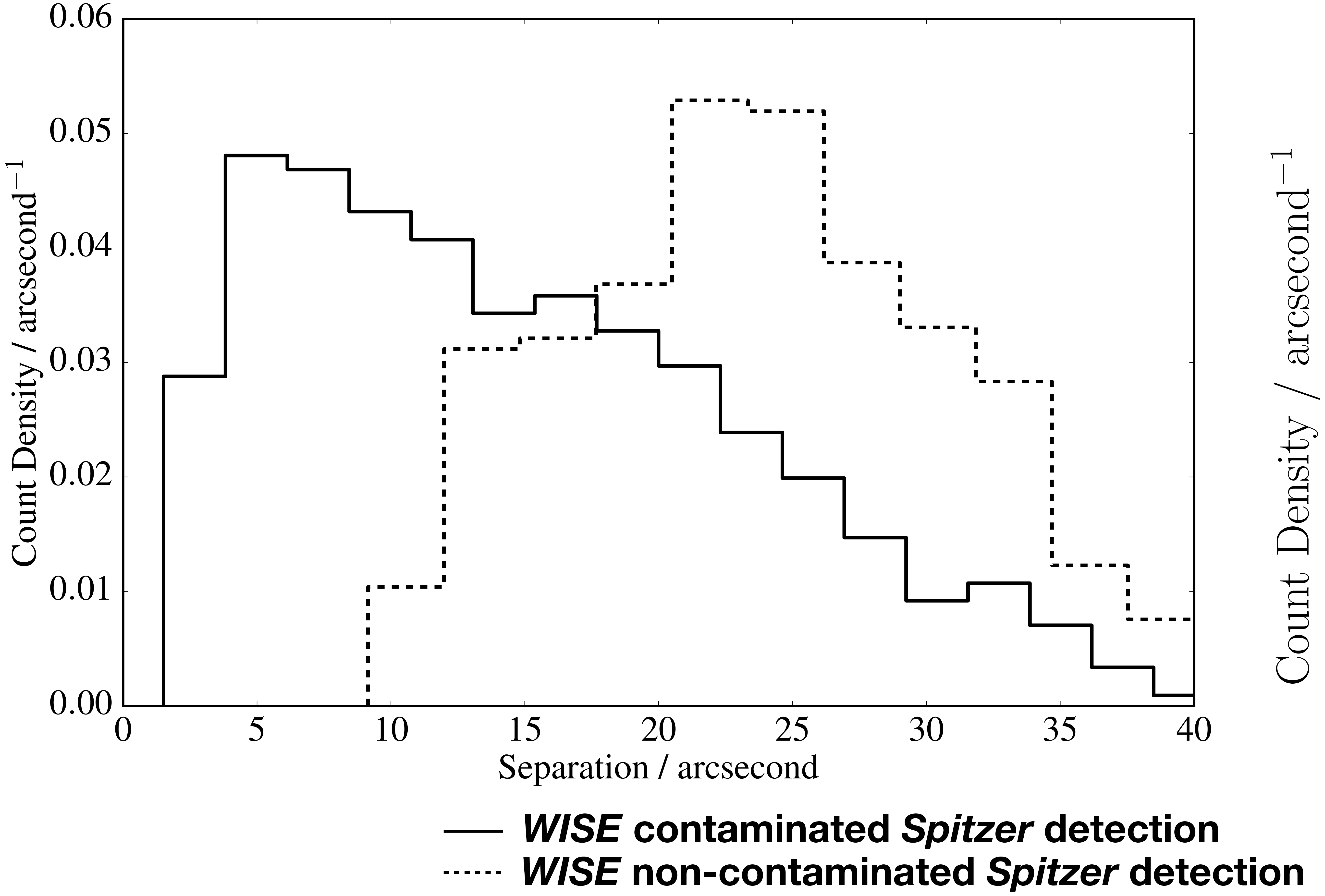
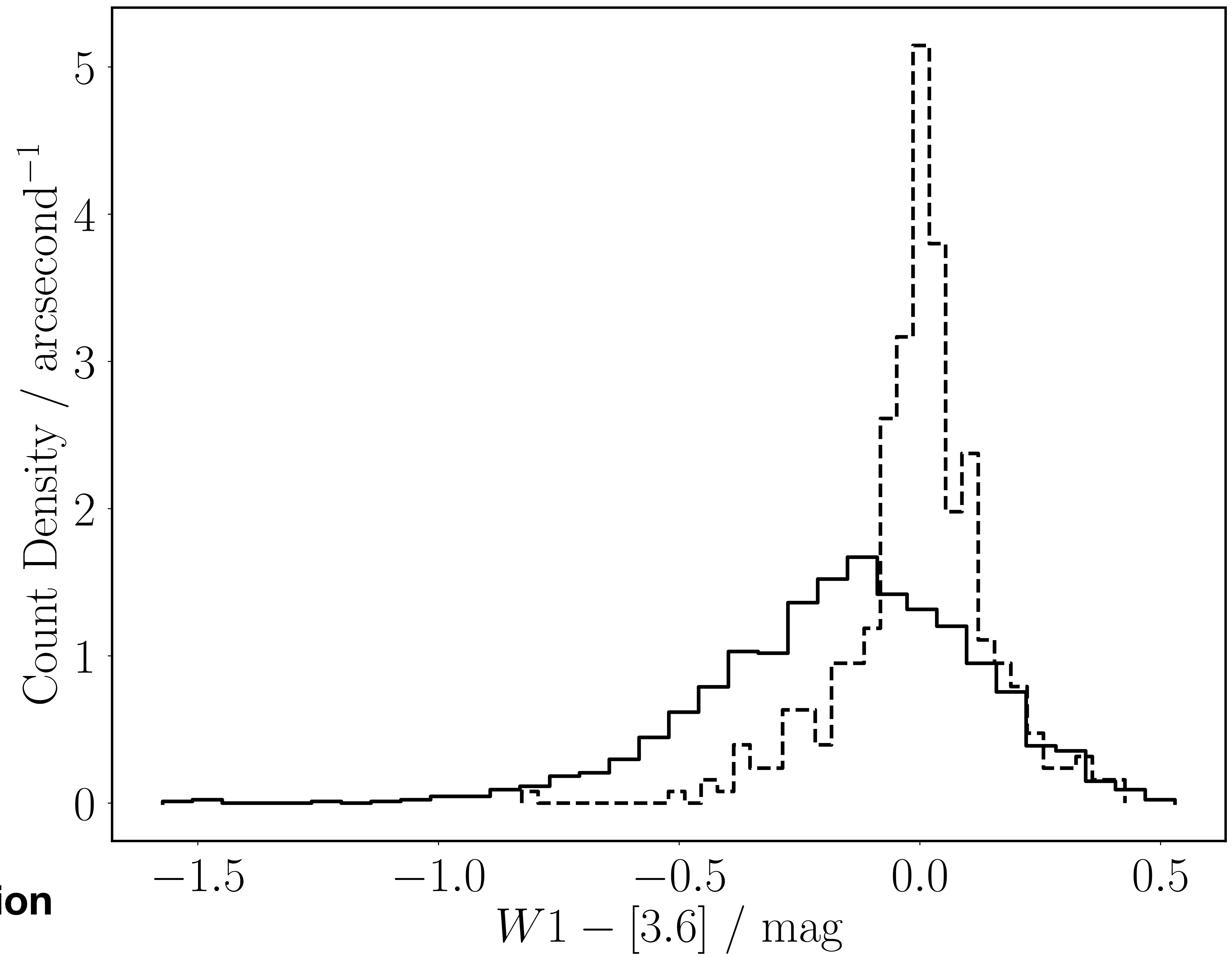
Resolving Contaminants



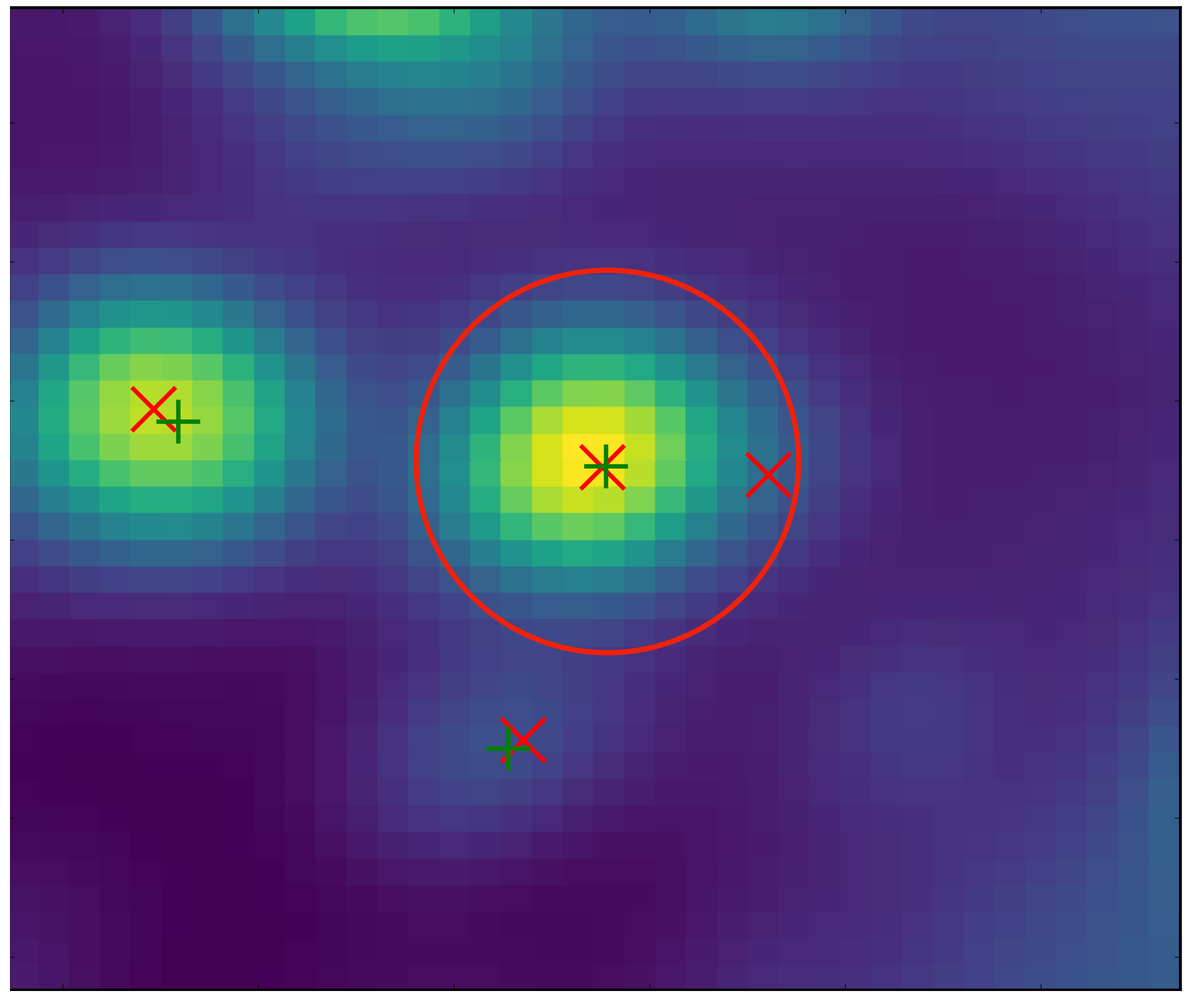


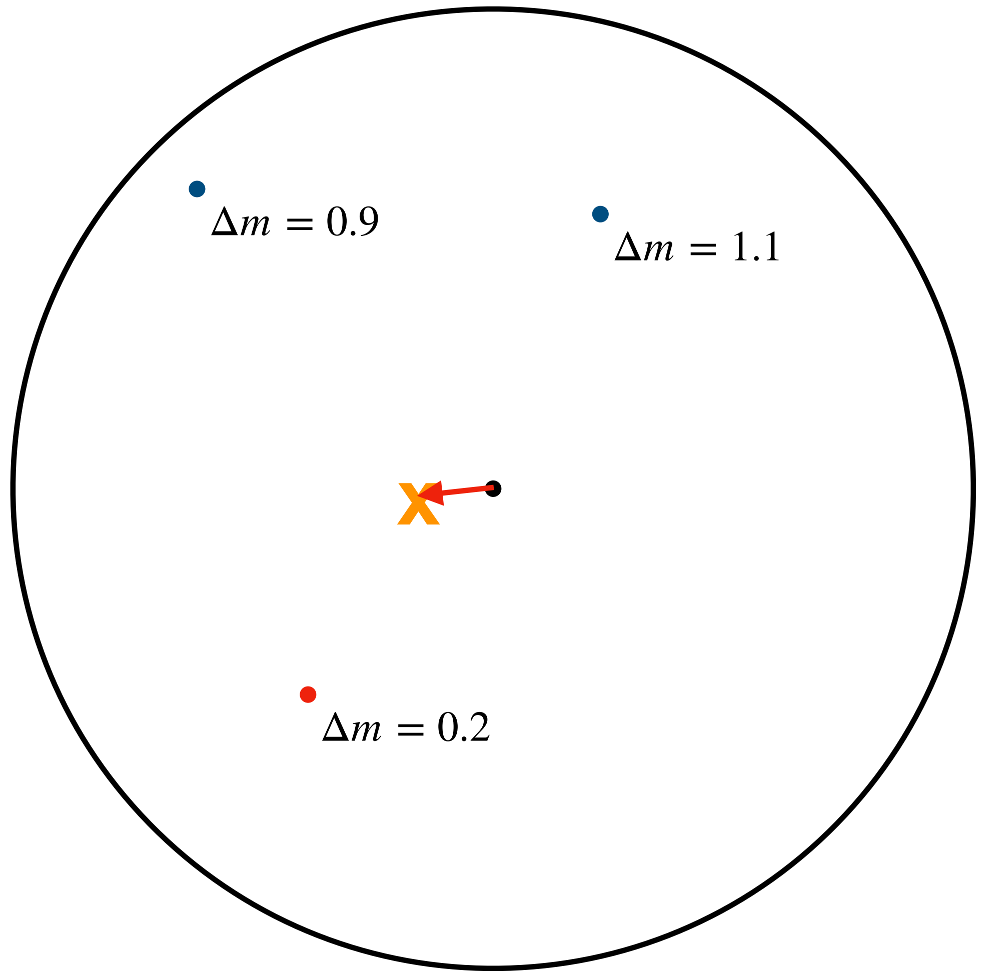
Spitzer - Werner et al. (2004)
IRAC - Fazio et al. (2004)
WISE - Wright et al. (2010)
Wilson & Naylor (2018b)

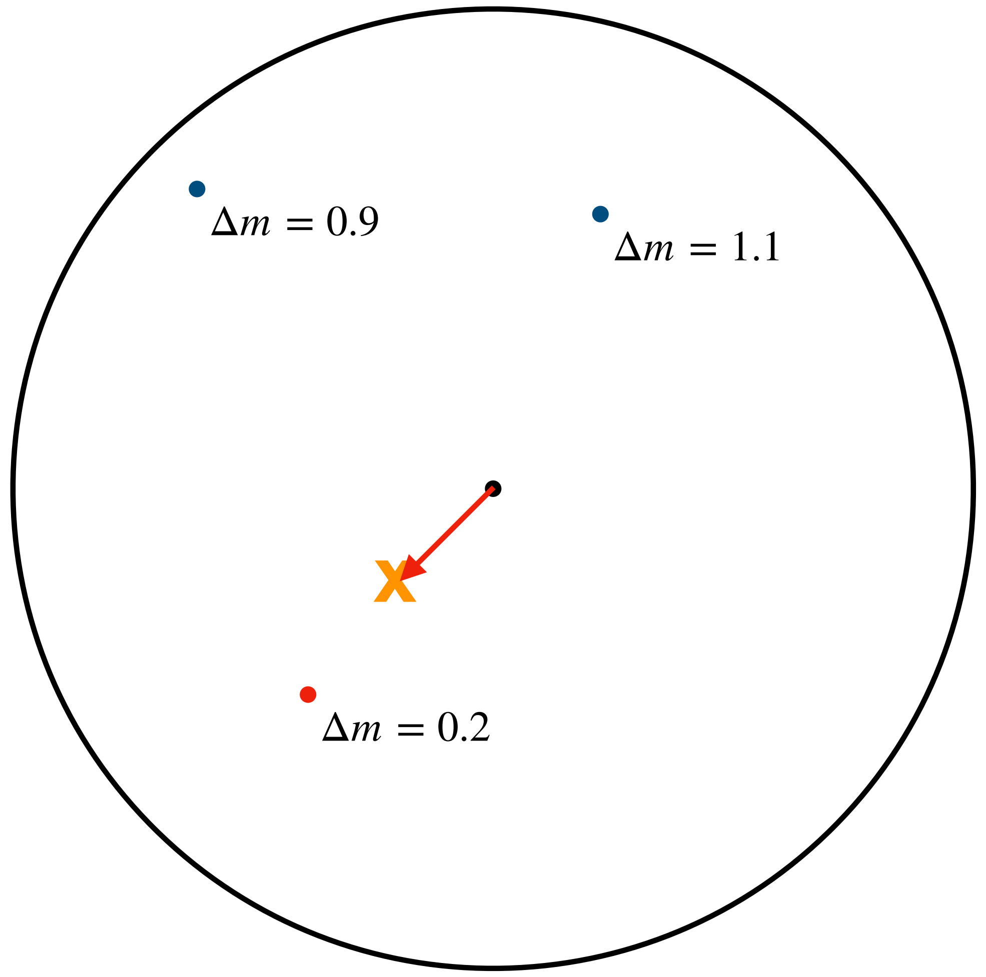
Modelling Crowded-Field Flux Brightening

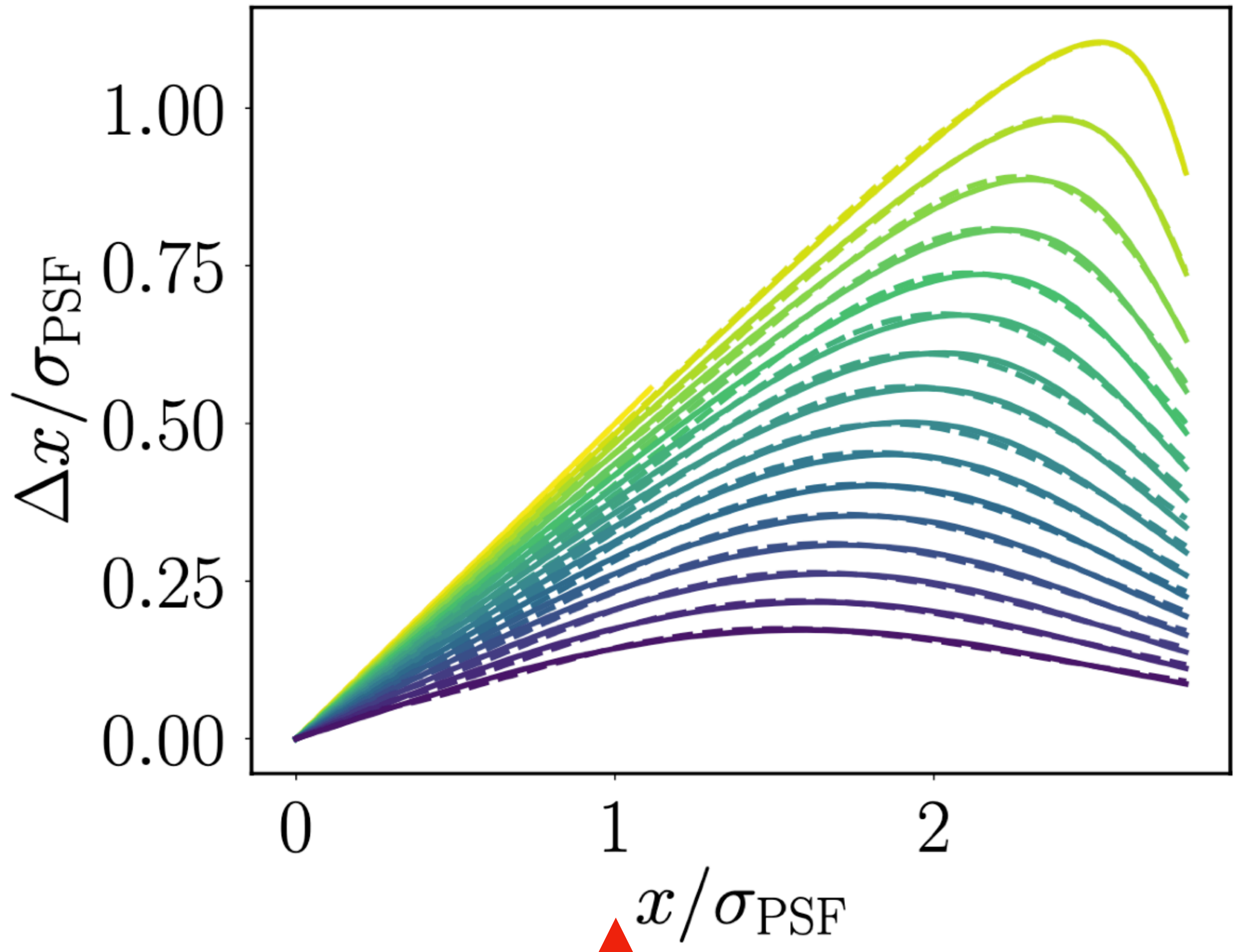
High SNR PSF or Aperture Photometry

Low SNR PSF Photometry









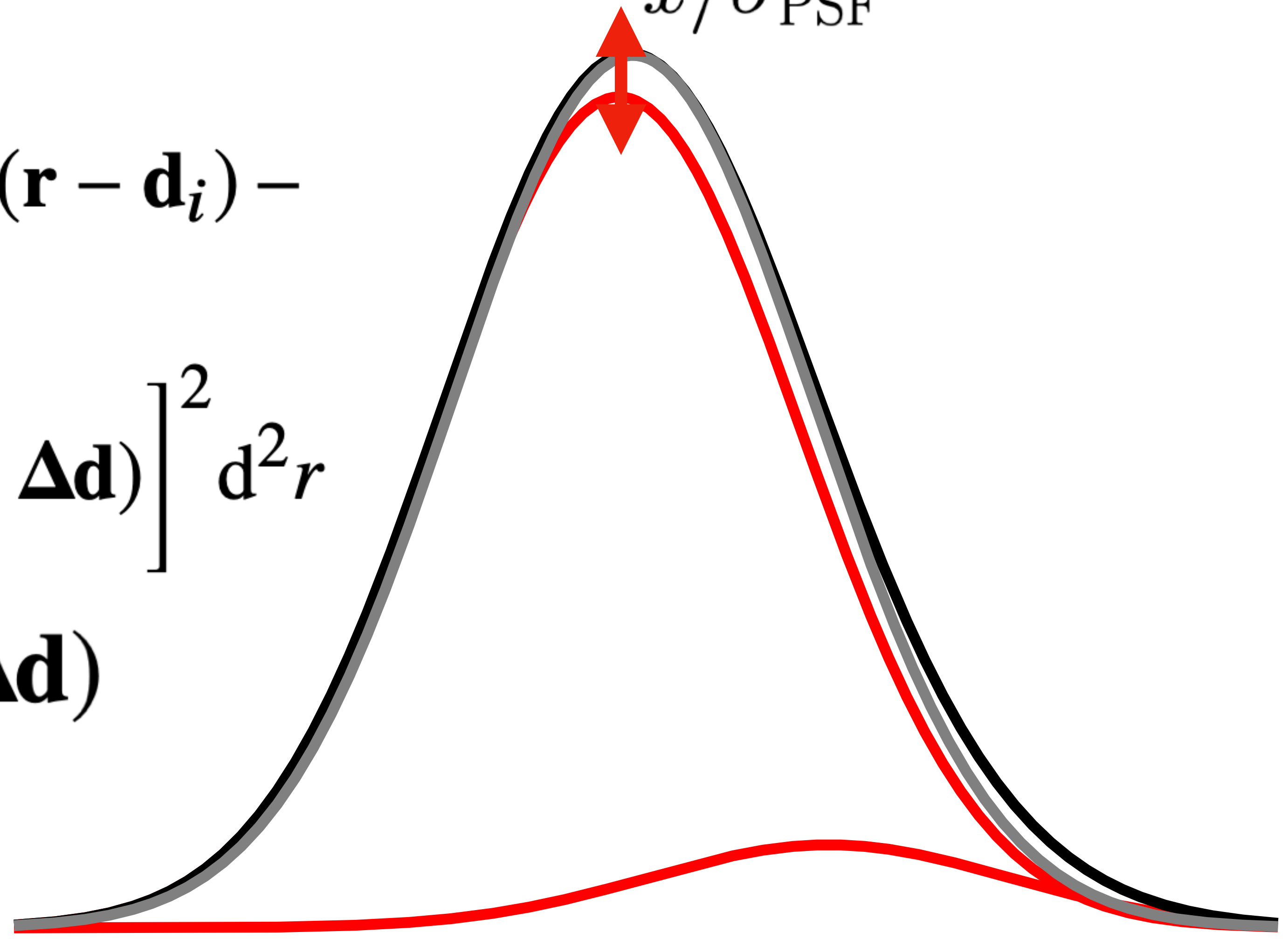
(This raises questions about the validity of quoting photometric statistical precisions if objects are systematically biased, and SED fitting in general in crowded fields)

$$\Delta x = \frac{\sum_i f_i x_i}{1 + \sum_i f_i}$$

$$\Delta f = \sum_i f_i$$

$$\log \mathcal{L} = -\frac{1}{2} \times L \int_{-\infty}^{\infty} \left[\phi(\mathbf{r}) + \sum_i f_i \phi(\mathbf{r} - \mathbf{d}_i) - (1 + \Delta f) \phi(\mathbf{r} - \mathbf{\Delta d}) \right]^2 \mathrm{d}^2 r$$

$$\Delta f = \psi'(\mathbf{\Delta d}) - 1 + \sum_i f_i \psi'(\mathbf{d}_i - \mathbf{\Delta d})$$



Wilson & Naylor (2018b; in prep.)
Plewa & Sari (2018)

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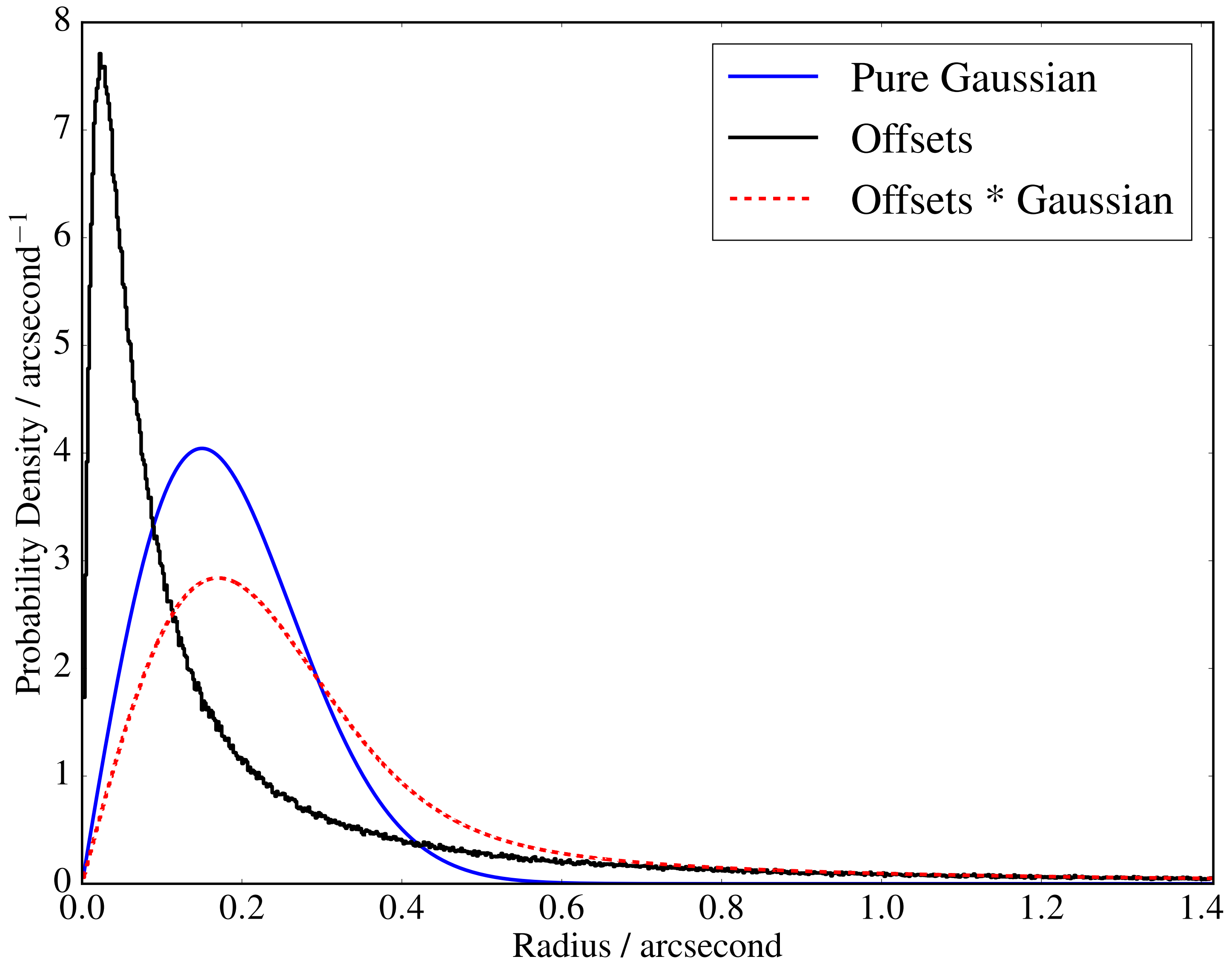
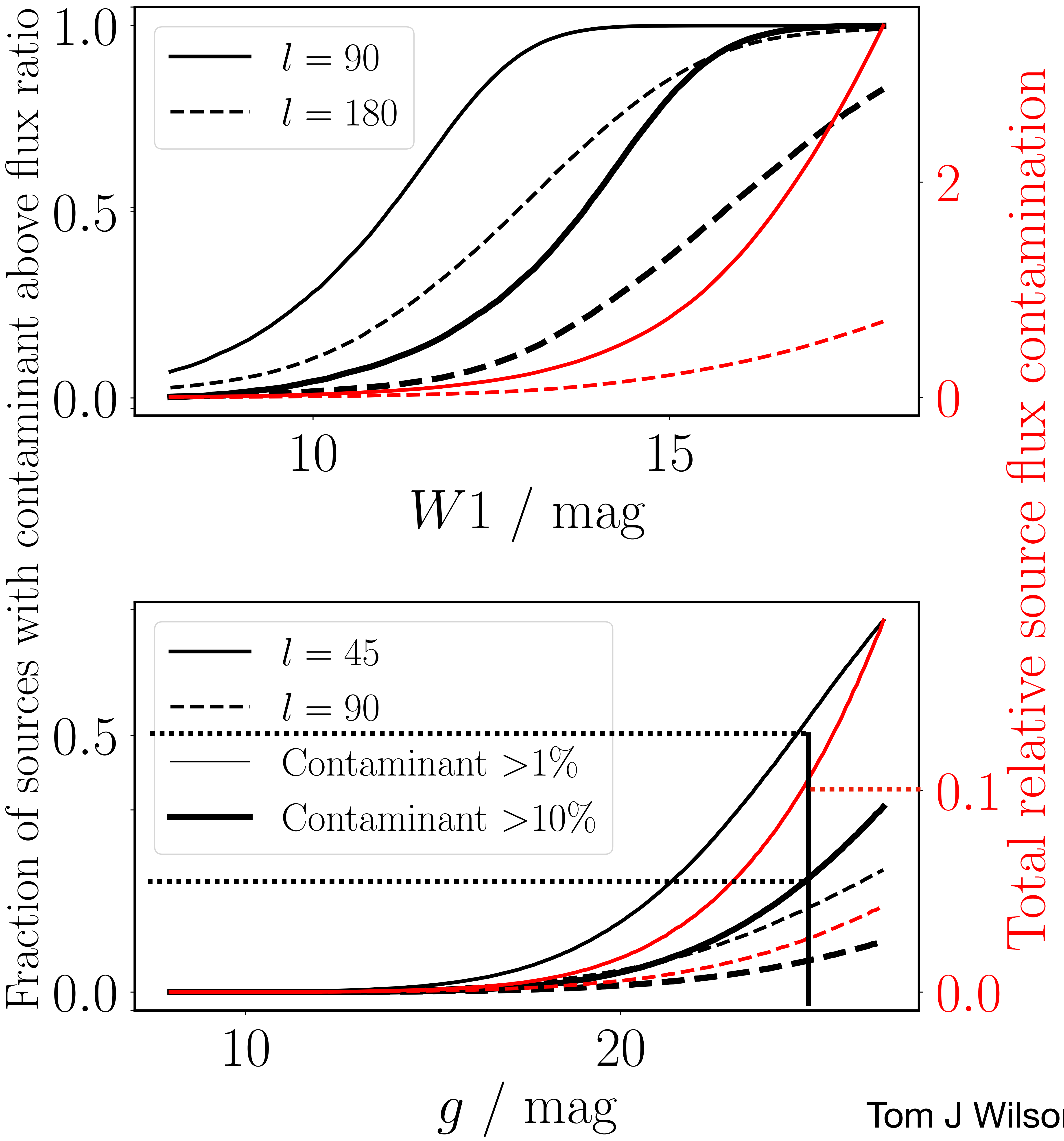
Photometry: Contamination Rates and Amounts





Typical, single visit images in near-Bulge regions of the Plane will have:

- 50% of objects with at least one >1% flux object in their PSF

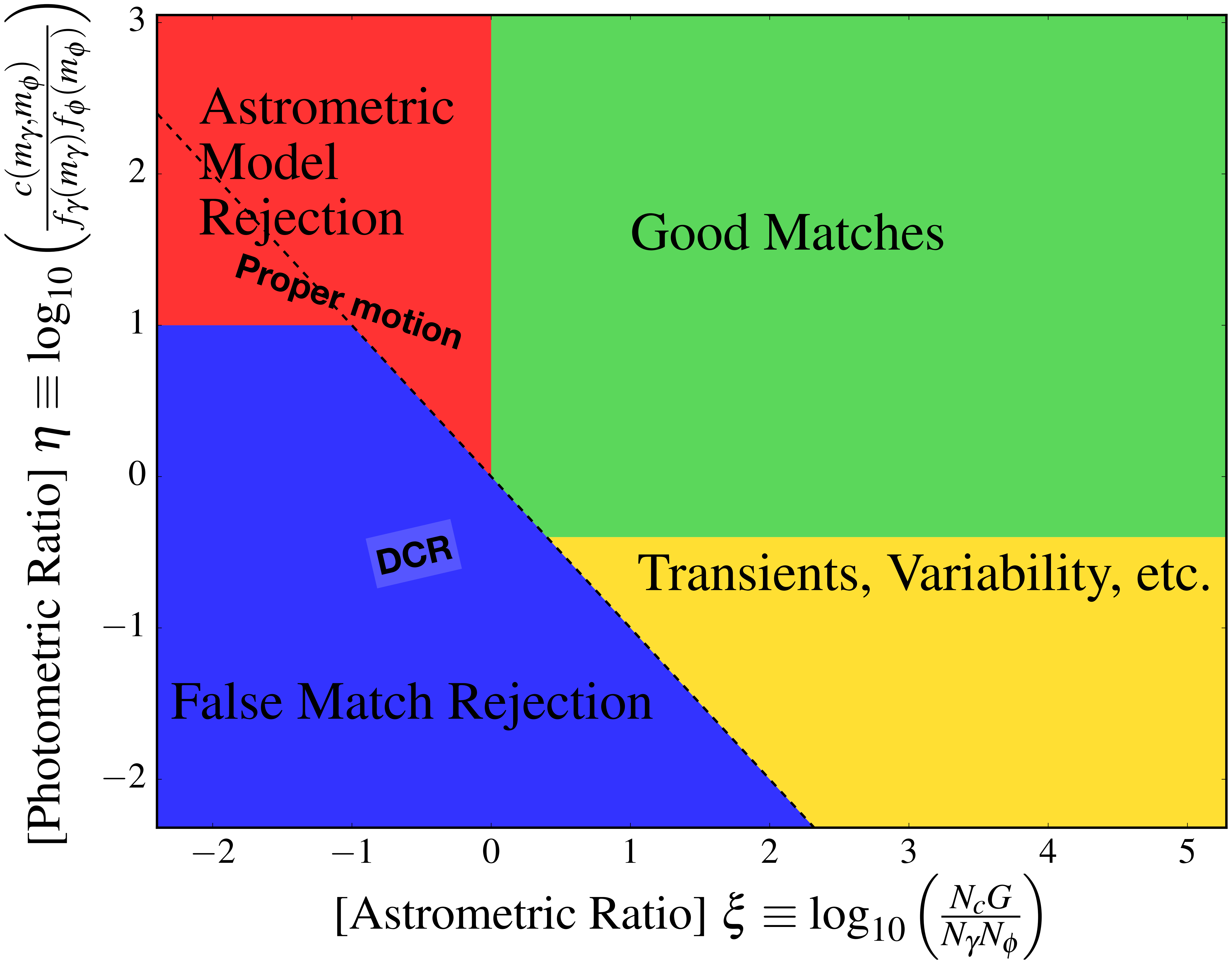
- 20% of objects with a >10% relative flux object contaminating them

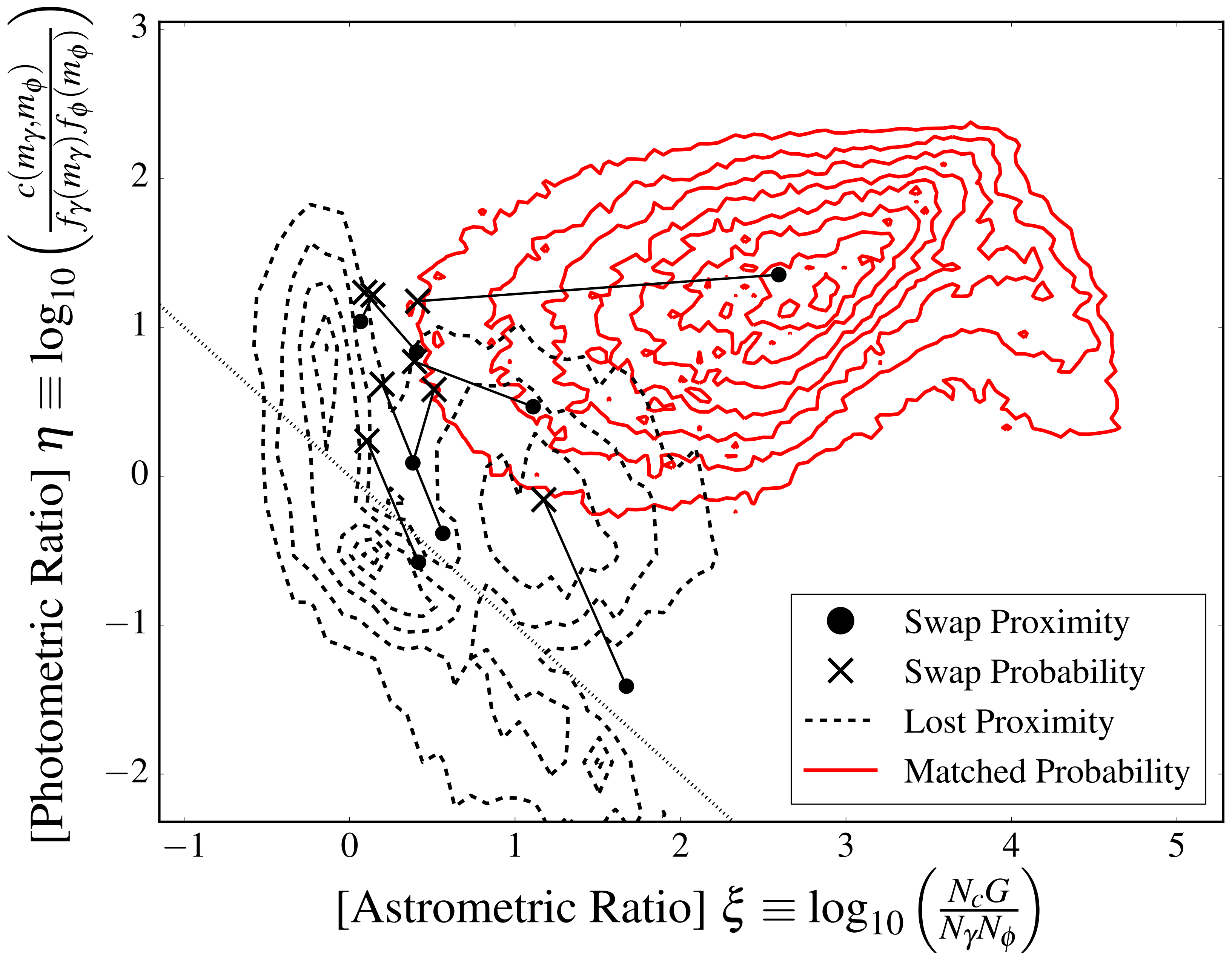
- an average 10% total "extra" flux

(the Bulge will be much more crowded! Nearest-neighbour matching won't work there, but neither will probabilistic matching without taking this effect into account…)

TRILEGAL - Girardi et al. (2005)
Wilson & Naylor (2018b)

The Likelihood Ratio Space





Wilson & Naylor (2018b)

Open Source Code: macauff

Matching Across Catalogues using the Astrometric Uncertainty Function and Flux

https://github.com/macauff/macauff

LSST:UK Consortium

(Points if you know your tartans!)

Tom J Wilson @onoddil

Verifying Astrometry: Accounting For Systematics

In each sightline (10s of sq deg for good bright source counting N):

1. **Cross-match your high angular resolution, high astrometric precision data to LSST to obtain separation distributions**
2. **Create systematics model for all non-centroid astrometric components of uncertainty**
3. **Fit full AUF to data, allowing centroid Gaussian uncertainty to be fit**
4. **Repeat for each brightness (and effectively different astrometric uncertainty)**
5. **Derive fit-quoted astrometric uncertainty relations**

Create crowding-caused perturbation model, for example:

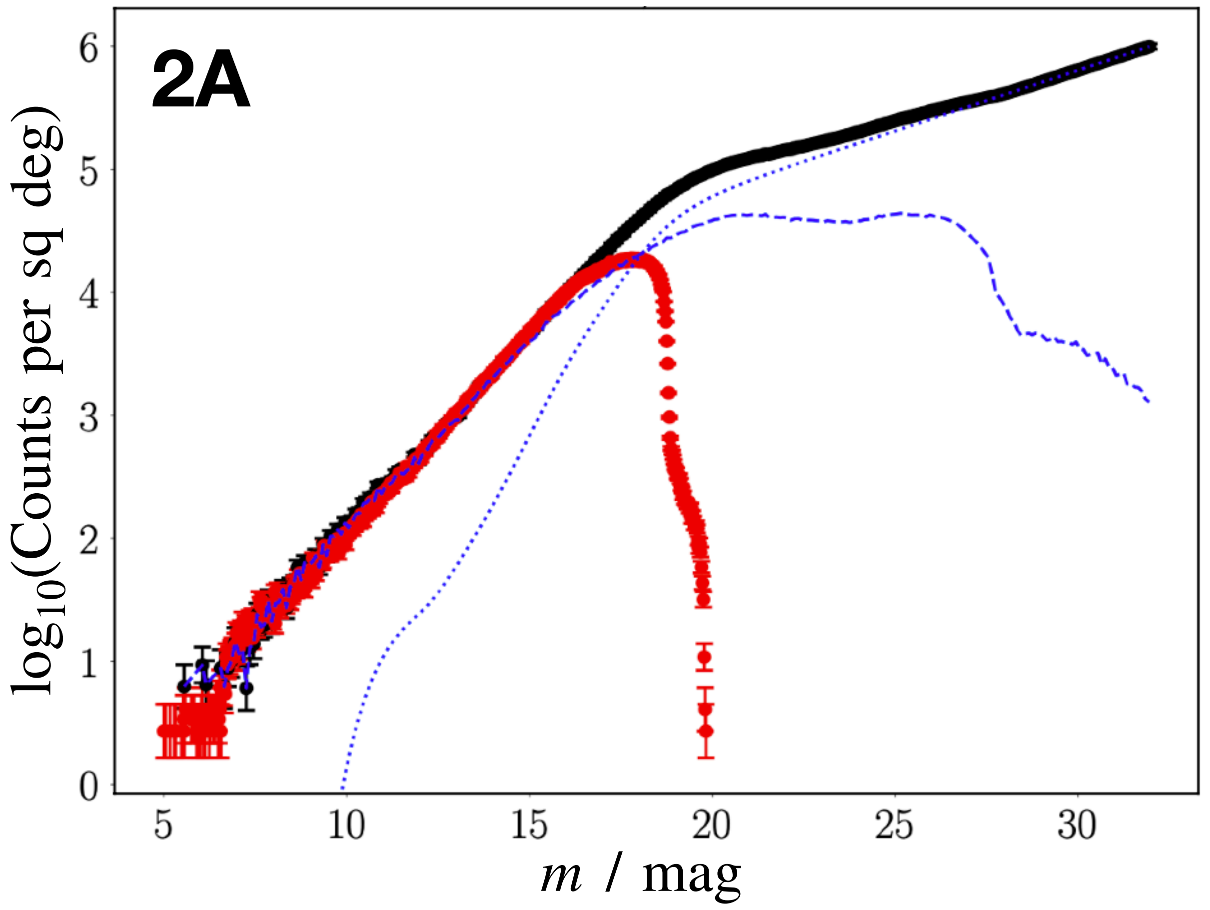
A. **Verify model source count densities match observed data**

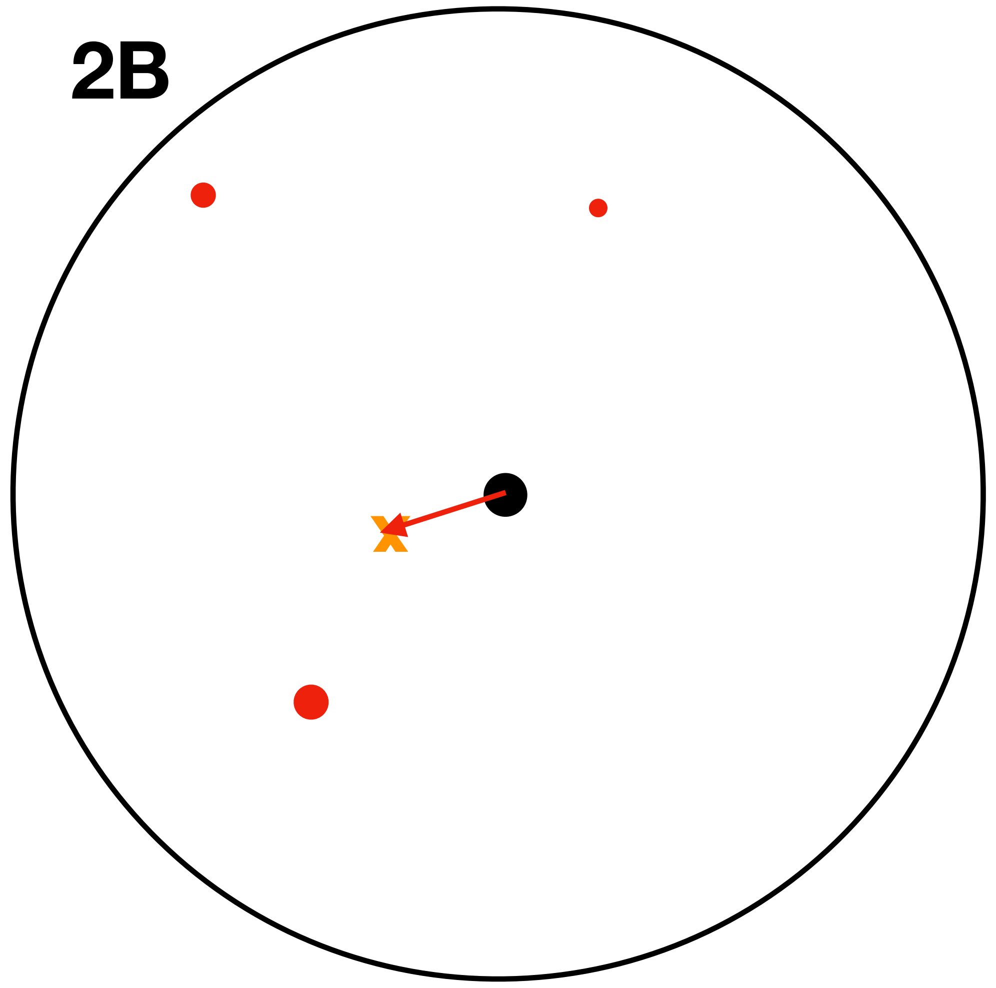
B. **Randomly draw perturbing sources within your PSF ("darts at a dartboard")**

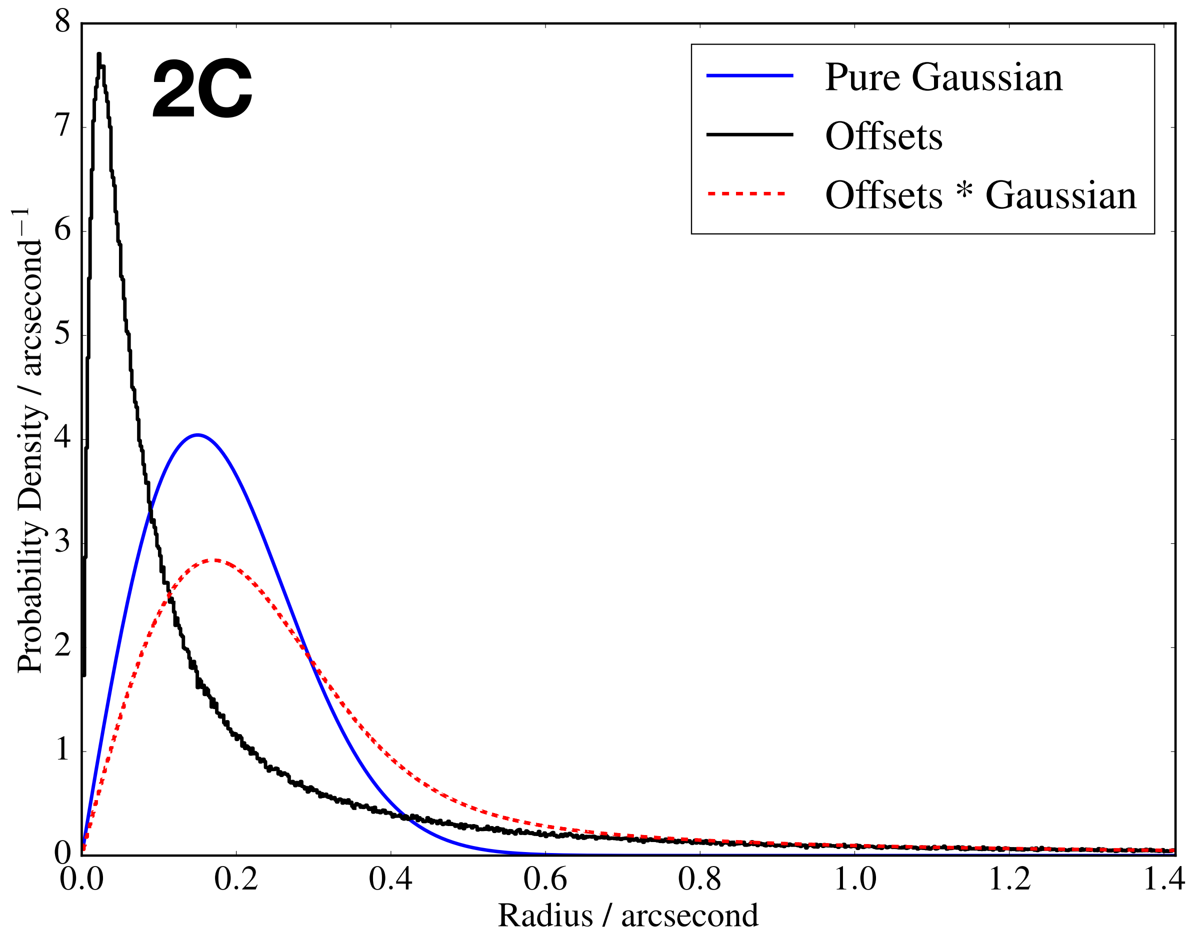
C. **Repeat lots of times to get a distribution of perturbation offsets**

D. **Repeat however many times you have different perturbation algorithms**

E. **Combine your perturbation algorithms**







2D

$$\bar{x} = \frac{1 \times 0 + \sum_i f_i x_i}{1 + \sum_i f_i}$$

- or -

$$\log \mathcal{L} = -\frac{1}{2} \times L \int_{-\infty}^{\infty} \left[\phi(\mathbf{r}) + \sum_i f_i \phi(\mathbf{r} - \mathbf{d}_i) - (1 + \Delta f)\phi(\mathbf{r} - \mathbf{\Delta d}) \right]^2 \mathrm{d}^2 r$$

$$\Delta x(x, y, f) = \begin{cases} f\, x \exp\left(-\frac{1}{4}\frac{x^2+y^2}{\sigma_\psi^2}\right) & f < 0.15 \\ \Omega(x, f) & f \geq 0.15, \end{cases}$$

where

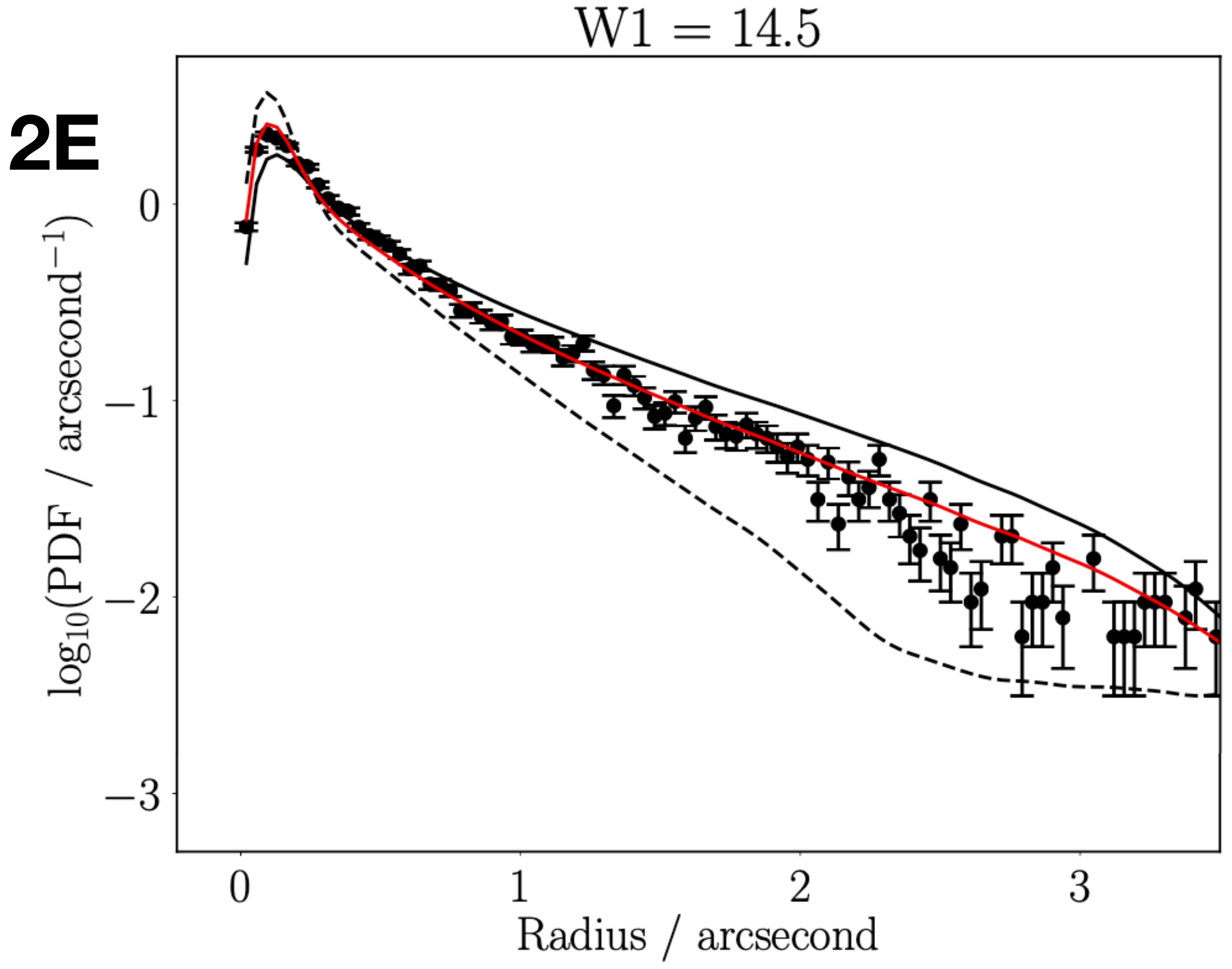
$$\Omega(x, f) = \Omega(x, f, \sigma, \mu, \alpha, T, r_c)$$

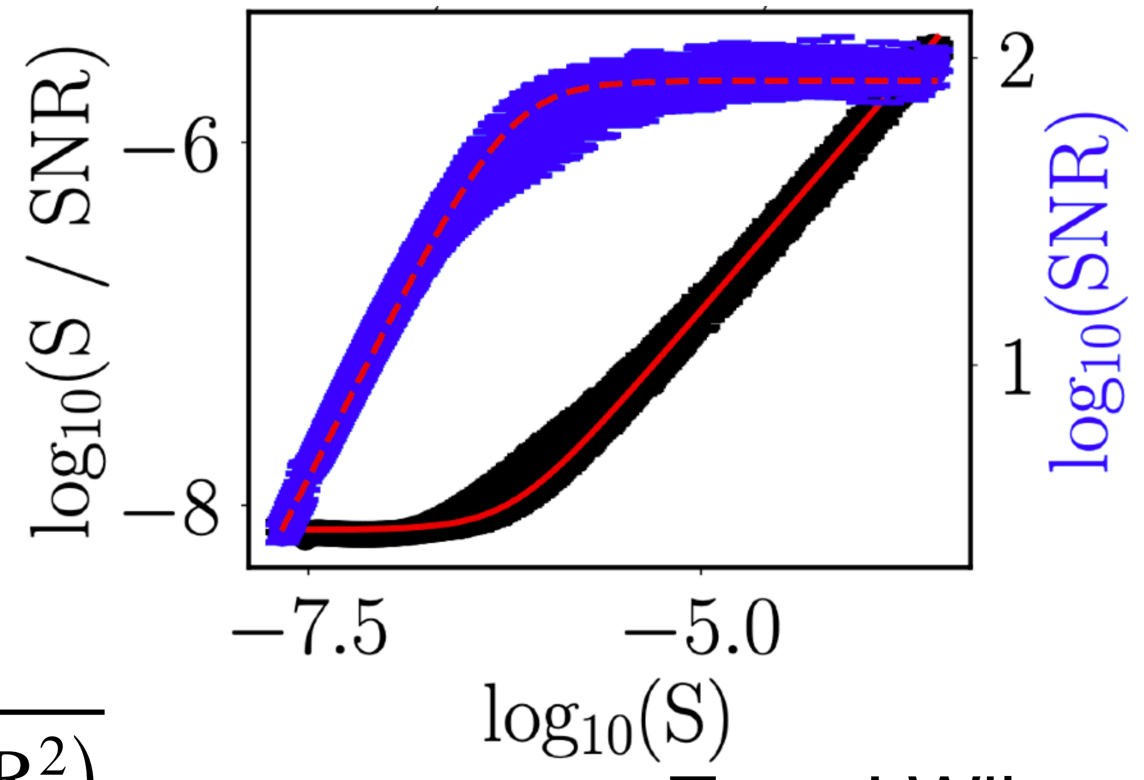
$$= \begin{cases} x f/(1+f) & x < r_c \text{ or } f > 0.9975 \\ 2f\frac{T}{\sigma}\lambda\left(\frac{x-\mu}{\sigma}\right)\Lambda\left(\alpha\frac{x-\mu}{\sigma}\right) & x > r_c \text{ and } f \leq 0.9975 \end{cases}$$

Wilson & Naylor (in prep.)
cf. Plewa & Sari (2018)

$$\mathrm{SNR} = \frac{S}{\sqrt{c \times S + b + (a \times S)^2}}$$

$$H = 1 - \sqrt{1 - \min\left(1,\, a \times \mathrm{SNR}^2\right)}$$





Tom J Wilson @onoddil

Verifying Astrometry: Accounting For Systematics

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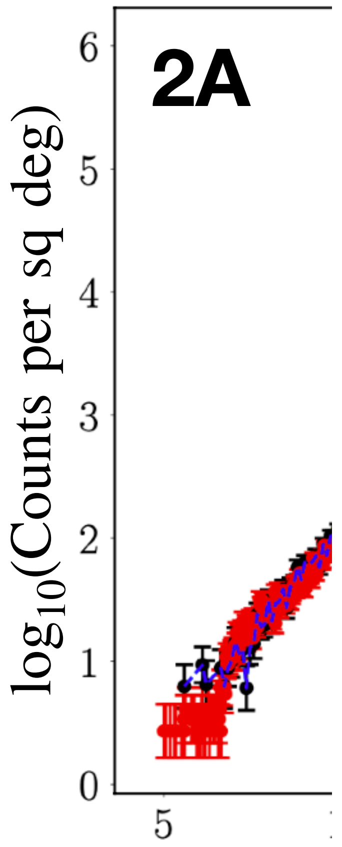
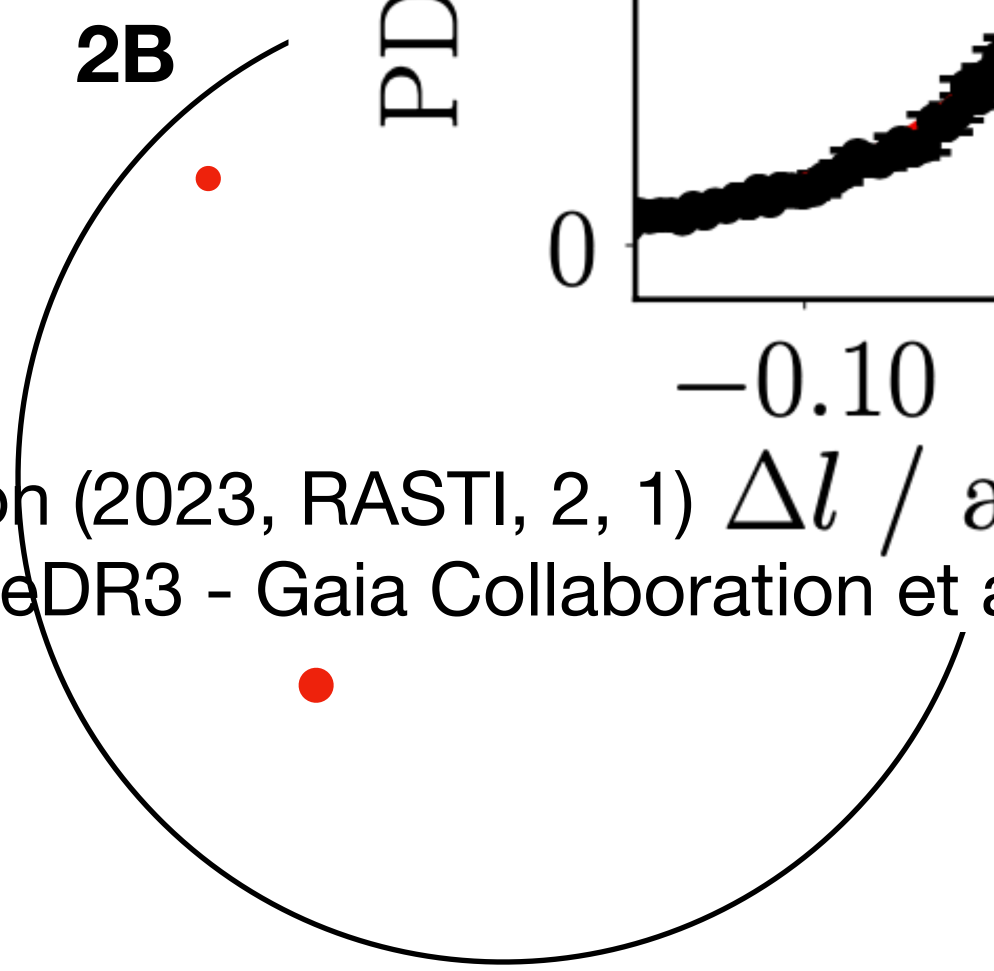
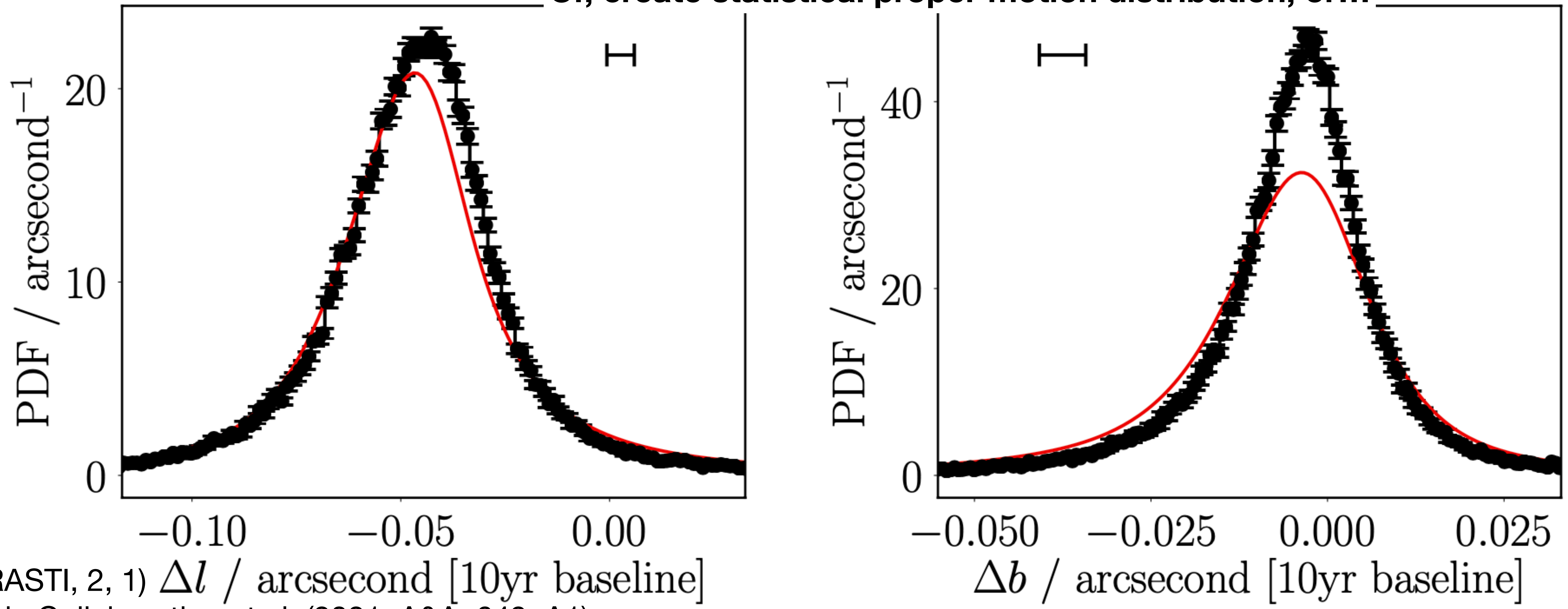
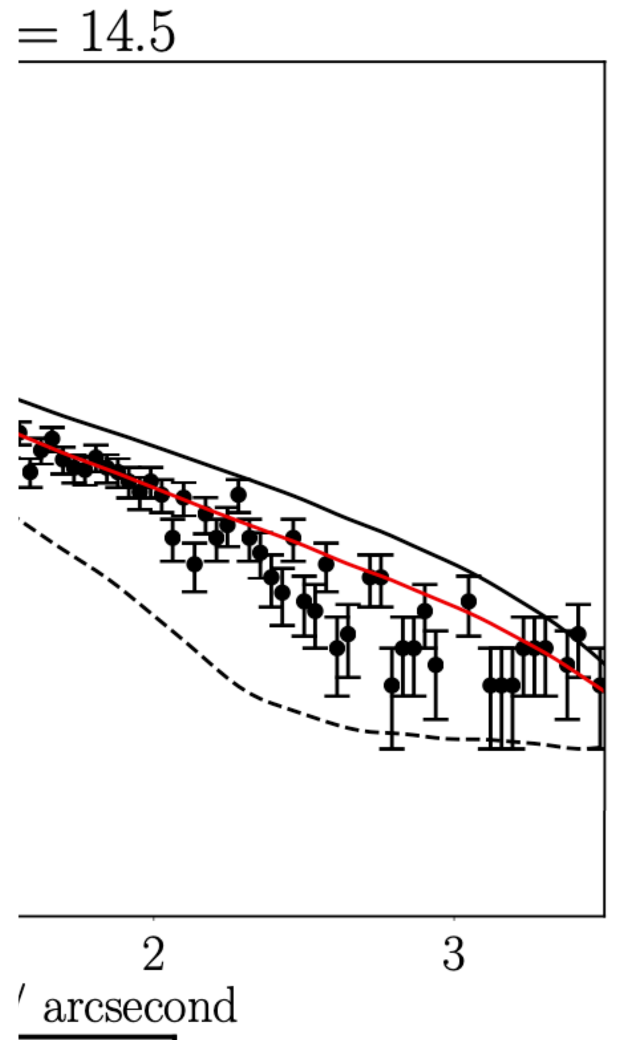
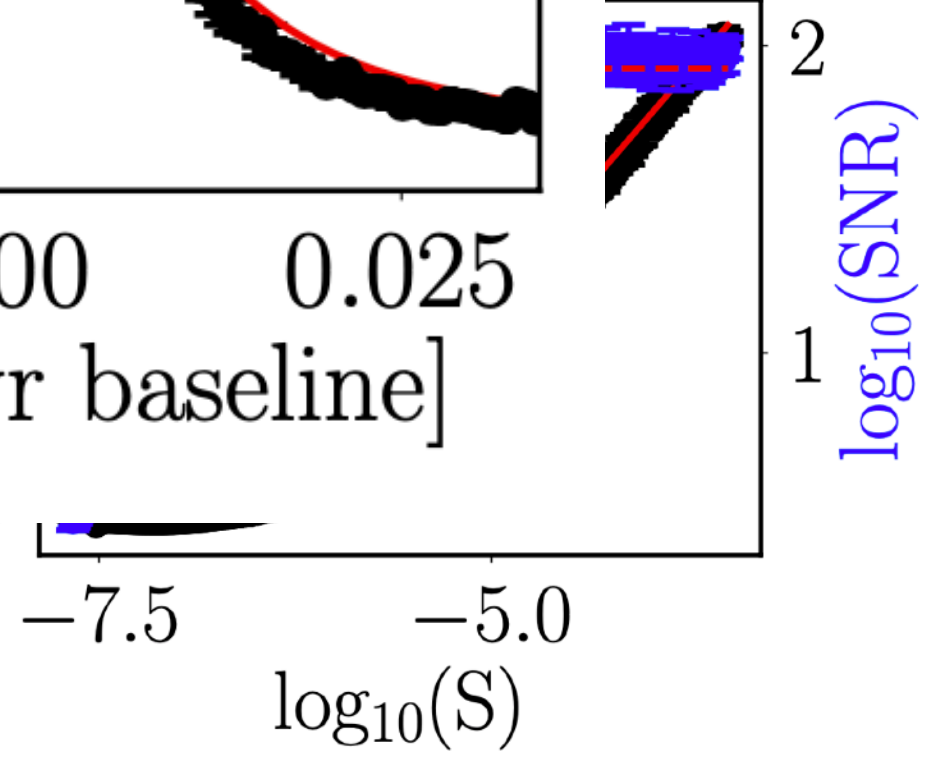


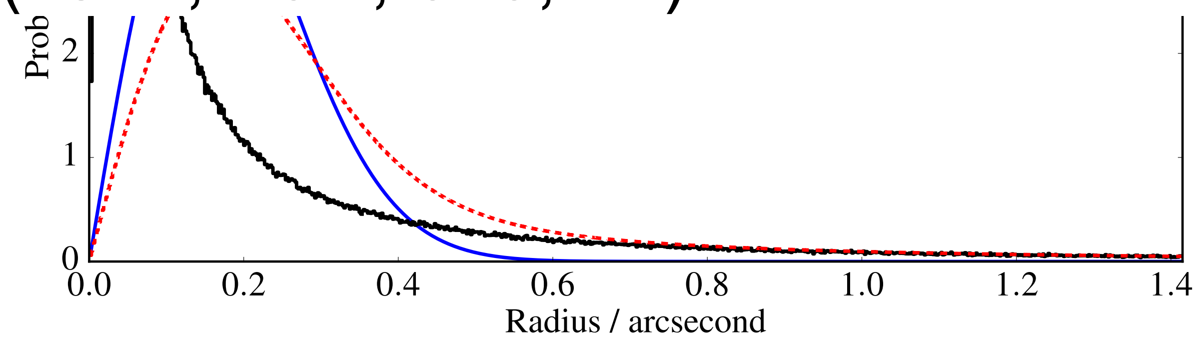


Or, create statistical proper motion distribution, or…









Wilson (2023, RASTI, 2, 1)
Gaia eDR3 - Gaia Collaboration et al. (2021, A&A, 649, A1)

$$\mathrm{SNR} = \frac{}{\sqrt{c \times S + b + (a \times S)^2}}$$

$$H = 1 - \sqrt{1 - \min\left(1,\, a \times \mathrm{SNR}^2\right)}$$

Tom J Wilson @onoddil

Verifying Astrometry: Fitting Centroid Uncertainty

In each sightline (10s of sq deg for good bright source counting N):

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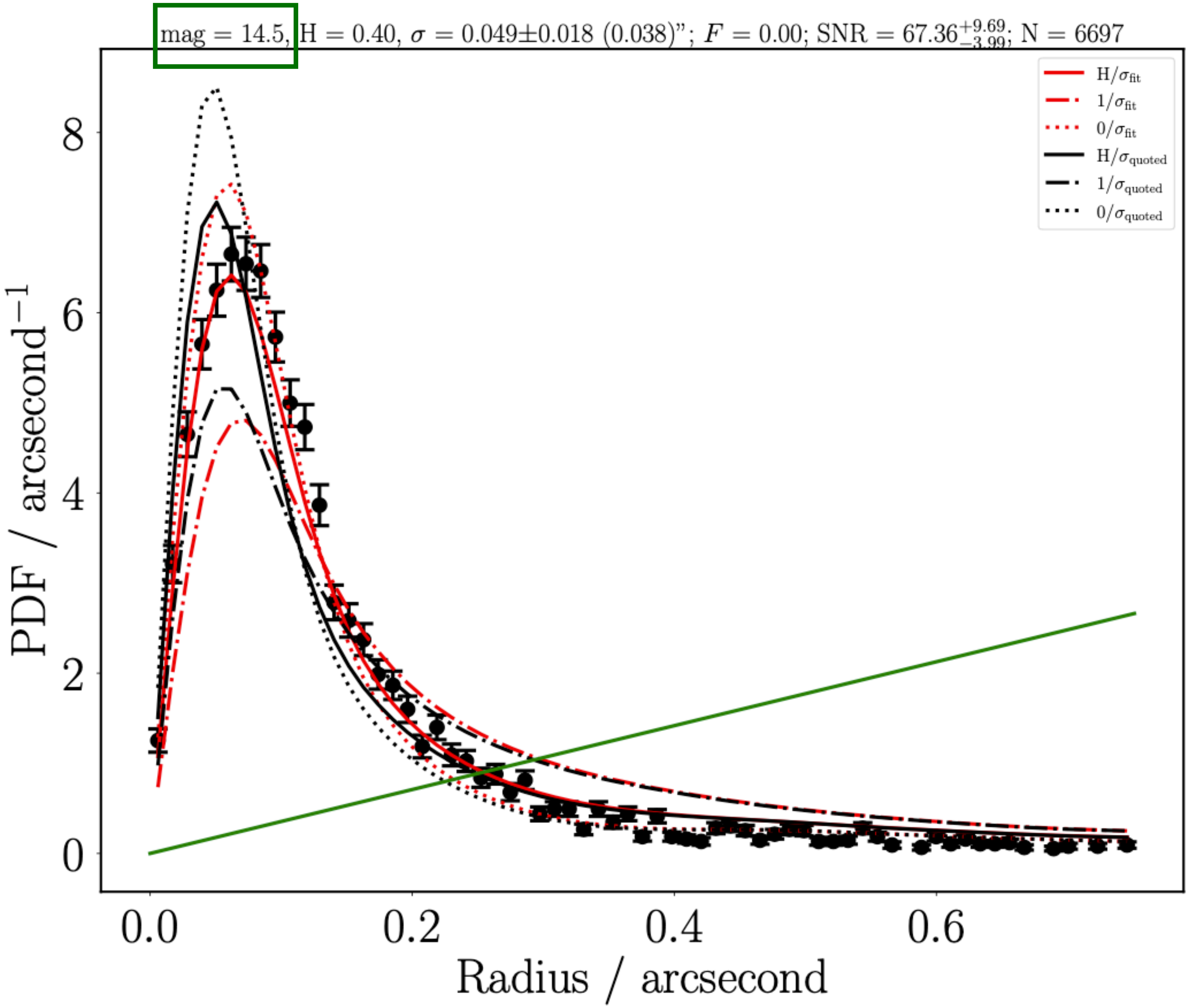
3. **Fit full AUF to data, allowing centroid Gaussian uncertainty to be fit**
4. **Repeat for each brightness (and effectively different astrometric uncertainty)**
5. **Derive fit-quoted astrometric uncertainty relations**

For each magnitude (uncertainty) slice in a given sightline, combine centroid uncertainty (Gaussian) and other AUF components (empirical) and fit for best-fitting sigma-value.

$$h_\gamma = h_{\gamma,\mathrm{centroiding}} * h_{\gamma,\mathrm{perturbation}} * \ \ldots$$

$$g(\Delta x, \Delta y, \sigma) = (2\pi\sigma^2)^{-1}\exp\left(-\frac{1}{2}\frac{\Delta x^2 + \Delta y^2}{\sigma^2}\right)$$

Also include false positive match rate (*F*) in case simple match case was not perfect



WISE - Wright et al. (2010)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

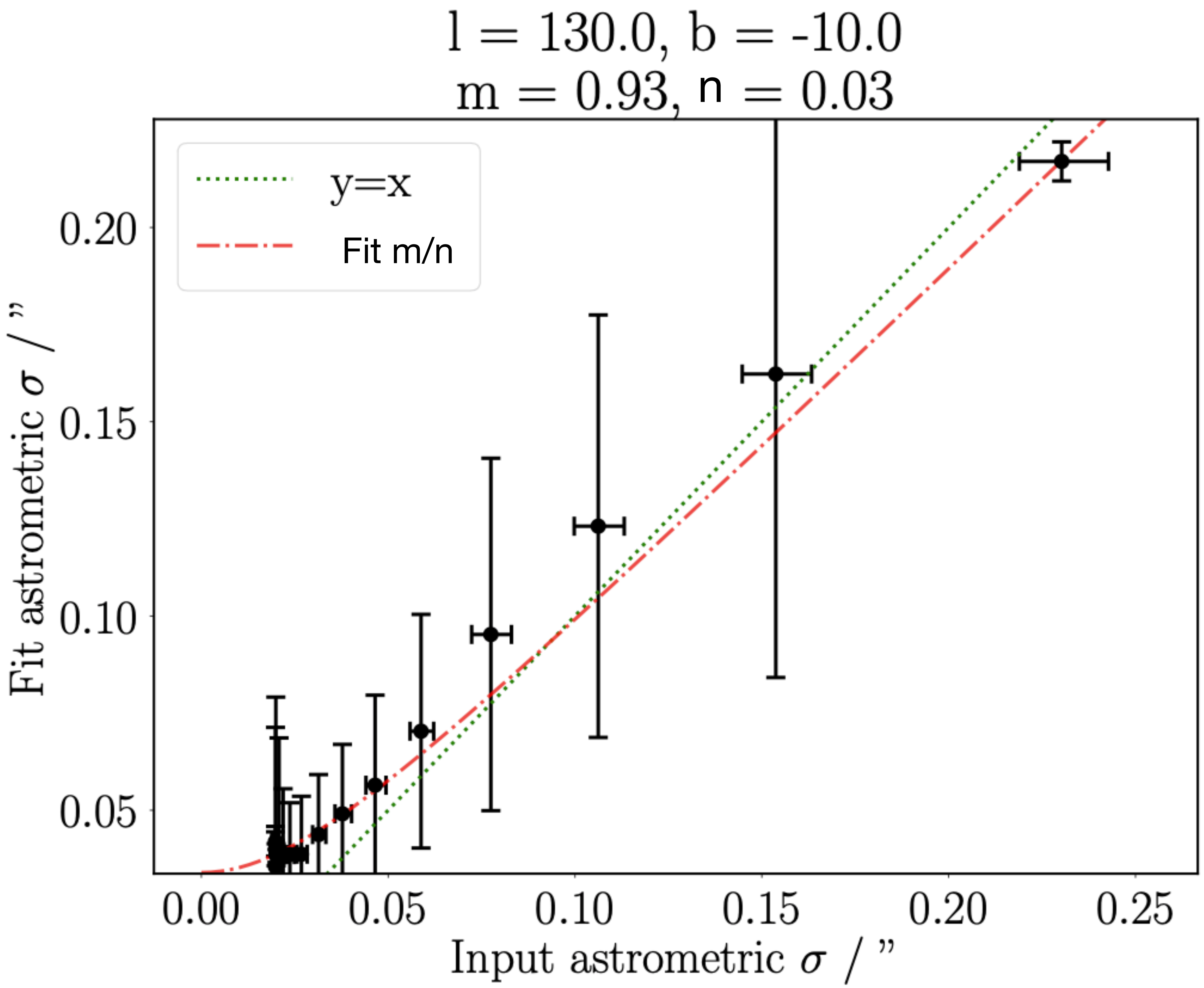
Tom J Wilson @onoddil

Verifying Astrometry: Characterisation

In each sightline (10s of sq deg for good bright source counting N):

1. **Cross-match your high angular resolution, high astrometric precision data to LSST to obtain separation distributions**
2. **Create systematics model for all non-centroid astrometric components of uncertainty**
3. **Fit full AUF to data, allowing centroid Gaussian uncertainty to be fit**
4. **Repeat for each brightness (and effectively different astrometric uncertainty)**
5. **Derive fit-quoted astrometric uncertainty relations**

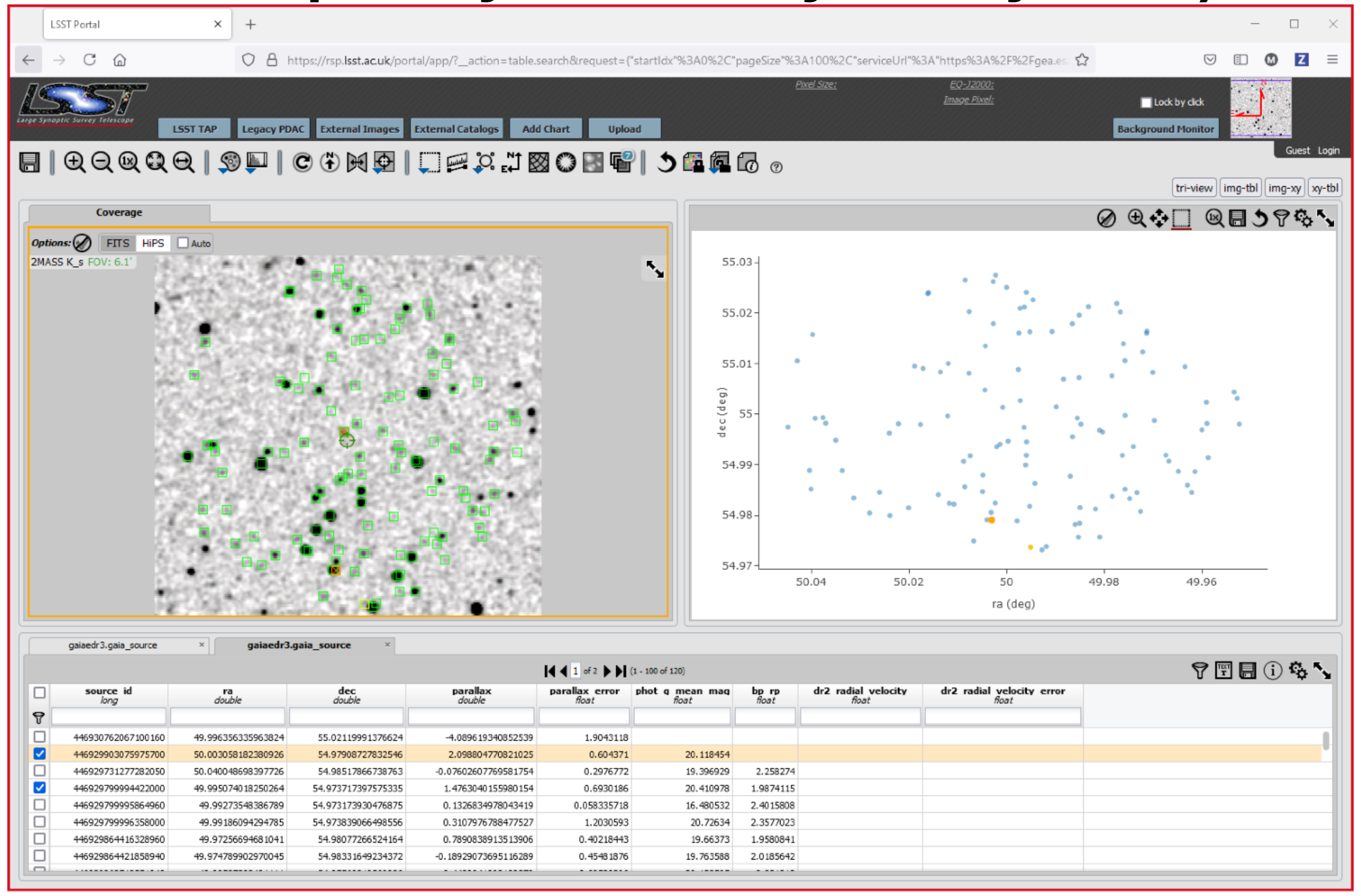
Fit for $y = \sqrt{(mx)^2 + n^2}$ (or, optionally, $y = mx + n$) to account for simple systematic bias n missing and compensating scaling factor m at lower SNR data



Tom J Wilson @onoddil

How To Use Our Cross-Matches

(Or, how this impacts you on a day-to-day basis)



Three tables per cross-match: merged catalogue dataset, and 2x non-match dataset (one per catalogue)

Example columns:

- **Designations of the two sources (e.g., WISE J… and *Gaia* EDR3…)**
- **RA and Dec (or Galactic l/b) of the two sources**
- **Magnitudes (corrected for necessary effects, such as e.g. *Gaia*) in all bandpasses for both objects**
- **Match probability — probability of the most likely permutation (see equation 26 of Wilson & Naylor 2018a)**
- **Eta - Photometric likelihood ratio (counterpart vs non-match probability, just for brightnesses; see eq37 of WN18a)**
- **Xi - Astrometric likelihood ratio (just position match/non-match comparison; see eq38 of WN18a)**
- **Average contamination - simulated mean (percentile) brightening of the two sources, based on number density of catalogue**
- **Probability of sources having blended contaminant above e.g. 1% relative flux**

We will provide a two match runs per catalogue pair match: one with, and one without, the photometry considered, to allow for the recovery of sources with “weird” colours but otherwise agreeable astrometry

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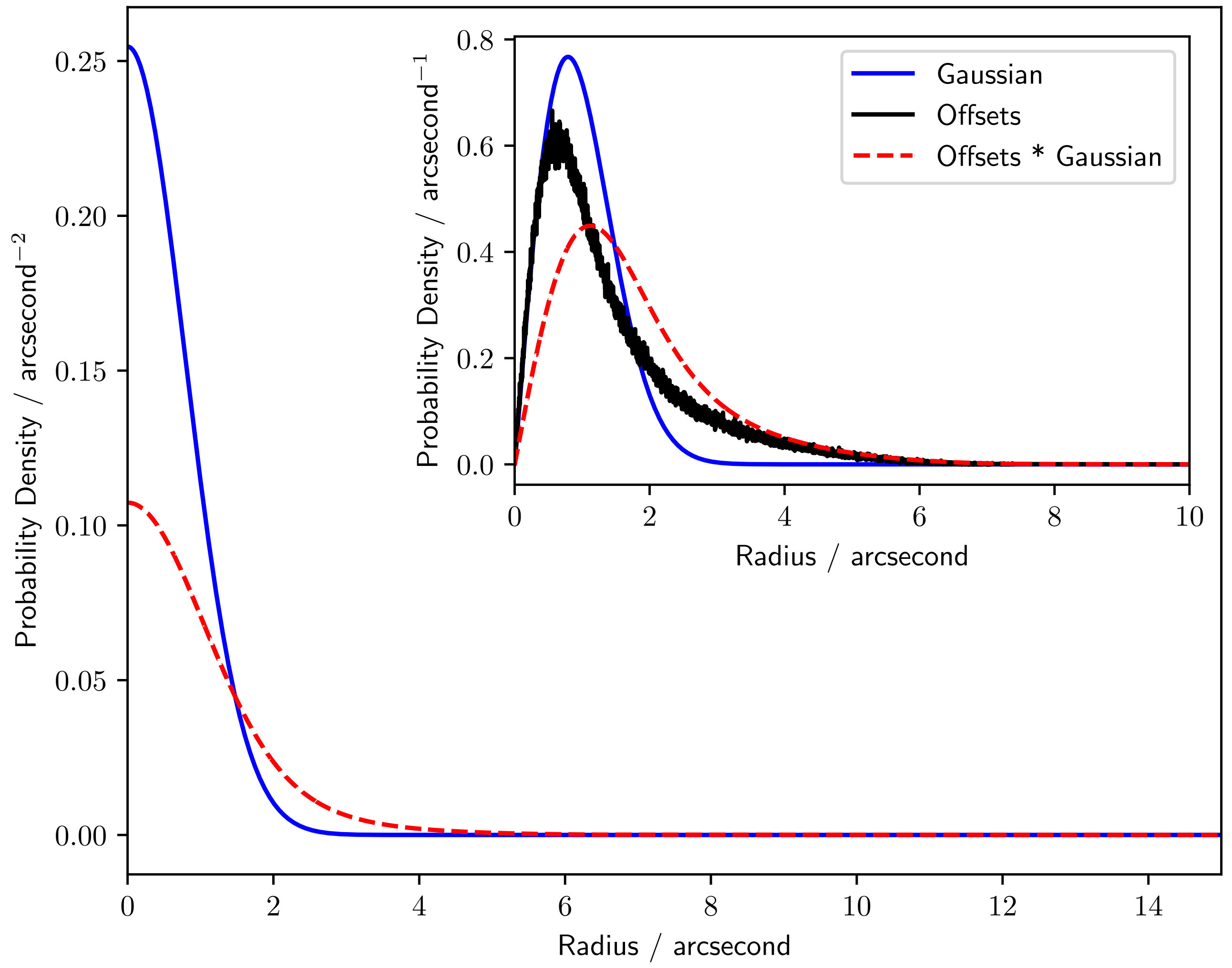
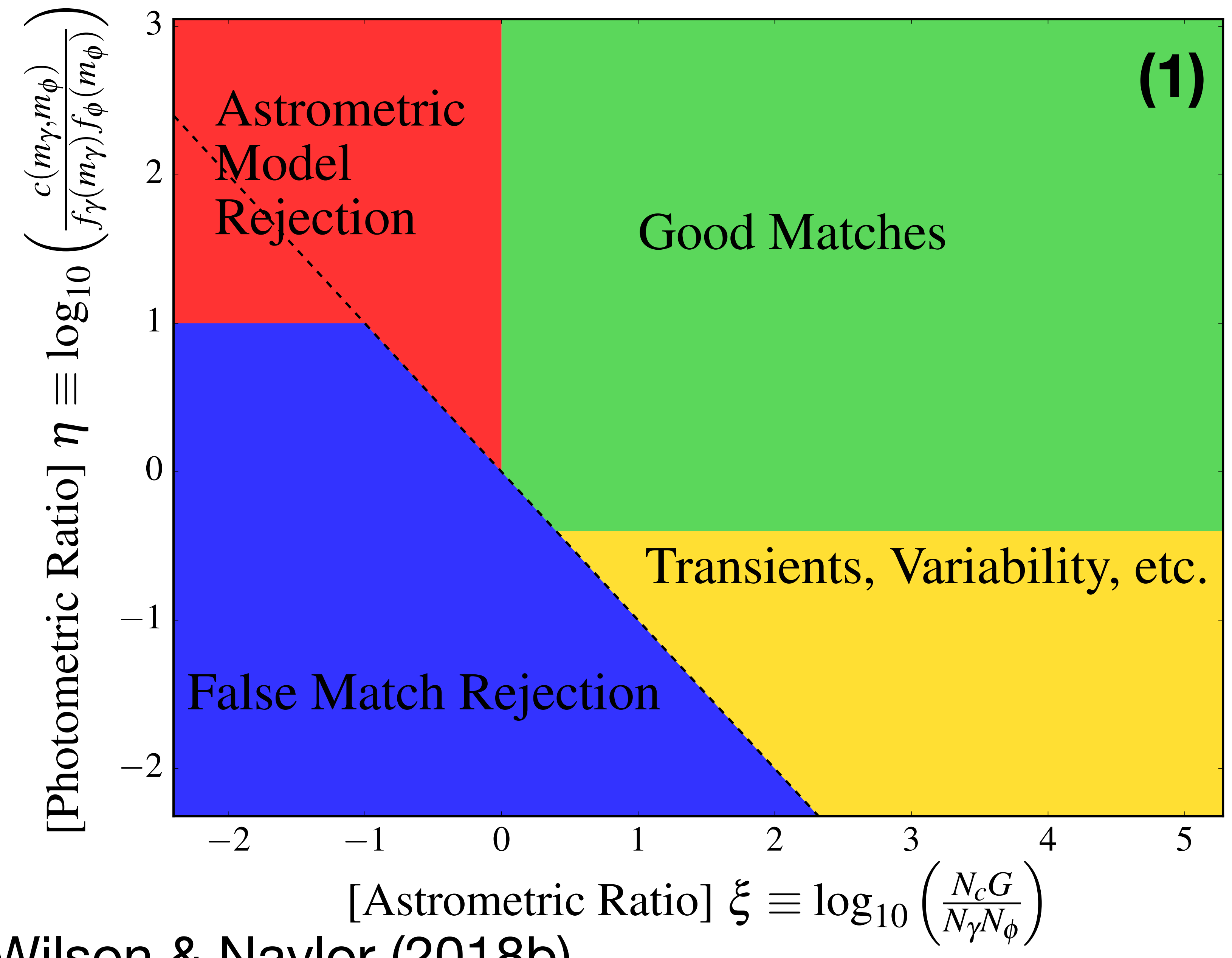
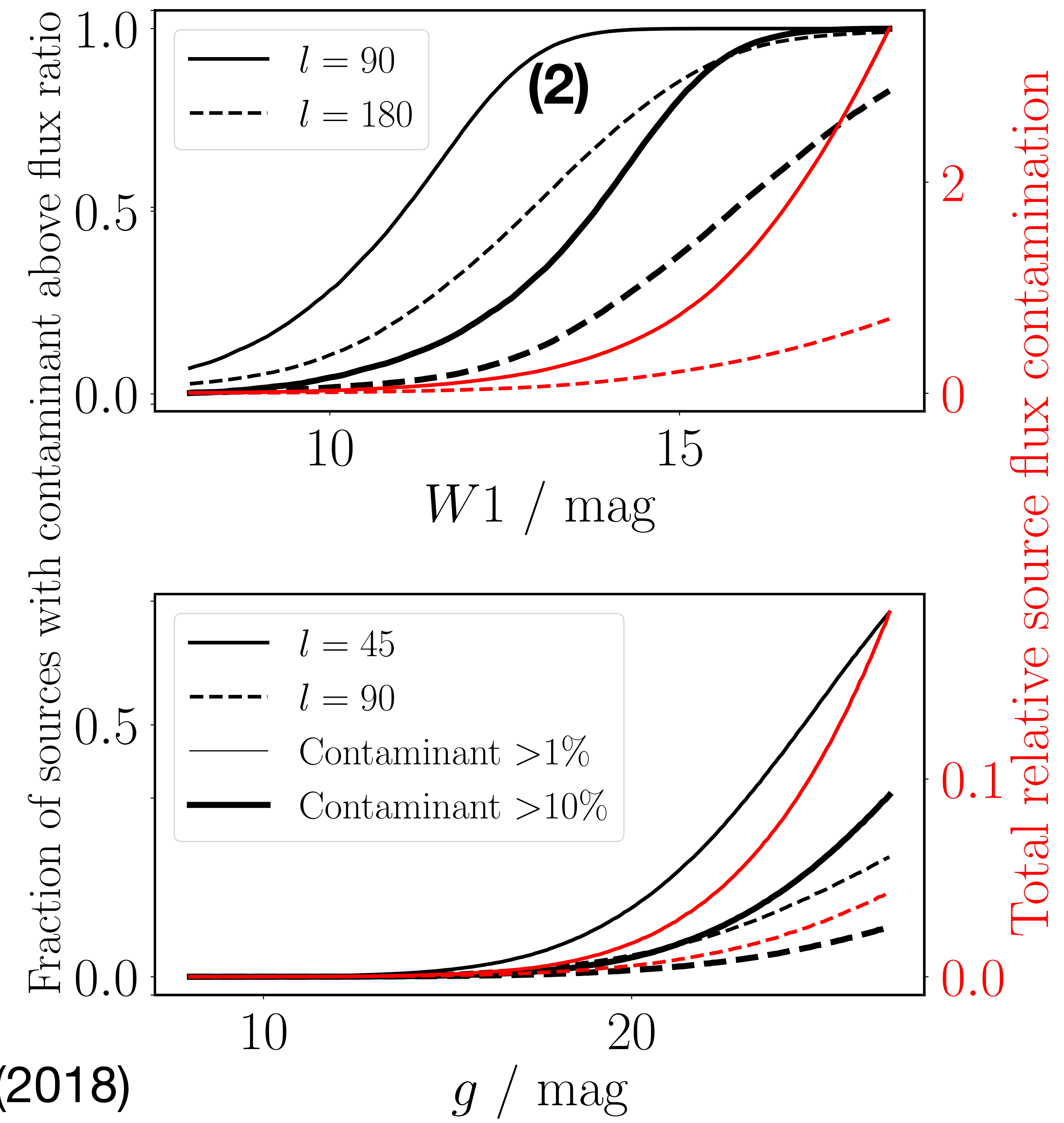
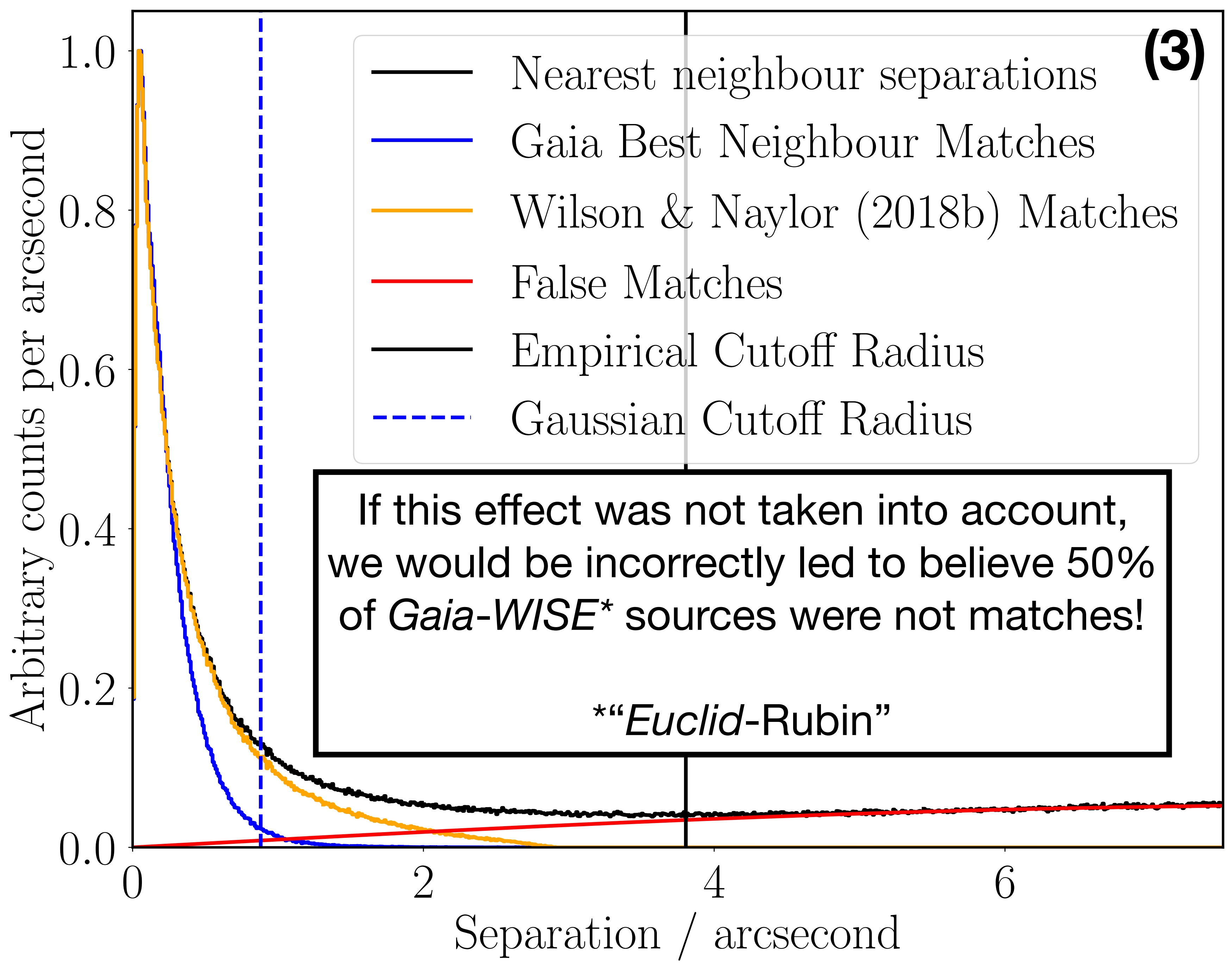
Why Use Macauff's Cross-Matches?

0) Getting cross-matches, even for "well behaved" fields
1) Finding "odd" objects, either using the inclusion vs non-inclusion of the photometry in the two match runs, or via the likelihood ratio space — separately-planned "real time" matching service for transient objects
2) Removing e.g. IR excess or correcting for extinction-like crowding brightening, through Average Contamination; crucial for "1% photometry" in both precision *and* accuracy
3) Recovering additional sources missed by other match services — either in crowded fields (we recover up to twice as many *Gaia*-*WISE* matches than the *Gaia* best neighbour matches), or with our extension to unknown proper motion modelling as an extra systematic









Wilson & Naylor (2018b)
WISE - Wright et al. (2010)
Gaia matches - Marrese et al. (2019)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)

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Conclusions

https://github.com/macauff/macauff

- **Blended star contamination causes positional shifts, now modelled robustly for the first time in the AUF**
- **Symmetric data-driven photometric likelihood now possible**
- **LSST will suffer of order 10% flux contamination in the future**
  - **Important for extinction/distance; "1% photometry"?**
  - **Modelling of statistical flux contamination allows for the recovery of "true" fluxes**
- **LSST will suffer at least one extra source (possibly up to 10!) in each 2" matching circle**
  - **Need to use astrometric uncertainty to reduce length scale over which matches are considered**
  - **Can use photometry in catalogues to break these false match degeneracies**
- **Can include other effects, like unknown proper motions or DCR, easily within AUF match framework**
- **High dynamic range matches must account for differential crowding matching to ancillary or historic data**

- **Accounting for these systematics, we can *confirm* quoted catalogue astrometric uncertainties in more extremes than would otherwise be possible, avoiding mistakenly thinking the pipeline values are "wrong"**

- **Upcoming LSST:UK cross-match service macauff — let me know your thoughts/needs/hopes/dreams**

Wilson & Naylor (2017, MNRAS, 468, 2517); Wilson & Naylor (2018a, MNRAS, 473, 5570);
Wilson & Naylor (2018b, MNRAS, 481, 2148); Wilson (2022, RNAAS); Wilson (2022, RASTI, 2, 1);
Wilson & Naylor (in prep.) — more AUF-related improvements!

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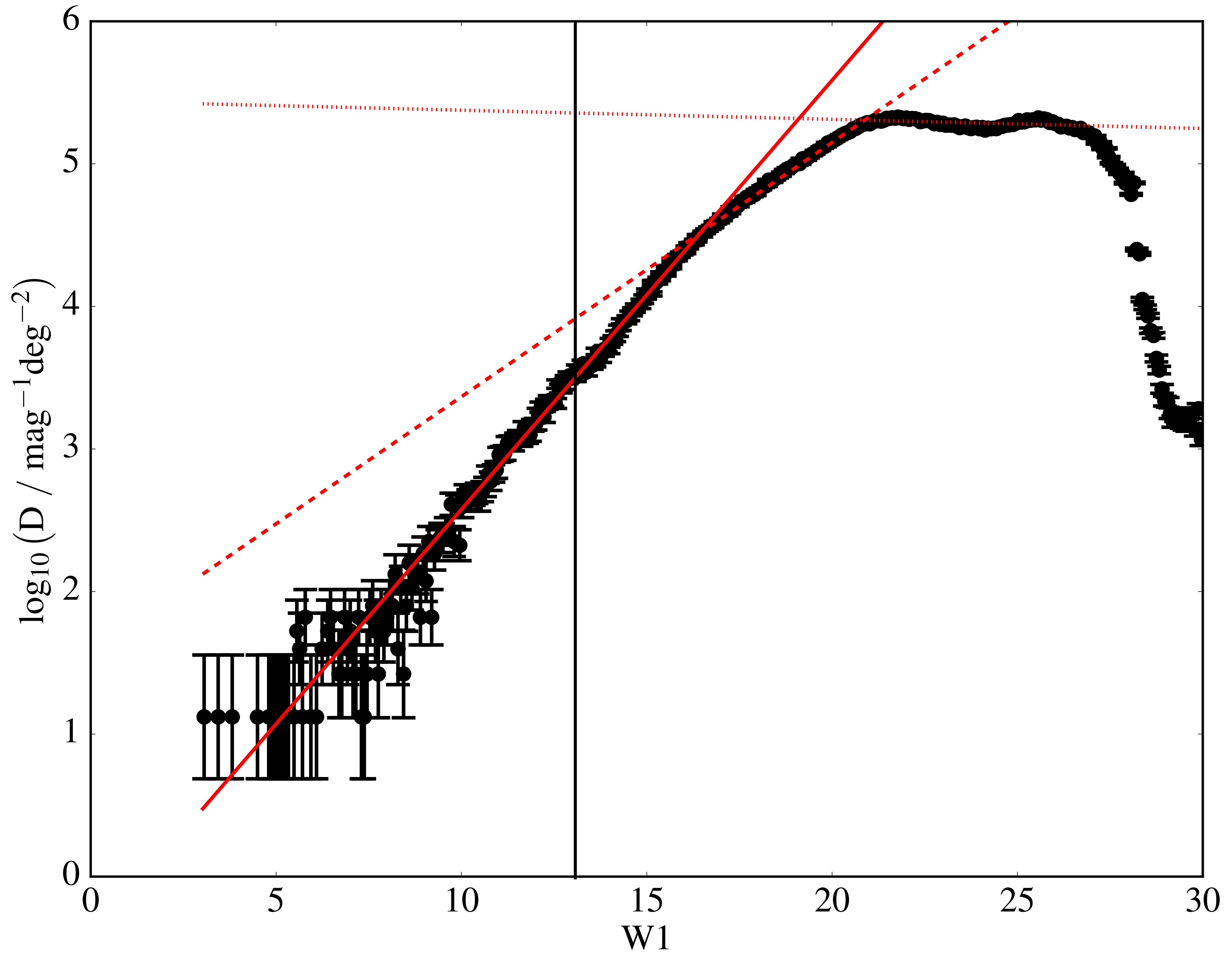
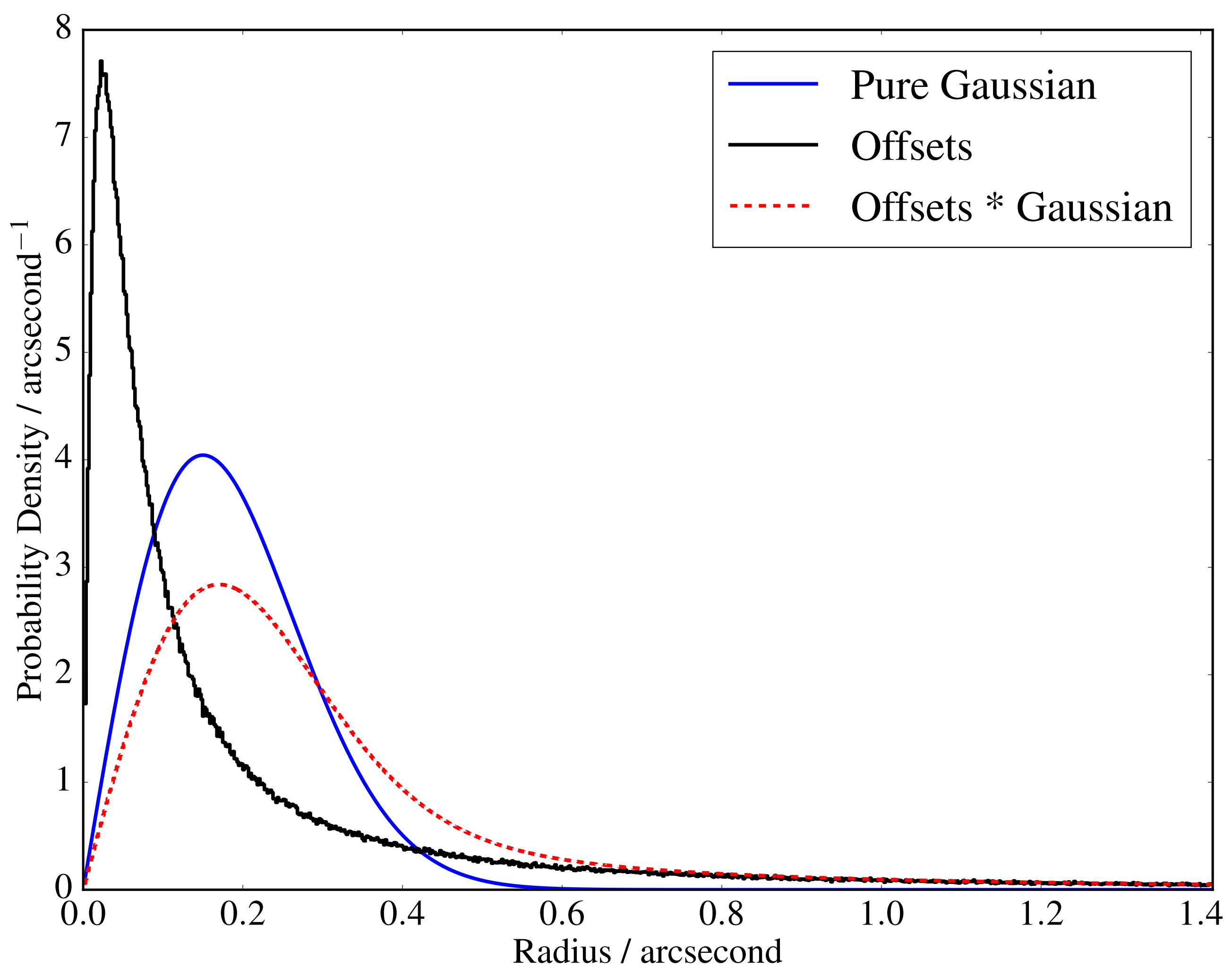
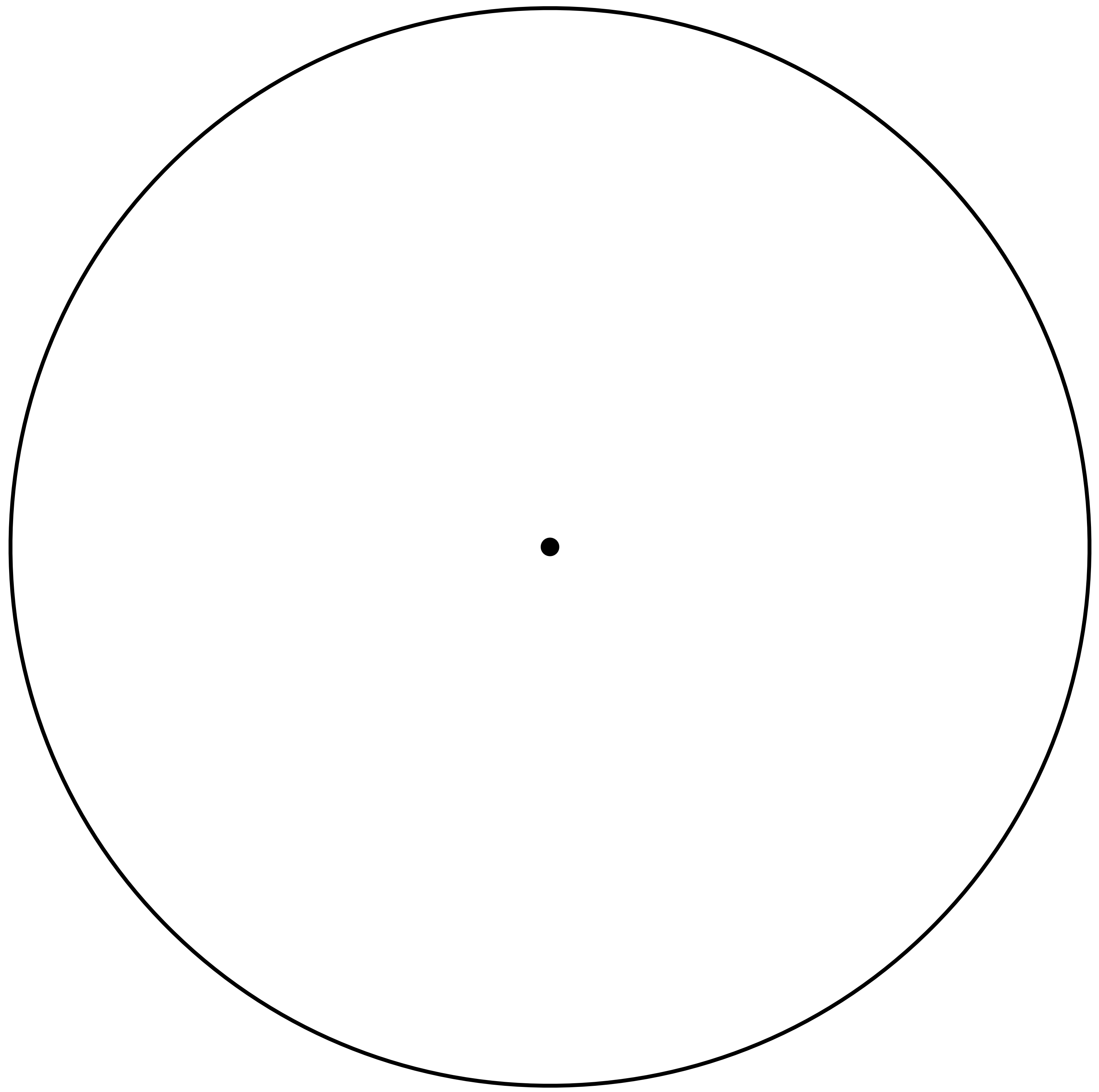
Building Empirical AUFs



(sources per PSF circle ~ 10^-6 sources per mag per sq deg)



PSF radius ~ 1.2 FWHM (Rayleigh criterion)



Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)

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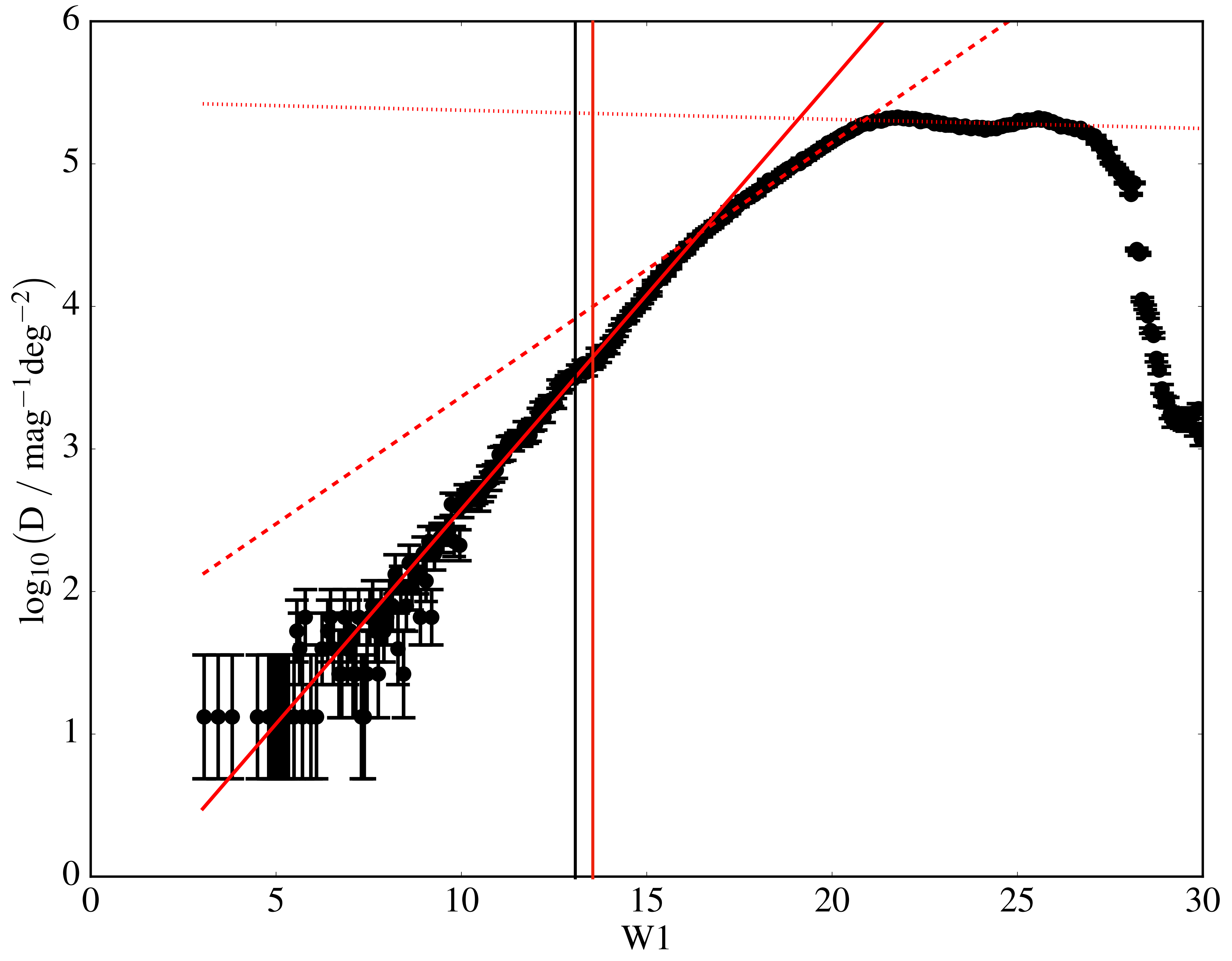

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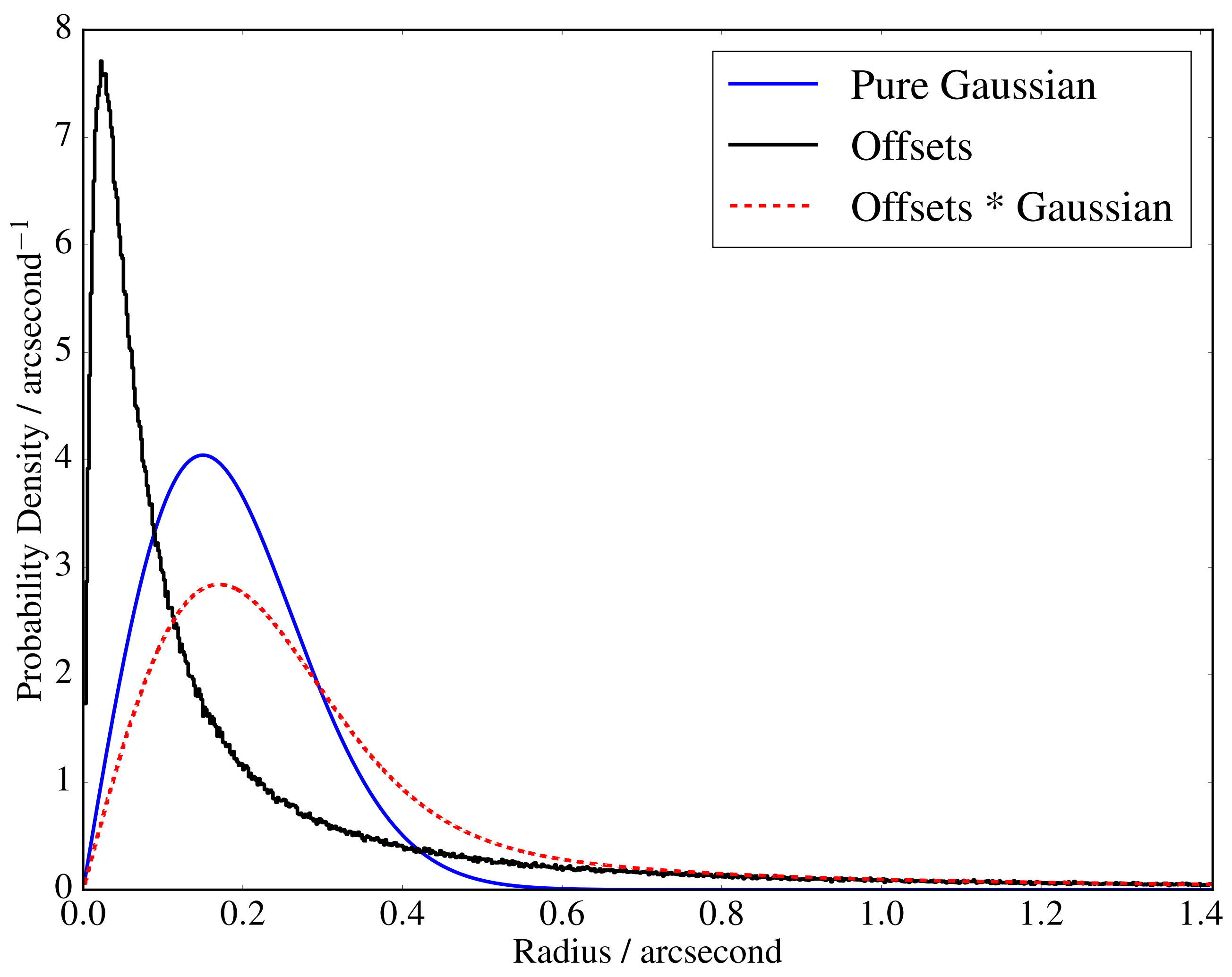

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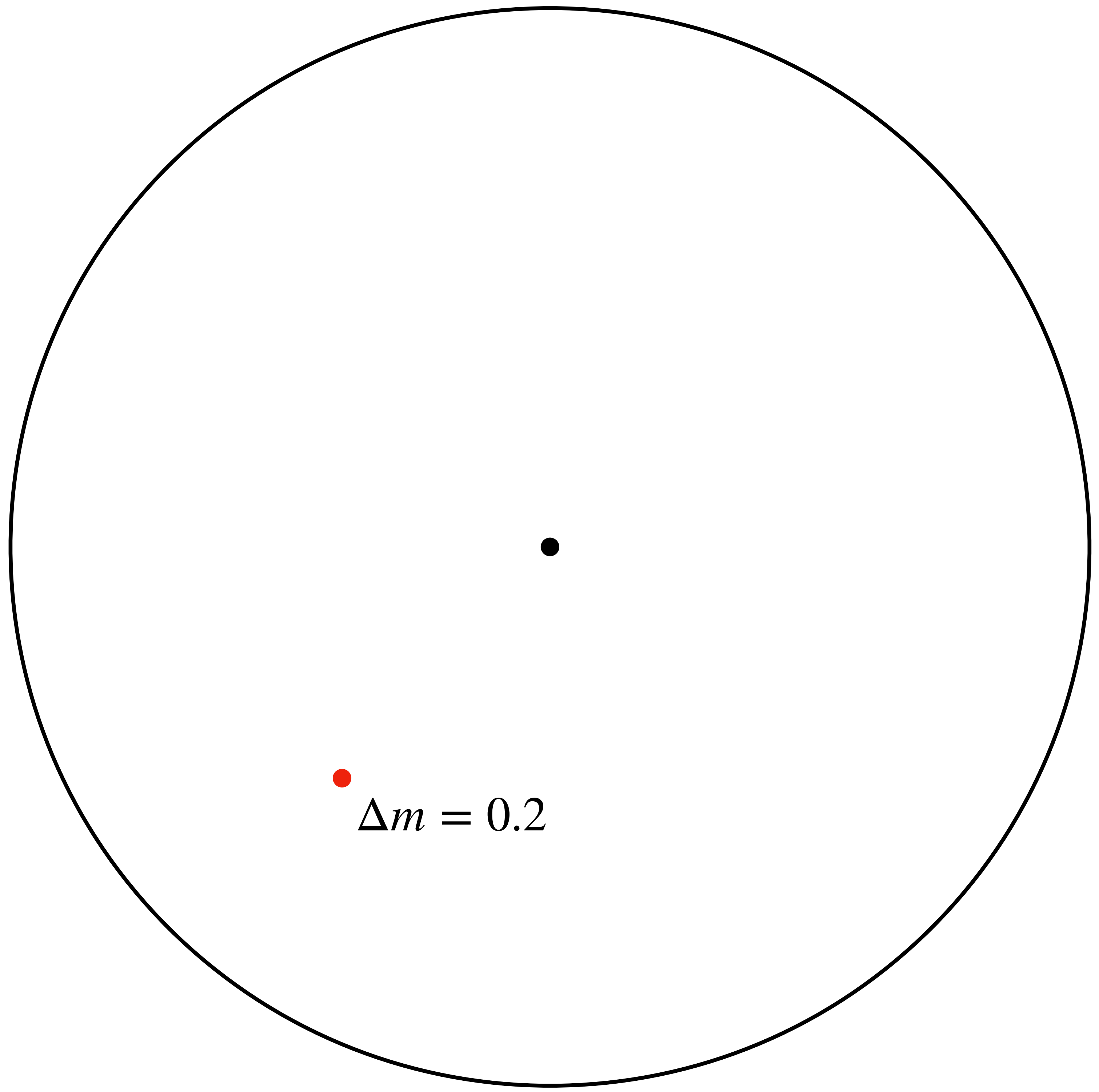

Wilson & Naylor (2018b)
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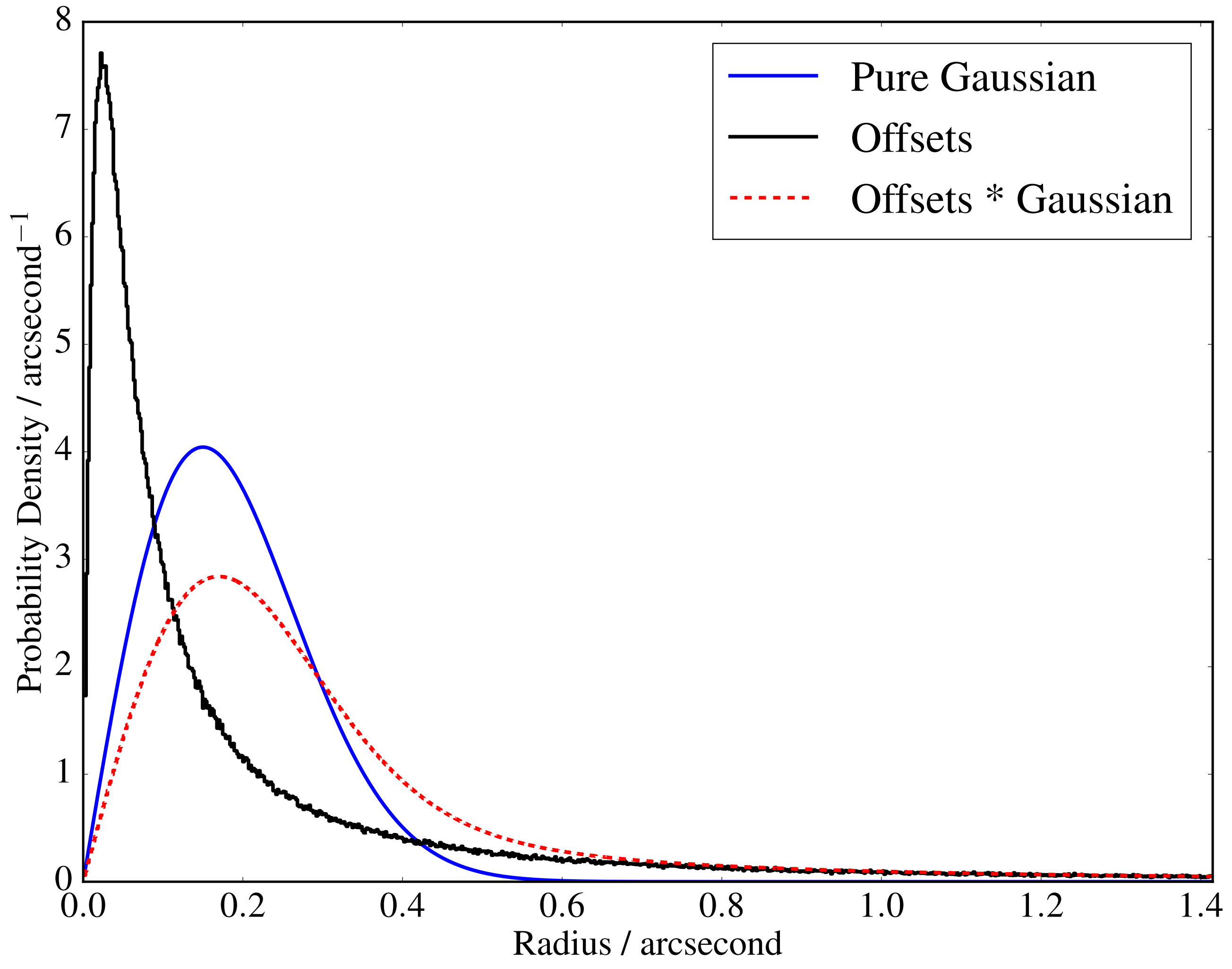

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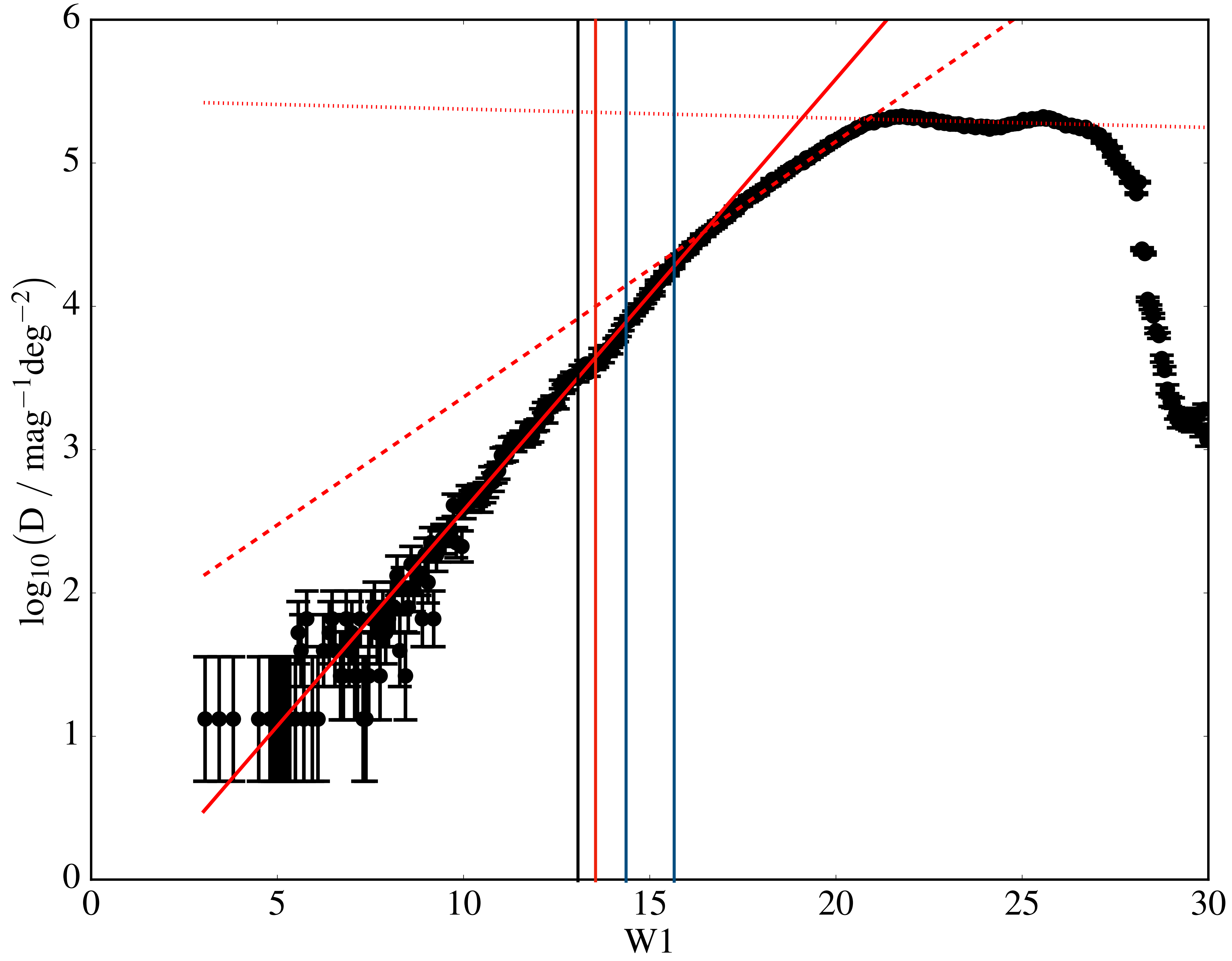

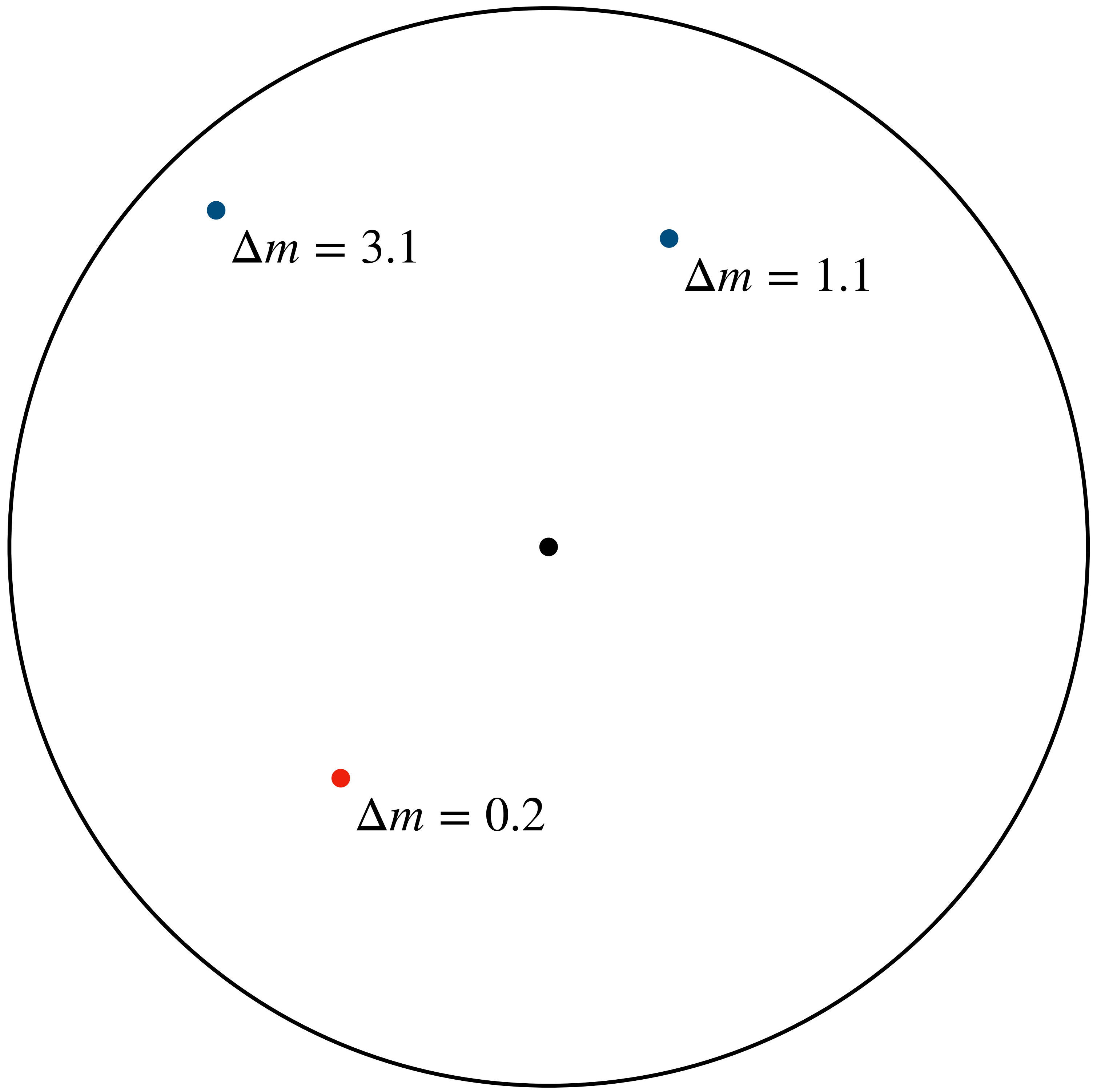

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Wilson & Naylor (2018b)
TRILEGAL - Girardi et al. (2005)

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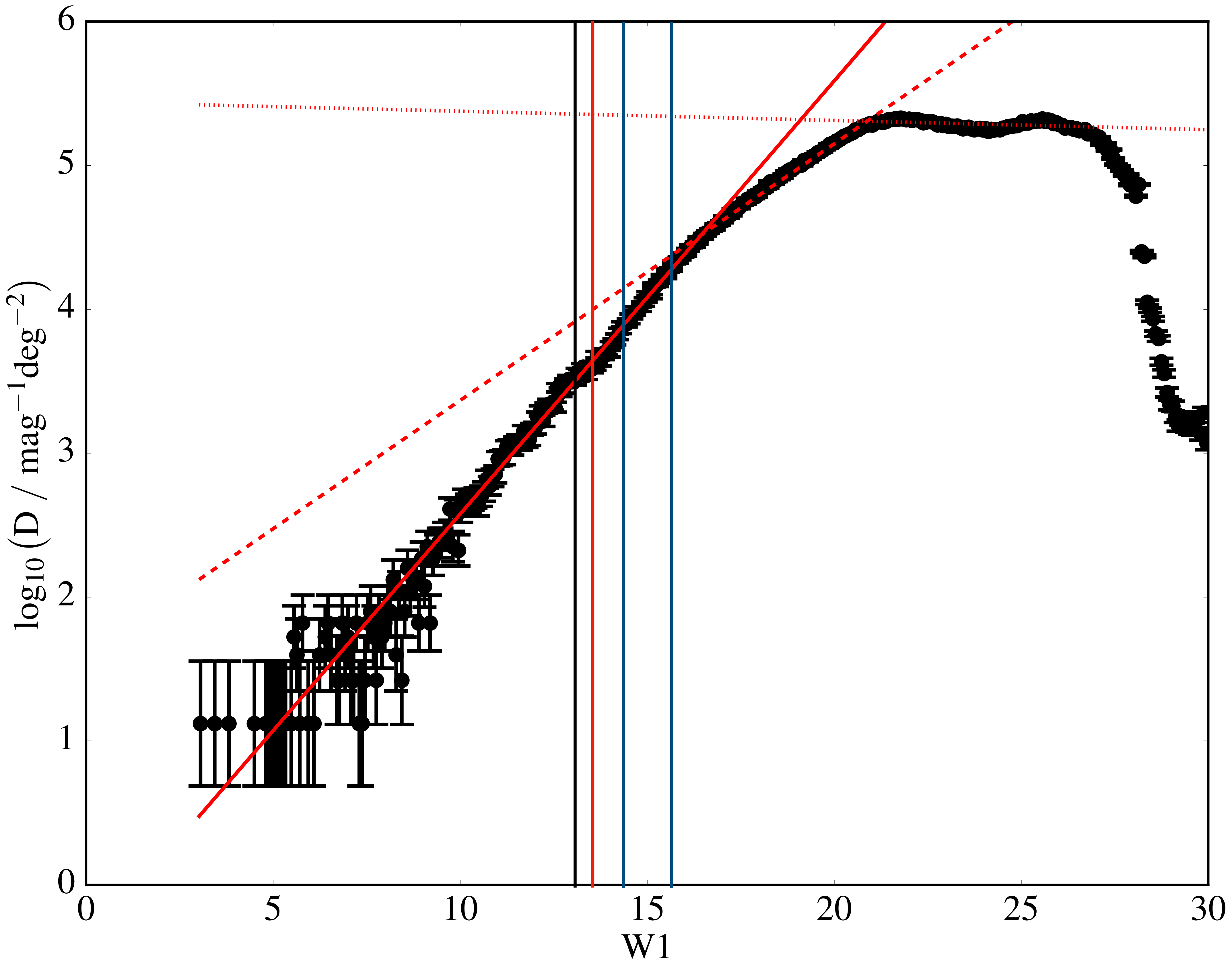
Building Empirical AUFs



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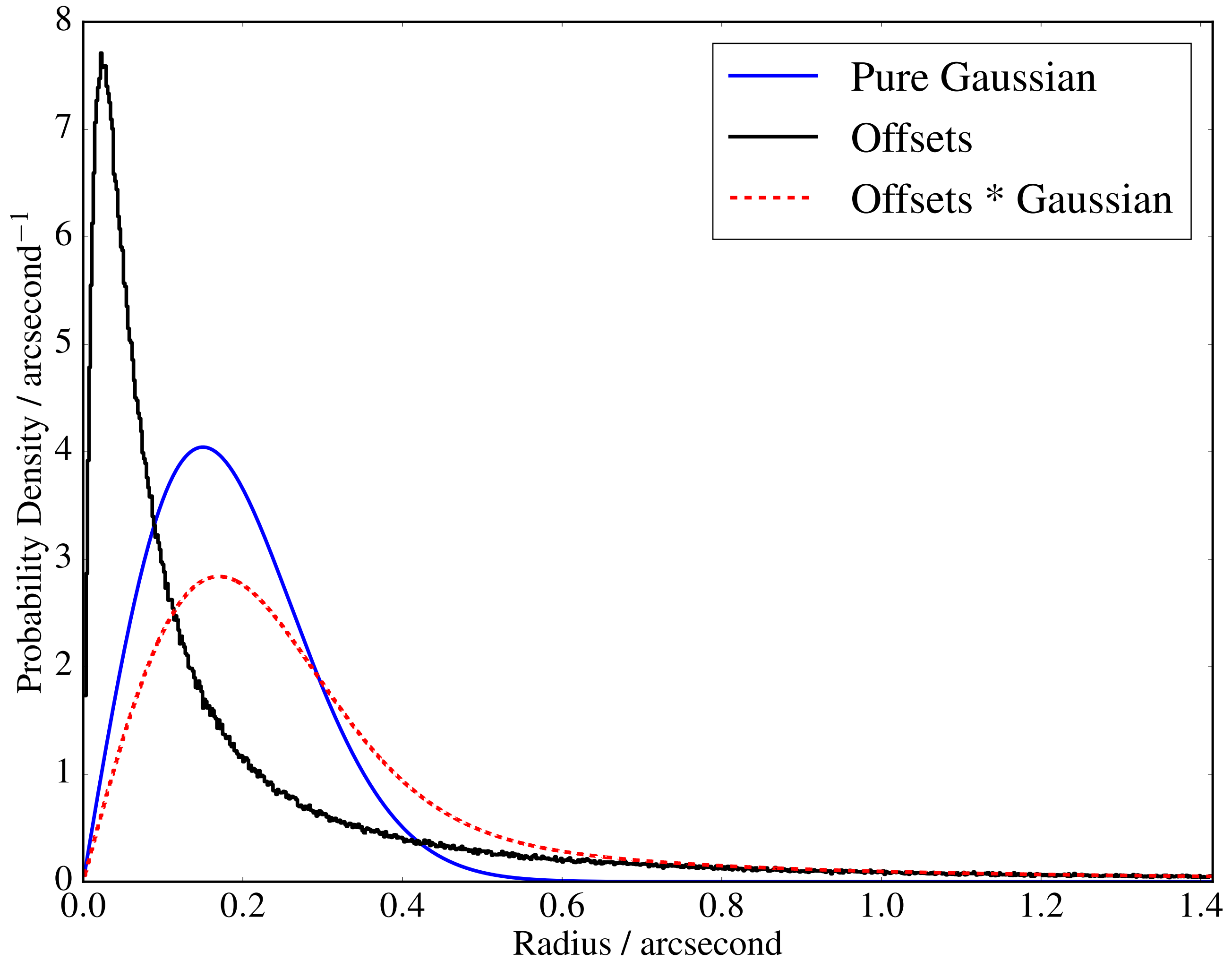

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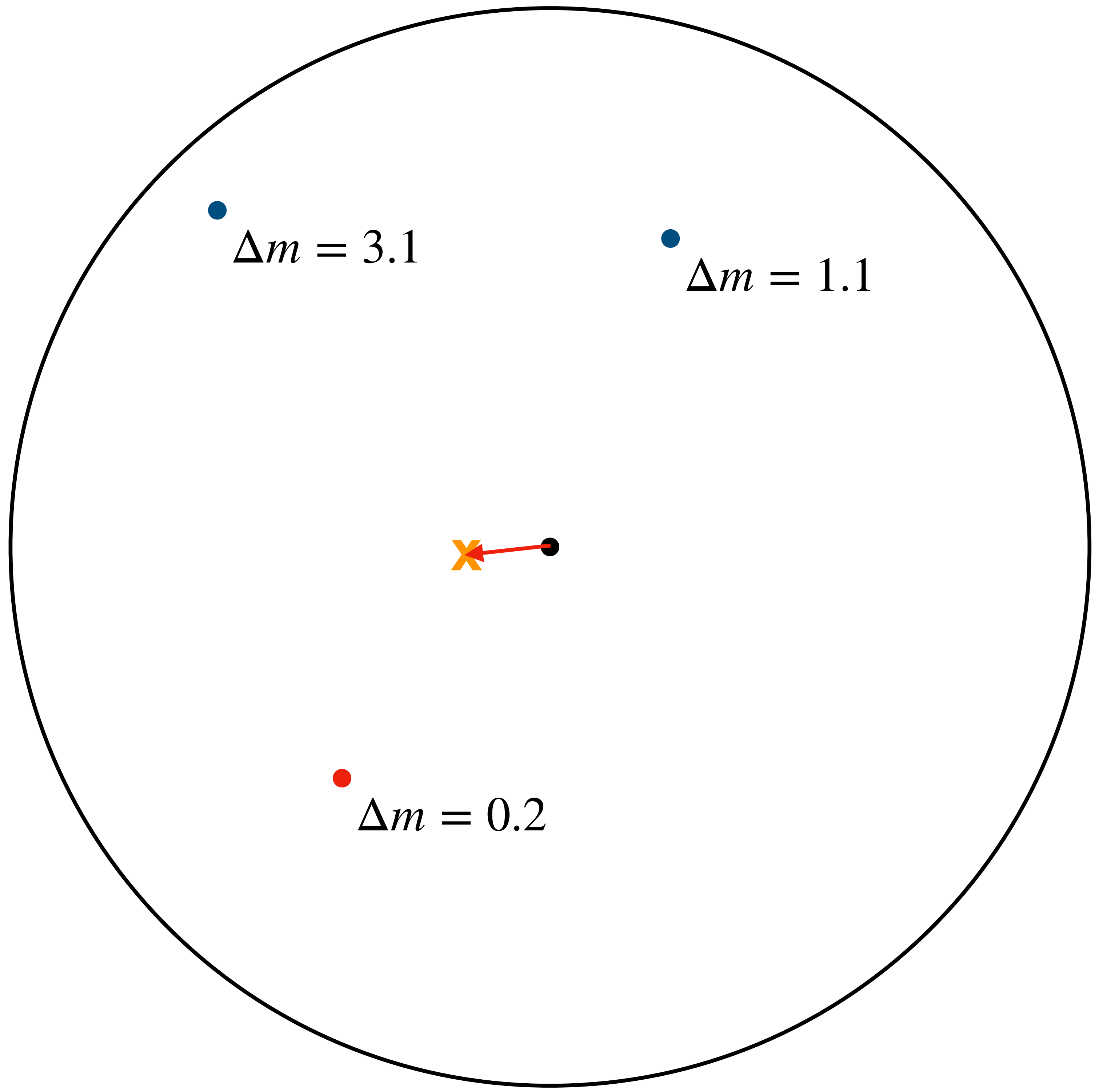

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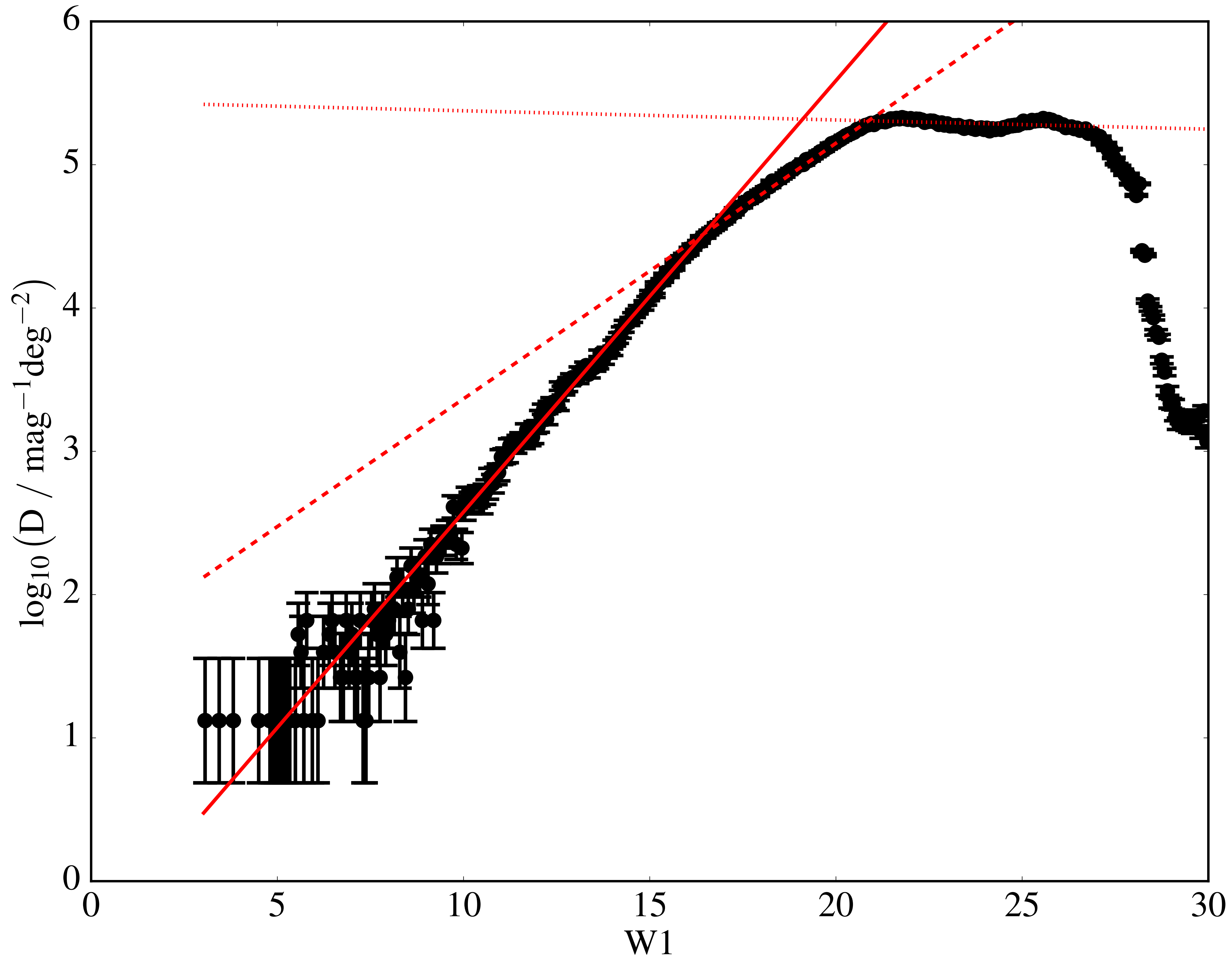
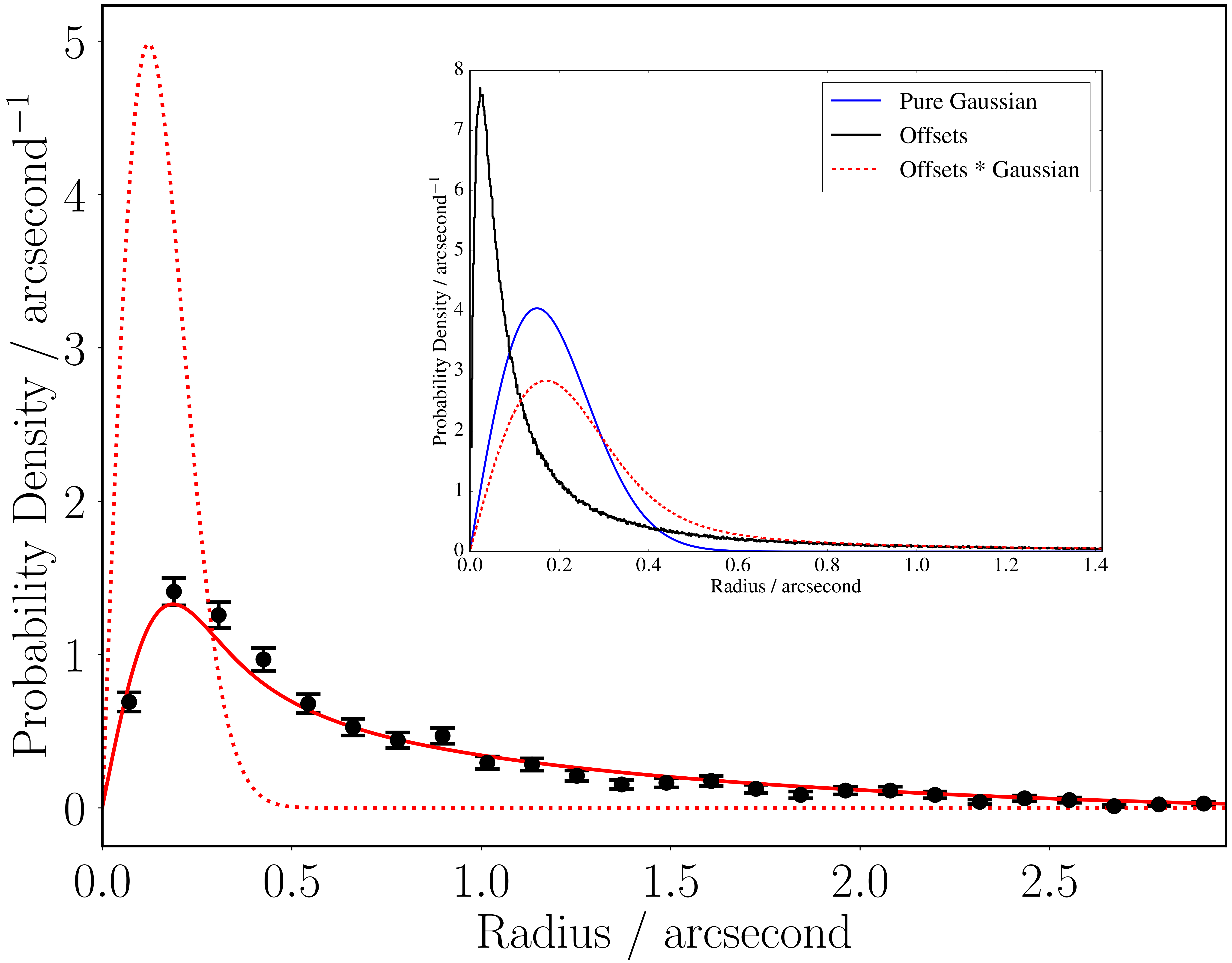
Building Empirical AUFs





WISE - Wright et al. (2010)
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